Approximation Algorithms for Minimum-Length Corridors

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Outline

- Introduction
- Preliminaries and Related Problems
- Parameterized Approximation Algorithm $\text{Alg}(S)$
- Selector Function $S$
- Constant Ratio Approximation
- Additional Results and Conclusion
Introduction:
Minimum-Length Corridor Problem (MLC)
Introduction:
Formal Definition of the MLC-R problem

- **INPUT:** A pair \((F,R)\), where \(F\) is a rectangular boundary partitioned into rectangles (or rooms) \(R=\{R_1,R_2,\ldots,R_r\}\).

- **OUTPUT:** A *corridor* consisting of a set of connected line segments each of which lies along the line segments that form \(F\) and/or the boundary of the rooms, that includes at least one point of \(F\) and at least one point from each of the rooms.

- **OBJECTIVE FUNCTION:** Minimize the total edge length of the corridor.
Introduction: Applications of the MLC Problem

Network Communication in Metropolitan Areas
Introduction:
Origin of the MLC Problem


- No polynomial time algorithm known
- Not even a constant ratio approximation algorithm
- Seems likely to be NP-complete but no proof known
Related Problems:
Outline

- Tree Errand Cover (TEC) problem
  - Generalization of the Group Steiner Tree (GSTP) Problem
Related Problems:

Formal Definition of the TEC problem

- **INPUT**: A connected undirected edge-weighted graph \( G=(V,E,w) \), where \( w:E \rightarrow \mathbb{R}^+ \) is an edge-weight function; a non-empty set \( C \subseteq V \), of terminals; a non-empty set \( \mathcal{E} = \{e_1,e_2,...,e_k\} \) of errands; a collection \( \mathcal{C} = \{C_1, C_2, ..., C_k\} \), where \( C_i \subseteq C \) specifies the vertices where errand \( e_i \) can be performed.

- **OUTPUT**: A tree \( T(G,\mathcal{C})=(V',E') \), where \( E' \subseteq E \) and \( V' \subseteq V \), such that for each errand \( e_i \) there is at least one vertex \( v \subseteq C_i \) and \( v \subseteq V' \), and the total length \( \sum_{e \in E'} w(e) \) is minimized.

GST problem: \( \mathcal{C} = \{C_1, C_2, ..., C_k\} \) is a partition of \( C \)
Related Problems:
MLC-R ∪ TEC

\[ \mathcal{E} = \{ R_i \mid 0 \leq i \leq 9 \} \]
\[ \mathcal{C} = \{ C_i \mid 0 \leq i \leq 9 \} \]

<table>
<thead>
<tr>
<th>i</th>
<th>( C_i )</th>
<th>a TEC solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( { v_1, v_2, v_3, v_4, v_8, v_9, v_{12}, v_{13}, v_{16}, v_{17}, v_{18}, v_{19}, v_{20} } )</td>
<td>( v_3 )</td>
</tr>
<tr>
<td>1</td>
<td>( { v_1, v_2, v_5, v_9, v_{10} } )</td>
<td>( v_5, v_{10} )</td>
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<tr>
<td>2</td>
<td>( { v_2, v_3, v_5, v_6 } )</td>
<td>( v_3, v_5, v_6 )</td>
</tr>
<tr>
<td>3</td>
<td>( { v_3, v_4, v_6, v_7, v_8 } )</td>
<td>( v_3, v_6, v_7 )</td>
</tr>
<tr>
<td>4</td>
<td>( { v_9, v_{10}, v_{13}, v_{14} } )</td>
<td>( v_{10}, v_{14} )</td>
</tr>
<tr>
<td>5</td>
<td>( { v_5, v_6, v_7, v_{10}, v_{11}, v_{14}, v_{15}, v_{18}, v_{19} } )</td>
<td>( v_5, v_6, v_7, v_{10}, v_{11}, v_{14}, v_{15} )</td>
</tr>
<tr>
<td>6</td>
<td>( { v_7, v_8, v_{11}, v_{12} } )</td>
<td>( v_7, v_{11} )</td>
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<tr>
<td>7</td>
<td>( { v_{13}, v_{14}, v_{17}, v_{18} } )</td>
<td>( v_{14} )</td>
</tr>
<tr>
<td>8</td>
<td>( { v_{11}, v_{12}, v_{15}, v_{16} } )</td>
<td>( v_{11}, v_{15} )</td>
</tr>
<tr>
<td>9</td>
<td>( { v_{15}, v_{16}, v_{19}, v_{20} } )</td>
<td>( v_{15} )</td>
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</tbody>
</table>
Related Problems:
TEC Problem Performance Ratio

