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ROUTING AROUND TWO RECTANGLES TO MINIMIZE THE LAYOUT AREA

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ABSTRACT

The problem of routing n two-terminal nets around two equal-width rectangles to minimize the total area is discussed. We develop an $O(n \log n)$ ($O(n)$ if the set of terminals is initially sorted) time approximation algorithm for this problem. Our algorithm generates a layout with area at most 2 OPT, where OPT is the area of an optimal area layout. We establish a lower bound for the area of an optimal layout from a lower bound for the size of the components, and a lower bound for the area occupied by the wires. The former lower bound is derived from the number of terminals on the sides of the rectangles, and the latter lower bound is based on the number of corners crossed by each wire. This suggests that nets should be connected by wires that "cross" the least number of corners (called lnc-wires). Nets for which all their lnc-wires cross the same rectangle corners are connected by lnc-wires, and a subset of the remaining nets is connected by lnc-wires that blend with previously introduced wires. To guarantee our approximation bound the remaining nets are connected in several ways, and a set of layouts is generated. The layout generated by the algorithm is a minimum area layout among these layouts.

Keywords: Rectangle routing, approximation algorithms, Manhattan routing, Knock-Knee routing, efficient algorithms.

1. Introduction

Let T (top) and B (bottom) be two rectangles with equal width ($w_T = w_B$) and possibly different heights (h_T and h_B). Assume the rectangles are placed on the same plane with the same orientation. The left side of T and B is placed along the same vertical line and rectangle T is above rectangle B. The distance between these two rectangles is at least $\lambda > 0$ units and the exact distance will be decided by our routing algorithm. Let N be a set of terminals (or terminal points) that lie on the sides of T and B. Set N is partitioned into n disjoint subsets N_i, $1 \leq i \leq n$, called *nets*. All the terminals in each net have to be made electrically common by interconnecting them with wires. The wires consist of a finite number of horizontal and vertical segments. All the horizontal segments are assigned to one layer and all the vertical segments are assigned to the other layer. Wire segments on different layers can be connected directly at any given point z by a wire perpendicular to the layers if both wire segments cross point z in their respective layers (i.e., the connection is made through a *contact cut* or *via*). Every pair of (distinct) parallel wire segments must be at least λ units apart and every wire segment must be at least λ units from each side of rectangles T and B, except in the region where the wire connects a terminal in N. Also, no wire segment is allowed inside of T and B on any of the layers. We assume that the distance between any two vertical lines each including a terminal located in the middle channel (the region between the bottom side of rectangle T and the top side of rectangle B) is at least λ. We shall refer to this assumption as the *middle-channel assumption*. Later on we explain why we make this assumption, and show how it can be eliminated at the expense of an additional layer for routing.

Problem $R2M$ (routing around two rectangles) consists placing T and B, vertically aligned, and connecting the terminals in each net by wires, that satisfy the restrictions imposed above, in such a way that the smallest enclosing rectangle, with the same orientation as T and B, has least area amongst all feasible layouts. This problem has applications in the bottom-up layout of integrated circuits.[1,2] The $R2M$ problem is referred to as the $2-R2M$ problem when each net consists of exactly two terminals. In this paper we focus on the $2-R2M$ problem under the wiring model just discussed which is referred to as the *Manhattan wiring model*. We also consider the $2-R2M$ problem without the middle-channel assumption under the *knock-knee* wiring model. Under this wiring model,[3,2] vertical and horizontal segments from different nets may be assigned to the same layer as long as they do not touch, and two wires may bend at a grid point rather than just cross as in the Manhattan wiring model. A Manhattan wiring is also a knock-knee wiring, but the converse is not necessarily true. When we refer to a wiring we mean a wiring under Manhattan model. When we wish to refer to knock-knee wirings we shall refer to them explicitly.

The $2-R2M$ problem without the middle channel assumption is an NP-hard problem because the channel between the two rectangles corresponds to the Manhattan channel routing problem which is known to be NP-hard.[4] It is not known whether the $2-R2M$ is NP-hard. With respect to the knock-knee model the $R2M$ problem without the middle channel assumption is an NP-hard problem because the channel between the two rectangles corresponds to the knock-knee channel routing problem which is known to be NP-hard,[5] but it is not known whether or not the $2-R2M$ with or without the middle-channel assumption or the $R2M$ are NP-hard. As we shall see later on, the reason for introducing the middle-channel assumption is not the complexity of the optimization problem, but rather the complexity of generating good area layouts for the channel routing problem (routing in the middle channel).

The $R1M$ problem is defined similarly, except that all terminals are located on the sides of one rectangle. Hashimoto and Stevens[6] present an $O(n \log n)$ algorithm to solve the $R1M$ problem for the case when all the points in N lie on one side of a rectangle. An $\Omega(n \log n)$ lower bound of time complexity for this problem has been established.[7] There are several algorithms to solve the $R1M$ problem when all nets have exactly two terminals.[8,9,10,2] Gonzalez and Lee's algorithm[10] is optimal with respect to the time complexity bound. Approximation algorithms for the $R1M$ problem have been developed by Gonzalez and Lee.[11,12] The time complexity for these algorithms is $O(m(n+\log m))$ and the best one[12] generates a layout with area at most 1.6 OPT, where OPT is the area of an optimal layout, m is the number of terminals and n is the number of nets. If more than two layers are allowed and wire overlap is permitted, the $R1M$ problem becomes an NP-hard problem,[13] even when the size of all nets is two.

Chandrasekhar and Breuer[14] studied a restricted version of the $2 - R2M$ problem in which some type of nets have to be connected by a special type of wires. This limitation on the set of feasible solutions simplifies the routing problem considerable and makes it polynomially solvable. Unfortunately, an optimal area layout for this problem with the additional restrictions is in general larger than that of an optimal area layout for the unrestricted $2 - R2M$ problem. In our problem we allow all possible type of connections and as a result of this we compare our layouts with the area of a "true" optimal area layout. Baker[15] presents an $O(n \log n)$ algorithm for the $2 - R2M$ problem in which optimality is measured with respect to the perimeter of the resulting enclosing rectangle. The algorithm generates a layout whose perimeter is within 1.9 times the perimeter of an optimal layout. The perimeter is simpler to approximate than the area mainly because the perimeter is a linear objective function, but layout area is considered to be one of the most important objective functions in VLSI optimization. Our approximation bound is two and Baker's[15] is 1.9; however, Baker[15] does not require the middle-channel assumption. Without the middle-channel assumption we can still approximate within two the layout area, but at the expense of an extra layer in the middle channel. Without the extra layer the problem is difficult to approximate. To convince yourself of this fact let us just consider the middle channel. The best known approximation algorithm for this problem has an approximation bound which is bounded by a constant (the constant is greater than two).[16] Even if such algorithm is used directly, it does not imply that we can generate a constant times optimal wiring because it may use a set of additional columns to the right of the channel, i.e., the algorithm approximates with respect to channel width, but not necessarily with respect to the channel area. This is the main reason we have made the middle-channel assumption. We should note that the middle-channel assumption is not too restrictive, since we show that such assumption may be eliminated by simply adding an extra layer and wiring under the knock-knee model.