  - The TEC problem can be approximated to within a ratio of $2\rho$ in polynomial time, when each errand is assigned to at most $\rho$ vertices.
  - For the MLC problem there are errands that may be assigned to an arbitrary number of vertices.

- GST problem
  - $(k-1) \text{OPT}$ (Ihler, E., 1991)
  - $(1 + \ln(k/2)) \cdot k^{0.5} \text{OPT}$ (Bateman, C. D. et al., 1997)
  - Polynomial time $O(k^\alpha)$-approximation algorithms, for arbitrarily small values of $\alpha > 0$ (Helvig, C. S. et al., 2001)

- These results do not generate a constant ratio approximation for the MLC and MLC-R problems.
Parameterized Approximation Algorithm: $Alg(S)$
Hierarchy of the MLC Problem
Parameterized Approximation Algorithm $Alg(S)$: Outline

MLC-R $\alpha$ p-MLC-R

- Parameterized algorithm $Alg(S)$ for the $p$-MLC-R problem.
Parameterized Approximation Algorithm $Alg(S)$: Selector Function $S$ and the $p$-MLC-$R_S$ problem

$I \in p$-MLC-$R$

$I \in p$-MLC-$R_S$

$S(2OC+)$
Parameterized Approximation Algorithm $Alg(S)$: Approximation Technique

- Feasible solutions of the $p$-MLC-R problem
- Feasible solutions after relaxing (LP) the set of feasible solutions of the $p$-MLC-$R_S$ problem
- Feasible solutions after rounding the solution found
Parameterized Approximation Algorithm $Alg(S)$:
For $p$-MLC-R problem

$t(I) \leq 2 k_S r_S \text{opt}(I)$

$I = (p, F, R) \in p$-MLC-R

$S$

$I_S = (p, F, R, S) \in p$-MLC-R$_S$

$J = (G = (V, E, w), C) \in \text{TEC}$

$J_S = (G = (V, E, w), C_S) \in \text{TEC}_S$

Invoke Slavik’s Algorithm:
$t(I) \leq 2 k \text{opt}(I); k = \max \{|V(R_i)|\}$

Invoke Slavik’s Algorithm:
$t(I_S) \leq 2 k_S \text{opt}(I_S); k_S = |S|$

$t(I_S) \leq r_S t(I)$

$\text{opt}(I_S) \leq t(I_S) \leq r_S \text{opt}(I)$

$\text{opt}(I_S) \leq r_S \text{opt}(I)$

$t(I) = t(I_S) \leq 2 k_S r_S \text{opt}(I)$
Selector Function $S$  

Outline

- $S$ selects four corners: $S(4C)$  
- Definition of **Special Point**  
  - $S$ selects special points: $S(+)  
  - $S$ selects two adjacent corners and one special point: $S(2AC+)$  
- $S$ selects two opposite corners and one special point: $S(2OC+)$
Selector Function $S$:
$S$ selects four corners: $S(4C)$

$$k_{S(4C)} = 4$$
$$t(I_{S(4C)}(j)) \leq r_{S(4C)} t(I(j))$$

$$opt(I(j)) = 4 + (j+2)(\epsilon - \delta)$$

$$opt(I_{S(4C)}(j)) > j$$

$$opt(I_{S(4C)}(j)) \sim \frac{j}{4+(j+2)(\epsilon - \delta)} opt(I(j))$$
How about if we select the middle vertex?

\[ t(I_S(j)) \leq r_S \ t(I(j)) \]

\[
\text{opt}(I(j)) = \text{opt}(I_{S(+)}(j))
\]
Selector Function S:
Definition of Special Point