Sarrafzadeh and Preparata[2] developed an algorithm that generates optimal area layouts for the case when the area of the layout is defined as the area of two nonoverlapping rectangles that enclose all the wires, i.e., one rectangle encloses T and the other encloses B. Under this objective function the problem can be solved efficiently, but then the resulting building block is not rectangular. This makes the wiring problem in the bottom-up approach difficult to solve. Our building block at each level in the bottom-up approach is a rectangle, which is simpler to handle.

In this paper we present an $O(n \log n)$ ($O(n)$ if the set of terminals is initially sorted) time approximation algorithm for the $2-R2M$ problem that generates a layout with area at most 2 OPT, where OPT is the area of an optimal layout. In the final section we show that this result also holds when the middle-channel assumption is removed provided knock-knees are allowed and three layers are available for routing.

Hereafter we assume that the terminals points are initially sorted, i.e., the terminal points are given by two lists each corresponding to the order in which terminals appear while traversing each of the rectangles in the clockwise direction starting at the bottom-left corner. We define some terms before outlining our algorithm. A net is called *local* if its terminals are located on the same side of the same rectangle. Otherwise, it is called *global*. If a corner s can be horizontally projected to a wire without intersecting any side of the two rectangles, then the wire *crosses horizontally corner s*. A net is said to be connected by an *lnc*-wire if the wire crosses horizontally the least number of rectangle corners amongst all wires connecting the net. A net is called *simple* if all *lnc*-wires connecting its terminals cross horizontally the same corners of T and B. Otherwise, the net is called *complex*. Note that in the definition for simple nets we use horizontal crossing rather than just crossing. The only place where this makes a difference is in the type of nets shown in Fig. 5. In our algorithm we always wire these nets as in Fig. 5(a).

The first few steps of our procedure correspond to the initial steps of previous algorithms.[9,10] In this step all local nets are connected by *lnc*-wires. The reason why this is a good decision is that any layout can be transformed to another layout without increasing its area in such a way that all local nets are connected by *lnc*-wires. The problem is then reduced to determining the type of wire connecting each global net in the presence of some previously introduced wires. We establish a lower bound for the area of an optimal layout from a lower bound for the size of the components and a lower bound for the area occupied by the wires. The former lower bound is derived from the number of terminals on the sides of the rectangles, and the latter lower bound is based on the number of corners crossed by each wire. This suggests that the global nets should be connected by *lnc*-wires. Simple nets are connected by *lnc*-wires and a subset of the complex nets is connected by *lnc*-wires that blend with previously introduced wires. All remaining unrouted nets have all their terminals on the top and bottom sides of the rectangles. Because the lower bound for the width of the rectangles is the number of terminals divided by four (rather than two for the total height of the rectangles), the lower bound is not large enough to establish an approximation bound of two when the remaining nets are routed in the obvious way. To achieve this approximation bound, the remaining nets are connected in several ways and a set of layouts is generated. Our algorithm selects the best of these layouts as its output. To establish our approximation bound we find the cost (in terms of area) of transforming an optimal solution to the solution generated by the algorithm. This is done for each type of net separately.

Let $2_S - R2M$ ($2_C - R2M$) denote a $2 - R2M$ problem with the property that each global net is simple (complex). In Sec. 2 we define our notation and present some basic results. In order to simplify the exposition of our results, we begin by presenting approximation algorithms for restricted versions of the $2-R2M$ problem. In Sec. 3 we present an approximation algorithm for the $2_S - R2M$ problem. An approximation algorithm $2_C - R2M$ problem is presented in Sec. 4 and in Sec. 5 we combine these results to obtain our approximation algorithm for the $2 - R2M$ problem.

2. Notation and Basic Results

We begin by defining our notation and proving some lemmas that will simplify our notation. Then we reduce our problem to only having to specify the type of path for the wire connecting each global net. This simplifies our analysis.

The four corners of T and B are labeled as follows. Starting with the bottom-left corner of T (B), traverse the sides of rectangle T (B) clockwise. The ith corner visited is labeled S_{i-1} (R_{i-1}). We use TL, TT, TR, and TB (BL, BT, BR, and BB); T_l, T_t, T_r and T_b (B_l, B_t, B_r and B_b); to represent the left, top, right and bottom sides of $T(B)$, respectively. Let X_jY_k represent the set of nets with one terminal located on side j of rectangle X and the other located on side k of rectangle Y, where $X, Y \in \{T, B\}$ and $j, k \in \{l, r, t, b\}$ (see Fig. A.1 in Appendix A). We use x_jy_k to represent the number of nets in set X_jY_k. For $1 \leq j \leq n$, the pairing function $C(j)$ is defined in such a way that T_j and $T_{C(j)}$ belong to the same net. We establish the following lemma to simplify our remaining notation.

Lemma 1. *Let W be any layout for an instance of the $2 - R2M$ problem. For each net $N_i \in T_bB_x$ (T_xB_t), where $x \in \{t, l, r, b\}$, if net N_i is connected by a wire that crosses the top (bottom) side of rectangle $T(B)$, then there exists another layout M such that net N_i is connected by a wire that does not cross the top (bottom) side of rectangle $T(B)$, and the area of M is not larger than the area of W.*

Proof. The proof is based on a simple interchange argument. Let M be minimum area layout in which the wire for each net crosses the same corners of the rectangles as the one in layout W, except for the wire for the net in Fig. 1(a) which is replaced by the one in Fig. 1(b). It is simple to see that the area of layout M is less than or equal to the area of layout W. □

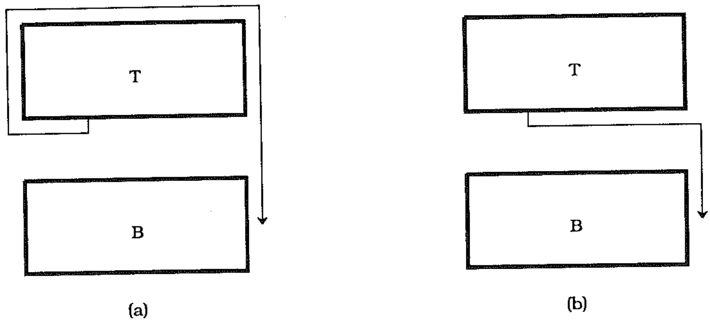


Fig. 1. (a) Layout W, and (b) layout M.

Hereafter, whenever we refer to an optimal area layout we assume, without loss of generality, that it cannot be transformed by applying the interchange argument given in Lemma 1, nor that it has wires with excessive segments (wires which can be trivially replaced by other wires without increasing the layout area). Note that this assumption does not make the problem easier; however, it allows us to use a simpler notation. The layouts constructed by our algorithm also satisfy these properties.