$p$

SpP
Selector Function $S$: Definition of Special Point

<table>
<thead>
<tr>
<th>$R_j$</th>
<th>Candidates to be a special point $u$</th>
<th>Min-connectivity distance $CD(u,R)=CD(R_j,R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>${v_2, v_5}$</td>
<td>2</td>
</tr>
<tr>
<td>$R_3$</td>
<td>${v_3, v_6, v_7, v_8}$</td>
<td>2</td>
</tr>
<tr>
<td>$R_5$</td>
<td>${v_5, v_6, v_7, v_{11}, v_{15}, v_{19}}$</td>
<td>2</td>
</tr>
</tbody>
</table>
Selector Function $S$:
$S$ selects special point: $S(+)\newline\newline$k_{S(+)}=1\newline$\newline$t(I_{S(+)}(j)) \leq r_{S(+)\cdot} t(I(j))\newline\newline$\begin{align*}
opt(I(j)) = \opt(I_{S(+)}(j))
\end{align*}
Selector Function $S$:
$S$ selects special points: $S(+)\  
\begin{align*}
  k_{S(+) } &= 1 \\
  t(I_{S(+) (j)}) &\leq r_{S(+) } t(I(j))
\end{align*}
\[ w \gg h \]

\[
  \text{opt}(I_{S(+) (j)}) \sim \frac{i}{2} \cdot \text{opt}(I(j))
\]
Selector Function $S$:
$S$ selects two adjacent corners and one special point: $S(2AC+)$

\[ k_{S(2AC+)} = 3 \]
\[ t(I_{S(2AC+)}(j)) \leq r_{S(2AC+)} t(I(j)) \]

$w >> h$

\[ \text{opt}(I_{S(2AC+)}) \sim \frac{1}{2} \cdot \text{opt}(I(j, k)) \]
Selector Function S:
Gathering properties to construct S

Special Point

Corners: Opposite ones
Constant Ratio Approximation:
Outline

- S selects two opposite corners and a special point: $S(2OC+)$
- $k_{S(2OC+)}=3$ and prove that $r_{S(2OC+)} \leq 5$
  - Ncpe/cpe rectangles and tour
  - Paths type-1 and type-2 for adjacent ncpe rectangles
  - CD($SpP_i,R$) is bounded by a portion of the corridor
    - Type-1
    - Type-2
Constant Ratio Approximation:
S selects two opposite corners and a special point: S(2OC+)

\[ k_{S(2OC+)} = 3 \]
\[ t(I_{S(2OC+)}) \leq r_{S(2OC+)} t(I) \]

TR, BL, SpP

I ∈ \( p\)-MLC-R
Constant Ratio Approximation: 
Ncpe/cpe rectangles and tour
Constant Ratio Approximation:

\[ k_{S(2OC+)} = 3 \]
\[ t(I_{S(2OC+)}) \leq r_{S(2OC+)} t(I) \]

For \( 1 \leq i \leq q \), it is possible to connect at least one of the critical points of every ncpe rectangle \( n_i \in VR \) to the corridor, by adding line segments of length at most \( l_{i-1} + h_i + l_i \).

\[ l_0 + \sum_{j=1}^{q} h_j + 2 \sum_{j=1}^{q-1} l_j + l_q < 4t(I) \]

\[ r_{S(2OC+)} \leq 5 \]
Constant Ratio Approximation:
Paths type-1 and type-2 for adjacent ncpe rectangles
**Constant Ratio Approximation:**
CD(SpP, R) is bounded by a portion of the corridor: Type-1

*If path \( T(X_{i-1}, Y_i) \) is type-1, then a critical point of \( n_i \) can be connected to the corridor by adding line segments of length at most \( l_{i-1} + h_i + l_i \).*
Constant Ratio Approximation:
CD(SpP_i,R) is bounded by a portion of the corridor : Type-1

If path $T(X_{i-1},Y_{i+1})$ is type-1, then a critical point in $n_i$ can be connected to the corridor by adding line segments of length at most $l_{i-1} + h_i + l_i$. 
Constant Ratio Approximation:
CD(SpP_i, R) is bounded by a portion of the corridor: Type-1

If path $T(X_{i-1}, Y_{i+1})$ is type-1, then a critical point in $n_i$ can be connected to the corridor by adding line segments of length at most $l_{i-1} + h_i + l_i$. 
Constant Ratio Approximation:
CD(SpPᵢ, R) is bounded by a portion of the corridor: Type-1

If path $T(X_{i-1}, Y_{i+1})$ is type-1, then a critical point in $n_i$ can be connected to the corridor by adding line segments of length at most $l_{i-1} + h_i + l_i$. 
Constant Ratio Approximation:
CD(SpP_i,R) is bounded by a portion of the corridor: Type-2
Constant Ratio Approximation:
$CD(SpP_i, R) = CD(n_i, R) \leq l_{i-1} + h_i + l_i$
Additional Results and Conclusions:

Outline

- $MLC_k$ problem, $c$-gons, $c \leq k$
- Rectangular group-TSP
  - Edges may be visited more than once
- Other results
Additional Results and Conclusions:

$\text{MLC}_k$ problem

Rectilinear $c$-gons for $c \leq k$, and $c \geq 6$

$k_{S(C^+)} = \frac{3}{2}n - 1; r_{S(C^+)} = 5.$

$2k_{S(C^+)} \cdot r_{S(C^+)} = 2 \cdot (\frac{3}{2}n - 1) \cdot 5 = 15n - 10.$
Additional Results and Conclusions: New results

- Hans Bodlaender\textsuperscript{1}, Corinne Feremans\textsuperscript{2}, Alexander Grigoriev\textsuperscript{2}, Eelko Penninkx\textsuperscript{1}, René Sitters\textsuperscript{3}, Thomas Wolle\textsuperscript{4}. On the Minimum Corridor Connection Problem and Other Generalized Geometric Problems. In 4\textsuperscript{th} Workshop on Approximation and Online Algorithms (WAOA) Zurich, Switzerland, September 2006.

- NP-completeness of MLC problem
- Geographic Clustering Problem:
  - NP-complete?
  - PTAS: $(1+\varepsilon)\ \text{OPT in time } n(\log n)^O(1/\varepsilon)$
- $\alpha$-fatness rooms:
  - NP-complete?
  - $(16/\alpha)-1\ \text{OPT}; 0\leq \alpha\leq 1$
  - If all the rooms are squares then $\alpha=1$ and the solution is 15 times the optimal one.

\textsuperscript{1}Utrecht University,\textsuperscript{2}Maastricht University,\textsuperscript{3}Max-Planck-Institute for Computer Science\textsuperscript{4}National ICT Australia Ltd.
Additional Results and Conclusions:
Rectangular group-TSP

- Instead a tree we have a tour: $2 \times 30 = 60$
  - The Errand (Tour) Scheduling problem can be approximated to within a factor of $3 \rho / 2$ in polynomial time, when each errand is assigned to at most $\rho$ vertices.

\[
\frac{3}{2} \cdot k_{S(2OC+)} \cdot r_{S(2OC+)}
\]

\[
k_{S(2OC+)} = 3 \text{ and } r_{S(2OC+)} = 5.
\]

This results in the approximation ratio 22.5
Additional Results and Conclusions: Rectangular group-TSP

• Mark de Berg\textsuperscript{a}, Joachim Gudmundsson\textsuperscript{b}, Mathew J. Katz\textsuperscript{c}, Christos Levcopoulos\textsuperscript{d}, Mark H. Overmars\textsuperscript{e}, A. Frank van der Stappen\textsuperscript{e}. \textit{TSP with neighborhoods of varying sizes}. J. of Algorithms 57 (2005) 22-36.
  - 1200 $\alpha^3$ times the optimal, $\alpha \geq 1$
  - 93 times the optimal when the regions are squares

\textsuperscript{a}TU Eindhoven, \textsuperscript{b}NICTA Sydney, \textsuperscript{c}Ben-Gurion University, \textsuperscript{d}Lund University, \textsuperscript{e}Utrecht University
Additional Results and Conclusions:

- TRA-MLC, TRA-MLC-R, p-MLC-R, MLC-R problems are NP-complete
- p-MLC-R_S for S(2OC+), S(4C+) are NP-complete
- Solution of the p-MLC-R and MLC-R problem is at most $2k_S r_S = 2 \times 3 \times 5 = 30$ times the optimal solution.
- Solution of the MLC_k problem is at most $15k - 10$ times the optimal solution.
- The approximation ratio is a constant!
Publications…


Thank you!