Since we are only concerned with layouts that cannot be transformed by the rule given by Lemma 1 and which do not have excessive segments, the type of wire connecting net $N_j = \{T_i, T_{C(i)}\}$ in a layout can be characterized by a triple (i, x, y), for $x, y \in \{+, -\}$ as follows.

1. If T_i and $T_{C(i)}$ are located on the same rectangle X, then $x \neq y$ and the wire connecting net N_j consists of the following sequence of wire segments: the first wire segment is incident to T_i and it is perpendicular to the side where terminal T_i is located; if $x =$ '$+$' ($x =$ '$-$') this segment is followed by a sequence of wire segments parallel to the boundary segments of X encountered while traversing the boundary of rectangle X in the clockwise (counter-clockwise) direction starting at T_i and ending at $T_{C(i)}$; the final segment is perpendicular to the side where $T_{C(i)}$ is located and it is incident at $T_{C(i)}$.
2. If T_i and $T_{C(i)}$ are located on different rectangles, then the wire is more complex. Let h be a horizontal line that partitions the plane so that each rectangle is in a different half space. The wire connecting the net consists of three wire segments: the top segment, the middle segment (possibly empty) and the bottom segment. Assume without loss of generality that T_i ($T_{C(i)}$) is located on rectangle T (B). The top segment consists of the following sequence of wire segments: the first wire segment is incident to T_i and it is perpendicular to the side where terminal T_i is located; if T_i is located on the bottom side of T, then the wire terminates when it reaches line h,

otherwise if $x =$ '+' ($x =$ '−') the first segment is followed by a sequence of wire segments parallel to the boundary segments of T encountered while traversing the boundary of rectangle T in the clockwise (counter-clockwise) direction starting at T_i and ending at the bottom-right (bottom-left) corner of rectangle T; and the final segment extends the last wire segment until it reaches line h. The bottom segment is similar to the top segment. If the top and bottom segment are both located on the left or the right side of the rectangles, then the middle segment is empty because we add the constraint that both the top and the bottom segment must end at the same point on line h. Otherwise, the middle segment is a horizontal wire segment that overlaps with h and joins the two points on line h where the top and bottom wires segments end.

Set $D = \{(d_1, x_1, y_1), (d_2, x_2, y_2), \ldots, (d_n, x_n, y_n)\}$, where $1 \leq d_i \leq 2n$ and $x_i, y_i \in \{+, -\}$ for $1 \leq i \leq n$, is said to be an *assignment* if $|\{d_1, C(d_1), d_2, C(d_2), \ldots, d_n, C(d_n)\}| = 2n$. Any subset of an assignment is said to be a *partial assignment*. Each tuple in an assignment or partial assignment specifies the type of wire connecting a net. For any $(i, x_j, y_j) \in D$, we say that the type of wire connecting T_i and $T_{C(i)}$ specified by D *crosses terminal* z if the terminal z can be horizontally or vertically projected to it without intersecting any side of the rectangles. A wire *crosses corner* s if s can be horizontally and vertically projected to it without intersecting any side of the rectangles. If corner s can be vertically (horizontally) projected to a wire without intersecting any side of the two rectangles, then this wire *crosses vertically* (*horizontally*) *corner* s.

For any assignment (or partial assignment) D we define the *height function* H_D as follows:

$$H_D(X) = \max\{\text{number of wires given by } D \text{ that cross horizontally point } z \mid z \text{ is a terminal or a corner located on side } X\},\quad \text{for } X \in \{TL, TR, BL, BR\};$$

$$H_D(X) = \max\{\text{number of wires given by } D \text{ that cross vertically point } z \mid z \text{ is a terminal or a corner located on side } X\},\quad \text{for } X \in \{TT, BB\},$$

$$H_D(TB) = \max \text{ number of wires given by } D \text{ that cross vertically point } z \mid z \text{ is a terminal or a corner located on side } TB \cup BT\}, \text{ and}$$

$$H_D(TB) = H_D(BT)\ .$$

We shall refer to $H_D(X)$ as the *height of assignment D on side X*. For an assignment D, we define $H_D(LL)$, $H_D(RR)$, and $H_D(TMB)$ as follows:

$$H_D(LL) = \mathbf{max}\{H_D(TL), H_D(BL)\} ,$$
$$H_D(RR) = \mathbf{max}\{H_D(TR), H_D(BR)\} , \text{ and}$$
$$H_D(TMB) = H_D(TT) + H_D(BT) + H_D(BB) .$$

For any assignment D and corner $U \in \{S_0, S_1, S_2, S_3, R_0, R_1, R_2, R_3\}$, we define the functions C_D and CH_D as follows:

$$C_D(U) = \mathbf{max}\{\text{number of wires given by } D \text{ that a cross corner in set } U\}, \text{ and}$$

$$CH_D(U) = \mathbf{max}\{\text{number of wires given by that cross horizontally a corner in set } U\} .$$

It is simple to prove the following relationships between these values:

$$H_D(LL) + H_D(RR) \geq \frac{1}{4} \sum_{\substack{U \text{ is a corner} \\ \text{of } T \text{ or } B}} CH_D(U) , \text{ and}$$

$$H_D(TMB) \geq \frac{1}{2} \sum_{\substack{U \text{ is a corner} \\ \text{of } T \text{ or } B}} C_D(U) .$$

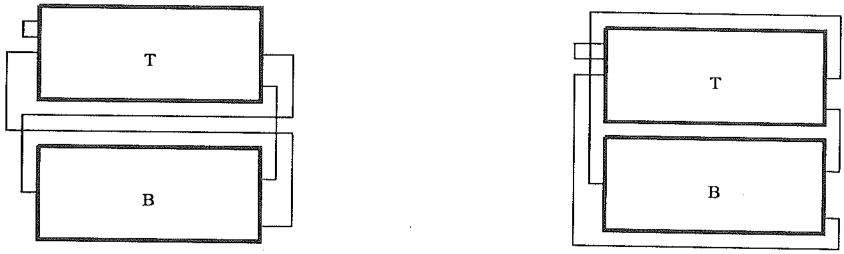
For any assignment D we define

$$h_D = h_T + h_B + \lambda H_D(TMB) , \text{ and}$$
$$w_D = w_T + \lambda(H_D(LL) + H_D(RR)) .$$

Assignment D is said to be an optimal assignment for problem instance I if D is an assignment with minimum $h_D \cdot w_D$ amongst all assignments for I. Every optimal area layout can be characterized by an optimal assignment, but not every optimal assignment characterizes an optimal area layout. Figure 2 gives two optimal assignments, but only one of them has an optimal area layout. Therefore, not every optimal assignment D can be wired inside a rectangle with area $h_D \cdot w_D$. However, every assignment D can be wired inside a rectangle with area $h_D \cdot (w_D + \lambda)$. Furthermore, such layout can be constructed in $O(n)$ time (remember that the we assumed that set of terminals is sorted). This facts are established in the following two lemmas.