- Questions
- Comments
- Suggestions
Introduction:
Applications of the MLC Problem

- Circuit Board Layout Design
  - Wires for power supply
  - Wires for clock signal

- Building Wiring Design
  - Optical Fiber for Data Communication Networks
  - Wires for Electrical Networks
Example...

\[ \{ x_1, \bar{x}_2 \} \quad \{ x_1, \bar{x}_4, x_5 \} \quad \{ \bar{x}_1, x_2 \} \]

\[ \{ \bar{x}_1, \bar{x}_2, x_4 \} \quad \{ x_2, \bar{x}_4 \} \quad \{ \bar{x}_2, x_3, x_4 \} \]

\[ \{ x_3, \bar{x}_4 \} \quad \{ \bar{x}_3, x_4, x_5 \} \quad \{ \bar{x}_4, \bar{x}_5 \} \]
Example...

\[ \{x_1, \overline{x}_2\} \ {x_1, \overline{x}_4, x_5}\ \{\overline{x}_1, x_2\} \]

\[\{\overline{x}_1, \overline{x}_2, x_4\} \ \{x_2, \overline{x}_4\} \ \{\overline{x}_2, x_3, x_4\}\]

\[\{x_3, \overline{x}_4\} \ \{\overline{x}_3, x_4, x_5\} \ \{\overline{x}_4, \overline{x}_5\}\]

TTTTTT
Example...

\[
\{x_1, \overline{x}_2\} \quad \{x_1, \overline{x}_4, x_5\} \quad \{\overline{x}_1, x_2\}
\]

\[
\{\overline{x}_1, \overline{x}_2, x_4\} \quad \{x_2, \overline{x}_4\} \quad \{\overline{x}_2, x_3, x_4\}
\]

\[
\{x_3, \overline{x}_4\} \quad \{\overline{x}_3, x_4, x_5\} \quad \{\overline{x}_4, \overline{x}_5\}
\]
Parameterized Approximation Algorithm Alg(S)
Parameterized Approximation Algorithm \textbf{Alg}(S)
Related Problems: Tree Vertex Cover

- **INPUT**: A connected undirected edge-weighted graph $G=(V,E,w)$, where $w:E \rightarrow \mathbb{R}^+$ is an edge-weight function.

- **OUTPUT**: A tree $T=(V’,E’)$, where $E’ \subseteq E$, and $V’ \subseteq V$ is a vertex cover (i.e. every edge in $G$ includes at least one vertex in $V’$) and the total edge-weight

$$\sum_{e \in E'} w(e)$$

is minimized.
Related Problems: Group Steiner Tree Problem
Generalization of the MLC Problem

- **INPUT**: A connected undirected edge-weighted graph $G=(V,E,w)$, where $w:E \rightarrow \mathbb{R}^+$ is an edge-weight function, a non-empty subset $S$, $S \subseteq V$, of *terminals*; and a partition $\{S_1,S_2,\ldots,S_k\}$ of $S$.

- **OUTPUT**: A tree $T(S)=(V',E')$, where $E' \not\subseteq E$ and $V' \not\subseteq V$, such that at least one terminal from each set $S_i$ is in the tree $T(S)$ and the total length $\sum_{e \in E'} w(e)$ is minimized.
Related Problems: Group Steiner Tree Problem
Reducing the TRA-MLC to the GST

(a)  
(b)
Related Problems: Group Steiner Tree Problem
Reducing the TRA-MLC to the GST
Related Problems: Group Steiner Tree Problem

<table>
<thead>
<tr>
<th>(k-1) OPT</th>
<th>Ihler, E. (1991)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 + ln(k/2)) ( k^{0.5} ) OPT</td>
<td>Bateman, C. D. et. al. (1997)</td>
</tr>
<tr>
<td>( O(k^{\varepsilon}) ), for any ( \varepsilon &gt; 0 )</td>
<td>Helvig, C. S. et. al. (2001)</td>
</tr>
</tbody>
</table>
MLC Problem: Hierarchy

MLC

Geometric Clustering Problem

α-fatness Rooms Problem

MLC - R

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α-fatness Rooms Problem

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Introduction:
Hierarchy of the MLC Problem
Related Problems:
Formal Definition of the TFNS Problem

- **INPUT**: A connected undirected edge-weighted graph $G=(V,E,w)$, where $w:E \rightarrow \mathbb{R}^+$ is an edge-weight function.

- **OUTPUT**: A tree $T=(V',E')$, where $E' \not\subseteq E$, and $V' \not\subseteq V$ is a *feedback node* set (i.e. every cycle in $G$ includes at least one vertex in $V'$) and the total edge-weight $\sum_{e \in E'} w(e)$ is minimized.
Related Problems:
TFNS-based algorithm for MLC (from TVC)

- Uses the approximation algorithm for the weighted vertex cover and then uses an approximation algorithm for the Steiner tree problem.
- Replace the approximation algorithm for the weighted vertex cover by a constant ratio approximation algorithm for the weighted FNS problem [Chudak, F. A. et. al. (1998)].
Related Problems:
Solving instances $I_j$ of MLC by TFNS-based algorithm

$$w \gg h$$

$$\text{opt}(I_{FNS}) \sim \frac{i}{2} \cdot \text{opt}(I(j))$$
$S(R_k)$: selecting $k$ points randomly, $k_{S(R_k)} = k$...

$t(I_S) \leq r_S t(I)$

- Middle vertex
- Rectangle with $2 \cdot k + 3$ vertices
- $\delta_{2k}$

\[
\frac{\binom{2k+2}{k-1}}{\binom{2k+3}{k}} = \frac{k}{2 \cdot k + 3} < \frac{1}{2}.
\]

\[
\text{opt}(I_{S(R_k)}) \geq \frac{1}{2} j
\]

\[
\text{opt}(I_{S(R_k)}) \sim \frac{j}{8 + (2j + 4)(\epsilon - \delta)} \cdot \text{opt}(I(j))
\]
$S(k+)$: selecting $k$ special points, $k_{S(k+)} = k$ ...

$t(I_S) \leq r_S \cdot t(I)$

$w >> h \quad \Rightarrow \quad \text{opt}(I_{S(k+)}) \sim \frac{i}{2} \cdot \text{opt}(I(j,k))$
**Additional Results and Conclusions:**

- **TRA-MLC**, \( \{x_1, \bar{x}_2\} \quad \{x_1, \bar{x}_4, x_5\} \quad \{\bar{x}_1, x_2\} \quad 1LC-R 
  \quad \text{problems are} \quad \{\bar{x}_1, \bar{x}_2, x_4\} \quad \{x_2, \bar{x}_4\} \quad \{\bar{x}_2, x_3, x_4\} 

- **p-MLC-R_S f** \( \{x_3, \bar{x}_4\} \quad \{\bar{x}_3, x_4, x_5\} \quad \{\bar{x}_4, \bar{x}_5\} \quad \text{P-completeness} 

- Solution of the **p-MLC-R** and **MLC-R** problem is at most \( 2k_s r_s = 2 \times 3 \times 5 = 30 \) times the optimal solution.

- Solution of the **MLC** problem is at most \( 15n - 10 \) times the optimal solution.

- The approximation ratio is a constant!
Parameterized Approximation Algorithm $\text{Alg}(S)$
NP-Completeness: TRA-MLC-R $\preceq$ MLC-R

$$|T_{\text{MLC-R}}| \leq B + 4Y + h + w + 8 \iff |T_{\text{TRA-MLC-R}}| \leq B$$

where $Y = B + h + w + 9$, and $B, w, h$ are greater than 2.
Example...

\[ \{ x_1, x_2 \} \quad \{ x_1, x_4, x_5 \} \quad \{ \bar{x}_1, x_2 \} \]

\[ \{ \bar{x}_1, \bar{x}_2, x_4 \} \quad \{ x_2, \bar{x}_4 \} \quad \{ \bar{x}_2, x_3, x_4 \} \]

\[ \{ x_3, \bar{x}_4 \} \quad \{ \bar{x}_3, x_4, x_5 \} \quad \{ \bar{x}_4, \bar{x}_5 \} \]