Lemma 2. *For every assignment D, there is a rectangle Q of size at most h by $(w + \lambda)$, where $h = h_T + h_B + \lambda H_D(TMB)$, and $w = w_T + \lambda(H_D(LL) + H_D(RR))$, with the property that rectangle T and B together with the interconnecting wires given by assignment D can be made to fit inside Q.*

Proof. The proof is a direct generalization of the proof for the $R1M$ problem,[9] and the T-shape routing problem.[17] Pinter's procedure[17] may require one additional



(a) Optimal assignment without an optimal area layout.

(b) Optimal assignment with an optimal area layout.

Fig. 2. Optimal assignments and optimal area layouts.

vertical track. For the case (see Fig. 2(a)) one additional vertical track is required on either the left or right side of the rectangles. □

Lemma 3. *For any assignment D a layout with the area given by Lemma 2 can be obtained in $O(n)$ time.*

Proof. The proof of this lemma is a straight forward generalization of the proof for the $R1M$ problem.[9] The algorithm that constructs the final layout uses as a subalgorithm the known algorithms.[7,6,17] □

For an assignment D we define the *area function* $A(D)$ as

$$(h_T + h_B + \lambda H_D(TMB)) \cdot (w_T + \lambda(H_D(LL) + H_D(RR))) .$$

Hereafter, we assume that $A(D)$ is the total area required for a layout of T and B and all interconnections given by D. As we showed before this is not always true, but since the difference is insignificant, we ignore it. Hereafter we corrupt our notation and say that every optimal assignment has an optimal area layout.

Net N_i is said to be a *local net* if its two terminals are located on the same side of the same rectangle, or both are located in the middle channel (see all nets labeled L in Fig. A.1 in Appendix A). Otherwise, net N_i is said to be *global*. Therefore, the set of nets L consists of the following set of nets.

$$L = T_bB_t \cup T_tT_t \cup B_tB_t \cup T_rT_r \cup B_rB_r \cup T_bT_b \cup B_bB_b \cup T_lT_l \cup B_lB_l \ .$$

For the set of nets L we define ML as the partial assignment in which each local net is connected by an Inc-wire. We show that every assignment can be transformed to another assignment without increasing its area and in which all local nets are connected by Inc-wires.

Lemma 4. *Every assignment D can be transformed to an assignment M such that $ML \subseteq M$ and $A(M) \leq A(D)$.*

Proof. The proof follows the same lines as the one for the $R1M$ problem.[9,10] □

The $2-R2M$ problem has been reduced to the problem of specifying the type of wire connecting each global net in the presence of the partial assignment ML, i.e., generate an assignment that includes ML. Remember that once we have an assignment, the proof of Lemma 3 (a constructive proof) can be used to find an optimal area layout for it. In the next two sections we present approximation algorithms for the $2_S - R2M$ problem and $2_C - R2M$ problem. In Sec. 5 we show how to combine these results to obtain our approximation algorithm that generates a layout with area at most 2 OPT, where OPT is the area of an optimal area layout.

3. Approximation Algorithm for the $2_S - R2M$ Problem

In this section we present an approximation algorithm for the $2_S - R2M$ problem. First we define the assignment from which our algorithm generates the final layout. In the final layout all simple nets are connected by *Inc*-wires. Then in Lemmas 5–9 we find a bound for the cost of transforming any optimal solution to our solution, and in Lemma 10 we establish a lower bound for the area of an optimal area layout. In Theorem 1 we establish the approximation bound for simple nets based on the previous lemmas.

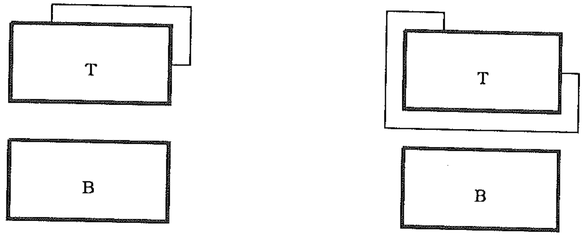
Let S be the set of global nets for the $2_S - R2M$ problem. By definition all nets in S are simple, i.e., all wires connecting a global net cross exactly the same rectangle corners. It is simple to show that the set S consists of the following subset of nets (see all nets labeled S in Fig. A.1 in Appendix A):

$$S = \left(\bigcup_{\substack{f,g \text{ are two} \\ \text{adjacent sides}}} T_f T_g \cup B_f B_g \right) \cup \left(\bigcup_{\substack{j \in \{l,t,b,r\} \\ k \in \{l,r\}}} T_j B_k \cup T_k B_j \right) .$$

For the set of nets in $L \cup S$, we define assignment MS as follows:

$$MS = ML \cup \{\text{each net in } S \text{ is connected by an } Inc\text{-wire}\} .$$

In Fig. 3 we give two layouts for a net in $T_t T_r$. The one in Fig. 3(a) shows the layout for the assignment constructed by our algorithm. Suppose that this net is routed as in Fig. 3(b) in an optimal assignment D. Let M be D except for the wire in Fig. 3(b) which is of the type given in Fig. 3(a). It is simple to show that:



(a) (b)

Fig. 3. A wire connecting a net in (a) $T_tT_r - Y_tY_r$, and (b) Y_tY_r.

$$\begin{aligned} H_M(TMB) &\le H_D(TMB) , \\ H_M(TL) &= H_D(TL) - 1 , \\ H_M(TR) &\le H_D(TR) + 1 , \\ H_M(BL) &= H_D(BL) , \text{ and} \\ H_M(BR) &= H_D(BR) . \end{aligned}$$

A straight forward generalization of the above observation is given by Lemmas 5–9.

Lemma 5. *Let D be an optimal assignment such that $ML \subseteq D$. Let M be D except that all nets in T_fT_g and B_fB_g, where f and g are two adjacent sides, are assigned as in our algorithm. Then*

$$H_M(TMB) \le H_D(TMB) ,$$

$$H_M(TL) \le H_D(TL) + \sum_{k\in\{t,b\}} y_ky_l - \sum_{k\in\{t,b\}} y_ky_r ,$$

$$H_M(TR) \le H_D(TR) - \sum_{k\in\{t,b\}} y_ky_l + \sum_{k\in\{t,b\}} y_ky_r ,$$

$$H_M(BL) \le H_D(BL) + \sum_{k\in\{t,b\}} z_kz_l - \sum_{k\in\{t,b\}} z_kz_r , \text{ and}$$

$$H_M(BR) \le H_D(BR) - \sum_{k\in\{t,b\}} z_kz_l + \sum_{k\in\{t,b\}} z_kz_r ,$$

where $Y_fY_g(Z_fZ_g)$ is the set of nets in $T_fT_g(B_fB_g)$ that are connected differently in assignments D and M. (It is important to keep in mind that the set of nets in $T_fT_g - Y_fY_g$ are connected by the same type of wires in D and M.)

Proof. For brevity the proof is not included. The wire connecting a net in set $T_tT_r - Y_tY_r$ and Y_tY_r is illustrated in Fig. 3. □

Lemma 6. *Let D be an optimal assignment such that $ML \subseteq D$. Let M be D except that all nets in T_jB_j, where $j \in \{l, r\}$ are assigned as in our algorithm. Then**

$$H_M(TMB) \leq H_D(TMB) ,$$
$$H_M(TL) \leq H_D(TL) + y_lb_l + y_lz_l - y_rb_r - y_rz_r ,$$
$$H_M(TR) \leq H_D(TR) - y_lb_l - y_lz_l + y_rb_r + y_rz_r ,$$
$$H_M(BL) \leq H_D(BL) + t_lz_l + y_lz_l - t_rz_r - y_rz_r , \text{ and}$$
$$H_M(BR) \leq H_D(BR) - t_lz_l - y_lz_l + t_rz_r + y_rz_r ,$$

where T_jZ_j (Y_jB_j) is the set of nets in T_jB_j that are connected differently only on the bottom (top) rectangle in assignments D and M, and Y_jZ_j is the set of nets in T_jB_j that are connected differently on the top and bottom rectangle in assignments D and M.

Proof. Since the proof is straight forward, it is omitted. The wire connecting a net in set (a) $T_lB_l - (T_lZ_l \cup Y_lB_l \cup Y_lZ_l)$, (b) T_lZ_l, (c) Y_lB_l, and (d) Y_lZ_l, are illustrated in Fig. 4. □

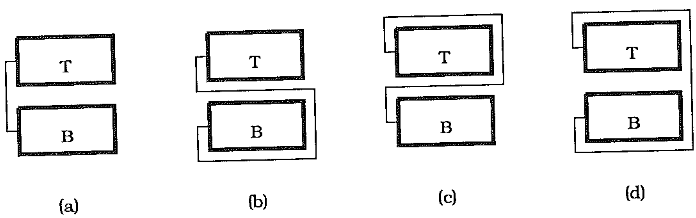


Fig. 4. A wire connecting a net in (a) $T_lB_l - (T_lZ_l \cup Y_lB_l \cup Y_lZ_l)$, (b) T_lZ_l, (c) Y_lB_l, and (d) Y_lZ_l.

Lemma 7. *Let D be an optimal assignment such that $ML \subseteq D$. Let M be D except that all nets in $T_lB_r \cup T_rB_l$ are assigned as in our algorithm. Then*

$$H_M(TMB) = H_D(TMB) - 2y_lz_r - 2y_rz_l ,$$
$$H_M(TL) \leq H_D(TL) + y_lb_r + y_lz_r - y_rb_l - y_rz_l ,$$
$$H_M(TR) \leq H_D(TR) - y_lb_r - y_lz_r + y_rb_l + y_rz_l ,$$
$$H_M(BL) \leq H_D(BL) - t_lz_r - y_lz_r + t_rz_l + y_rz_l , \text{ and}$$
$$H_M(BR) \leq H_D(BR) + t_lz_r + y_lz_r - t_rz_l - y_rz_l ,$$

where T_jZ_k (Y_jB_k) is the set of nets in T_jB_k that are connected differently only on the bottom (top) rectangle in assignments D and M, and Y_jZ_k is the set of nets in

*One can trivially establish a sharper bound; however, such bound is not needed to establish the approximation bound of two.

T_jB_k that are connected differently on the top and bottom rectangle in assignments D and M.

Proof. Since the proof is straight forward, it is omitted. The wire connecting a net in set (a) $T_lT_r - (T_lZ_r \cup Y_lB_r \cup Y_lZ_r)$, (b) T_lZ_r, (c) Y_lB_r, and (d) Y_lZ_r, is illustrated in Fig. 5. □

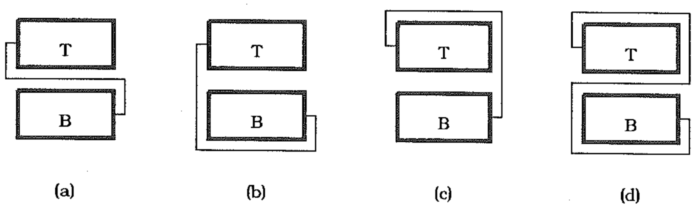


Fig. 5. A wire connecting a net in (a) $T_lB_r - (T_lZ_r \cup Y_lB_r \cup Y_lZ_r)$, (b) T_lZ_r, (c) Y_lB_r, and (d) Y_lZ_r.

Lemma 8. *Let D be an optimal assignment such that $ML \subseteq D$. Let M be D except that all nets in $T_bB_j \cup T_jB_t$, where $j \in \{l, r\}$, which are assigned as in our algorithm. Then*

$$\begin{aligned} H_M(TMB) &\le H_D(TMB) , \\ H_M(TL) &\le H_D(TL) + y_lz_t - y_rz_t , \\ H_M(TR) &\le H_D(TR) - y_lz_t + y_rz_t , \\ H_M(BL) &\le H_D(BL) + y_bz_l - y_bz_r , \text{ and} \\ H_M(BR) &\le H_D(BR) - y_bz_l + y_bz_r , \end{aligned}$$

where Y_jZ_k is the set of nets in T_jB_k that are connected differently in assignments D and M.

Proof. The proof of this lemma is similar to Lemma 6. For brevity the proof is not included. The wire connecting a net in set (a) $T_bT_l - (Y_bZ_l)$, and (b) Y_bZ_l, is illustrated in Fig. 6. □

Lemma 9. *Let D be an optimal assignment such that $ML \subseteq D$. Let M be D except that all nets in $T_tB_j \cup T_jB_b$, where $j \in \{l, r\}$, which are assigned as in our algorithm. Then*

$$\begin{aligned} H_M(TMB) &\le H_D(TMB) , \\ H_M(TL) &\le H_D(TL) + y_tb_l + y_tz_l - y_tb_r - y_tz_r + y_lb_b + y_lz_b - y_rb_b - y_rz_b , \\ H_M(TR) &\le H_D(TR) - y_tb_l - y_tz_l + y_tb_r + y_tz_r - y_lb_b - y_lz_b + y_rb_b + y_rz_b , \\ H_M(BL) &\le H_D(BL) + t_tz_l + y_tz_l - t_tz_r - y_tz_r + t_lz_b + y_lz_b - t_rz_b - y_rz_b , \text{ and} \\ H_M(BR) &\le H_D(BR) + t_tz_l - y_tz_l + t_tz_r + y_tz_r - t_lz_b - y_lz_b + t_rz_b + y_rz_b , \end{aligned}$$

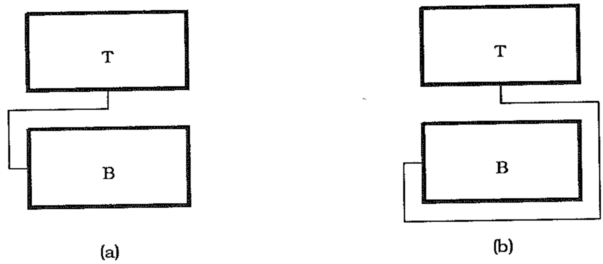


Fig. 6. A wire connecting a net in (a) $T_bB_l - (Y_bZ_l)$, and (b) Y_bZ_l.

where T_jZ_k (Y_jB_k) is the set of nets in T_jB_k that are connected differently only on the bottom (top) rectangle in assignments D and M, and Y_jZ_k is the set of nets in T_jB_k that are connected differently on the top and bottom rectangle in assignments D and M.

Proof. The proof of this lemma is similar to Lemma 6. For brevity the proof is not included. The wire connecting a net in set (a) $T_tT_l - (T_tZ_l \cup Y_tB_l \cup Y_tZ_l)$, (b) T_tZ_l, (c) Y_tB_l, and (d) Y_tZ_l, as illustrated in Fig. 7. □

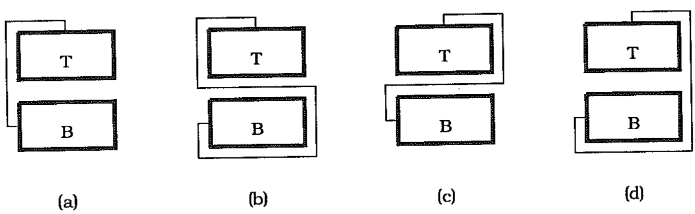


Fig. 7. A wire connecting a net in (a) $T_tB_l - (T_tZ_l \cup Y_tB_l \cup Y_tZ_l)$, (b) T_tZ_l, (c) Y_tB_l, and (d) Y_tZ_l.

Before proving our main result in this section we establish a lower bound for the area of an optimal area layout for the nets in set S. Note that from Lemmas 5–9 we know that our algorithm generates an optimal layout with respect to the height of the enclosing rectangle for the $2_S - R2M$ problem. The lower bounds are given in Table 1.

Lemma 10. *Let D be an optimal assignment such that $ML \subseteq D$. Assignment D, and rectangles T and B satisfy the lower bound given in Table 1.*

Proof. We only prove the lower bounds in the first column of Table 1, since the proof for the bounds in the second column is similar. The proof for these bounds can be obtained by adding the lower bounds given in Table 2. Therefore, we only

Table 1. Lower bounds for the $2_S - R2M$ problem.

set	Contribution to our lower bound for		
	$\frac{w_T}{\lambda} + H_D(LL) + H_D(RR)$	$\frac{h_T+h_B}{\lambda} + H_D(TMB)$	
T_fT_g	$\frac{1}{2}(t_ft_g + y_fy_g)$	$t_ft_g + y_fy_g$	f, g are adjacent
B_fB_g	$\frac{1}{2}(b_fb_g + z_fz_g)$	$b_fb_g + z_fz_g$	f, g are adjacent
T_jB_j	$\frac{1}{2}(t_jb_j + t_jz_j + y_jb_j) + y_jz_j$	$t_jb_j + 2(t_jz_j + y_jb_j + y_jz_j)$	$j \in \{l, r\}$
T_lB_r	$\frac{1}{2}(t_lb_r + t_lz_r + y_lb_r) + y_lz_r$	$2(t_lb_r + y_lz_r)$	—
T_rB_l	$\frac{1}{2}(t_rb_l + t_rz_l + y_rb_l) + y_rz_l$	$2(t_rb_l + y_rz_l)$	—
T_bB_j	$\frac{1}{2}(t_bb_j + y_bz_j)$	$t_bb_j + y_bz_j$	$j \in \{l, r\}$
T_jB_t	$\frac{1}{2}(t_jb_t + y_jz_t)$	$t_jb_t + y_jz_t$	$j \in \{l, r\}$
T_tB_j	t_tb_j	$t_tb_j + t_tz_j + y_tb_j + y_tz_j$	$j \in \{l, r\}$
T_jB_b	t_jb_b	$t_jb_b + t_jz_b + y_jb_b + y_jz_b$	$j \in \{l, r\}$

need to establish the lower bounds given in Table 2. To derive these bounds, we make the following observations.

(a) Each net in T_fT_g (B_fB_g), where f and g are adjacent sides, has exactly one terminal on the top or bottom side of T (B).

(b) Each net in T_jB_k (T_kB_j), where $j \in \{t, b\}$ and $k \in \{l, r\}$, has exactly one terminal on the top or bottom side of T (B).

By the above observations and the fact that every terminal is at least λ units away from each corner of T and B, we know that

$$\frac{w_T}{\lambda} \geq \frac{1}{4}\left(\sum_{\substack{f,g \text{ are} \\ \text{adjacent}}} (t_ft_g + b_fb_g) + \sum_{\substack{j\in\{t,b\}, \\ k\in\{l,r\}}} (t_jb_k + t_kb_j)\right).$$

This lower bound is given by the first column of Table 2.

Let us now establish a lower bound for the number of wires crossing the corners of T and B. For an optimal assignment D, we know that:

1. For adjacent sides f and g, every wire connecting a net in Y_fY_g (Z_fZ_g) crosses horizontally three corners of T (B) and every wire connecting a net in $T_fT_g - Y_fY_g$ ($B_fB_g - Z_fZ_g$) crosses horizontally one corner of T (B).
2. For $j \in \{l, r\}$, every wire connecting a net in $T_jZ_j \cup Y_jB_j$, crosses horizontally four corners of T and B; every wire connecting a net in $Y_lZ_l \cup Y_rZ_r$ crosses horizontally six corners of T and B; and every wire connecting a net in $T_jB_j - (T_jZ_j \cup Y_jB_j \cup Y_jZ_j)$ crosses horizontally two corners of T and B.

Table 2. Lower bounds for the $2_S - R2M$ problem.

set	Contribution to our lower bound for		
	$\frac{w_T}{\lambda}$	$H_D(LL)+H_D(RR)$	
T_fT_g	$\frac{1}{4}t_ft_g$	$\frac{1}{4}t_ft_g+\frac{1}{2}y_fy_g$	f,g are adjacent
B_fB_g	$\frac{1}{4}b_fb_g$	$\frac{1}{4}b_fb_g+\frac{1}{2}z_fz_g$	f,g are adjacent
T_jB_j	—	$\frac{1}{2}(t_jb_j+t_jz_j+y_jb_j)+y_jz_j$	$j\in\{l,r\}$
T_lB_r	—	$\frac{1}{2}(t_lb_r+t_lz_r+y_lb_r)+y_lz_r$	—
T_rB_l	—	$\frac{1}{2}(t_rb_l+t_rz_l+y_rb_l)+y_rz_l$	—
T_bB_j	$\frac{1}{4}t_bb_j$	$\frac{1}{4}t_bb_j+\frac{1}{2}y_bz_j$	$j\in\{l,r\}$
T_jB_t	$\frac{1}{4}t_jb_t$	$\frac{1}{4}t_jb_t+\frac{1}{2}y_jz_t$	$j\in\{l,r\}$
T_tB_j	$\frac{1}{4}t_tb_j$	$\frac{3}{4}t_tb_j$	$j\in\{l,r\}$
T_jB_b	$\frac{1}{4}t_jb_b$	$\frac{3}{4}t_jb_b$	$j\in\{l,r\}$

3. Every wire connecting a net in $T_lZ_r\cup T_rZ_l\cup Y_lB_r\cup Y_rB_l$, crosses horizontally four corners of T and B; every wire connecting a net in $Y_lZ_r\cup Y_rZ_l$ crosses horizontally six corners of T and B; and every wire connecting a net in

$$(T_lB_r-(T_lZ_r\cup Y_lB_r\cup Y_lZ_r))\cup(T_rB_l-(T_rZ_l\cup Y_rB_l\cup Y_rZ_l))$$

crosses horizontally two corners of T and B.
4. For $j\in\{l,r\}$, every wire connecting a net in $Y_bZ_j\cup Y_jZ_t$ crosses horizontally three corners of T and B and every wire connecting a net in $(T_bB_j-Y_bZ_j)\cup(T_jB_t-Y_jZ_t)$ crosses horizontally one corner of T and B.
5. For $j\in\{l,r\}$, every wire connecting a net in $T_tB_j\cup T_jB_b$, crosses horizontally three corners of T and B.

From the above observations (the ith observation implies the ith line below) we know that:

$$\begin{aligned}
&\sum_{i=0}^{3}(CH_D(S_i)+CH_D(R_i))\\
&\quad\geq \sum_{\substack{f,g\text{ are}\\ \text{adjacent}}}(t_ft_g+b_fb_g)+\sum_{\substack{f,g\text{ are}\\ \text{adjacent}}}2(y_fy_g+z_fz_g)\\
&\quad+\sum_{j\in\{l,r\}}2t_jb_j+\sum_{j\in\{l,r\}}(2(t_jz_j+y_jb_j)+4y_jz_j)\\
&\quad+2(t_lb_r+t_lz_r+y_lb_r)+4y_lz_r+2(t_rb_l+t_rz_l+y_rb_l)+4y_rz_l
\end{aligned}$$

$$+\sum_{j\in\{l,r\}}(t_b b_j + t_j b_t) + \sum_{j\in\{l,r\}} 2(y_b z_j + y_j z_t)$$

$$+\sum_{j\in\{l,r\}} 3(t_t b_j + t_j b_b)$$

Since $H_D(LL)$ ($H_D(RR)$) is at least as large as 1/4 times the sum of the horizontal height of the four left (right) corners of T and B, we know that

$$H_D(LL) + H_D(RR) \geq \frac{1}{4}(CH_D(R_0) + CH_D(R_1) + CH_D(S_0) + CH_D(S_1))$$
$$+\frac{1}{4}(CH_D(R_2) + CH_D(R_3) + CH_D(S_2) + CH_D(S_3)) .$$

The lower bounds in the second column of Table 2 follows from the above inequalities. The proof for the lower bounds for the first column of Table 1 is obtained by adding the bounds given by Table 2. This completes the proof for the lemma.

□

Theorem 1. *For the $2_S - R2M$ problem, let D be an optimal assignment such that $ML \subseteq D$ and let MS be the assignment generated by our algorithm. Then, $A(MS) \leq 2A(D)$.*

Proof. There is a simpler proof for this theorem, but in order to facilitate its incorporation in Sec. 5, we prove it in four cases.

Case 1: $H_{MS}(LL) = H_{MS}(TL)$ and $H_{MS}(RR) = H_{MS}(BR)$.

From Lemmas 5–10,[†] we know that $\frac{A(MS)}{A(D)} \leq (1 + \frac{p}{q}) \cdot (1 + \frac{r}{s})$, where

$$p = \sum_{k\in\{t,b\}} y_k y_l + \sum_{k\in\{t,b\}} z_k z_r$$
$$+ y_l b_l + t_r z_r + t_l z_r + y_l b_r + 2y_l z_r + y_b z_r + y_l z_t + y_t b_l + t_t z_r + y_l b_b + t_r z_b$$

$$q = \sum_{\substack{f,g \text{ are}\\ \text{adjacent}}} \frac{1}{2}(t_f t_g + y_f y_g) + \sum_{\substack{f,g \text{ are}\\ \text{adjacent}}} \frac{1}{2}(b_f b_g + z_f z_g)$$

$$+\sum_{j\in\{l,r\}} (\frac{1}{2}(t_j b_j + t_j z_j + y_j b_j) + y_j z_j) + \frac{1}{2}(t_l b_r + t_l z_r + y_l b_r) + y_l z_r$$

$$+\frac{1}{2}(t_r b_l + t_r z_l + y_r b_l) + y_r z_l + \sum_{j\in\{l,r\}} \frac{1}{2}\cdot(t_b b_j + y_b z_j)$$

$$+\sum_{j\in\{l,r\}} \frac{1}{2}\cdot(t_j b_t + y_j z_t) + \sum_{j\in\{l,r\}} t_t b_j + \sum_{j\in\{l,r\}} t_j b_b$$

[†] p and r are from Lemmas 5–9, and q and s are from Lemma 10.

$$r = -2y_l z_r - 2y_r z_l$$

$$s = \sum_{\substack{f,g \text{ are} \\ \text{adjacent}}} (t_f t_g + y_f y_g) + \sum_{\substack{f,g \text{ are} \\ \text{adjacent}}} (b_f b_g + z_f z_g)$$

$$+ \sum_{j \in \{l,r\}} (t_j b_j + 2(t_j z_j + y_j b_j + y_j z_j)) + 2(t_l b_r + y_l z_r) + 2(t_r b_l + y_r z_l)$$

$$+ \sum_{j \in \{l,r\}} (t_b b_j + y_b z_j) + \sum_{j \in \{l,r\}} (t_j b_t + y_j z_t)$$

$$+ \sum_{j \in \{l,r\}} (t_t b_j + t_t z_j + y_t b_j + y_t z_j) + \sum_{j \in \{l,r\}} (t_j b_b + t_j z_b + y_j b_b + y_j z_b)$$

A straight forward manipulation of the above inequalities[11,12] gives the bound of two.

Case 2: $H_{MS}(LL) = H_{MS}(BL)$ and $H_{MS}(RR) = H_{MS}(TR)$.

The proof for this case is similar to the proof for case 1.

Case 3: $H_{MS}(LL) = H_{MS}(TL)$ and $H_{MS}(RR) = H_{MS}(TR)$.

The proof for this case is trivial since assignment D is optimal, i.e.,

$$H_{MS}(TMB) \le H_D(TMB) \text{ , and } H_{MS}(TL) + H_{MS}(TR) \le H_D(TL) + H_D(TR) \text{ .}$$

Case 4: $H_{MS}(LL) = H_{MS}(BL)$ and $H_{MS}(RR) = H_{MS}(BR)$.

The proof for this case is similar to the proof for case 3. This completes the proof of the lemma. □

4. Approximation Algorithm for the $2_C - R2M$ Problem

In this section we present an approximation algorithm for the $2_C - R2M$ problem. Let C be the set of global nets. By definition all the nets in C are complex, i.e., each net can be connected by at least two *Inc*-wires that cross different rectangle corners (see nets labeled C in Fig. A.1 in Appendix A). Clearly,

$$C = \left(\bigcup_{\substack{f,g \text{ are two} \\ \text{opposite sides}}} T_f T_g \cup B_f B_g \right) \cup \left(\bigcup_{k \in \{t,b\}} T_t B_k \cup T_k B_b \right) .$$

In subsection 4.1 we outline our procedure to route the set of nets in $V = T_l T_r \cup B_l B_r$ and establish our approximation bound for that case. We show in subsection 4.2 how to route the remaining nets, and establish our approximation bound for the $2_C - R2M$ problem.

4.1. Assignment for the Set of Nets $V = T_lT_r \cup B_lB_r$ and the Analysis of its Approximation Bound

We only explain how the assignment $MV(T_lT_r)$ for all nets in T_lT_r is constructed, since the assignment $MV(B_lB_r)$ is constructed by a similar procedure. If the number of nets in T_lT_r is odd then delete one of the nets to make the cardinality of the set even. When the layout for all nets (except for the ones deleted which is at most two) has been constructed, the remaining nets is connected in all possible ways (this number is bounded by a constant) and the best layout is the one generated by the algorithm. The approximation bound will still hold because the nets deleted at this step are connected as in the optimal area assignment in one of the layouts. Figure 8a and 8b give a layout for the assignment $MV(T_lT_r)$ and $MV(B_lB_r)$ constructed by our algorithm for sets T_lT_r and B_lB_r each with four nets. For any permutation, π, of the nets in set T_lT_r we define an assignment ASG(T_lT_r, π) as follows: the wire connecting the kth net ($1 \leq k \leq t_lt_r$) in π, for k is even (odd), begins on the right (left) side of T, it crosses the bottom (top) side of T and ends on the left (right) side of T. Note that by the bottom side of T we mean the middle channel. We claim that there is a permutation, π, of the nets in set T_lT_r such that there is a layout for assignment ASG(T_lT_r, π) with the property that for each k ($1 < k \leq t_lt_r$) the wires connecting the kth and ($k-1$)st net in π can share the same track on the side where the wire connecting the ($k-1$)st net ends. In this case we say that π is a *good permutation* for the set of nets in T_lT_r.

Claim: There is a good permutation for the set of nets T_lT_r.

Proof. For brevity the proof is omitted. An interested reader can find the proof Gonzalez and Lee's paper.[12] □

A good permutation, π, can be constructed by a simple recursive procedure.[12] Once π is obtained, the assignment $MV(T_lT_r)$ can be easily constructed. Figure 8a and 8b give a layout for the assignment $MV(T_lT_r)$ and $MV(B_lB_r)$ constructed by our procedure for sets with four nets each.

Our approximation algorithm constructs the assignment $MV = ML \cup MV(T_lT_r) \cup MV(B_lB_r)$. Before proving that our algorithm generates a layout with area at most twice the area of an optimal layout, we need to establish upper bounds for the area of the layouts obtained from assignment MV and prove lower bounds for the area of an optimal area layout.

Lemma 11. *Let D be an optimal assignment such that $ML \subseteq D$. Let M be D except that all nets in $V = T_lT_r \cup B_lB_r$, which are assigned as in our algorithm. Then*

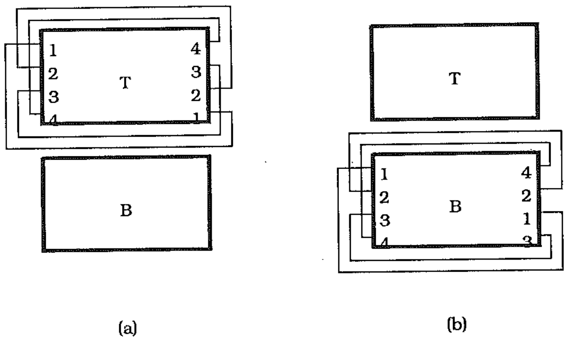


Fig. 8. (a) Layout for set $T_l T_r$, and (b) layout for set $B_l B_r$.

$$\begin{aligned}
H_M(TMB) &= H_D(TMB) \ , \\
H_M(TL) &\le H_D(TL) + \frac{1}{2} t_l t_r + 1 \ , \\
H_M(TR) &\le H_D(TR) + \frac{1}{2} t_l t_r \ , \\
H_M(BL) &\le H_D(BL) + \frac{1}{2} b_l b_r + 1 \ , \text{ and} \\
H_M(BR) &\le H_D(BR) + \frac{1}{2} b_l b_r \ .
\end{aligned}$$

Proof. For brevity the proof is not included. □

Before proving our main result in this subsection we establish a lower bound on the area required by an optimal layout. The lower bound is given in Table 3.

Table 3. Lower bounds for the set of nets $T_l T_r \cup B_l B_r$.

set	Contribution to our lower bound for	
	$\frac{w_T}{\lambda} + H_D(LL) + H_D(RR)$	$\frac{h_T + h_B}{\lambda} + H_D(TMB)$
$T_l T_r$	$\frac{1}{2} t_l t_r$	$2 t_l t_r$
$B_l B_r$	$\frac{1}{2} b_l b_r$	$2 b_l b_r$
—	1	2

Lemma 12. *Let D be an optimal assignment such that $ML \subseteq D$. Assignment D and rectangles T and B satisfy the lower bound given in Table 3.*

Proof. Since the proof for the bounds is similar to Lemma 10, it is omitted. We should note that the constants are introduced because every terminal must be located at least λ units from each corner of the rectangles, horizontally there is only one rectangle, and vertically there are two rectangles. □

Theorem 2. *For the $2_C - R2M$ problem, let D be an optimal assignment such that $ML \subseteq D$ and let MV be the assignment generated by our algorithm. Then $A(MV) \leq 2A(D)$.*

Proof. There are four cases that need to be considered:

Case 1: $H_{MV}(LL) = H_{MV}(TL)$ and $H_{MV}(RR) = H_{MV}(BR)$.

From Lemmas 11 (p and q) and 12 (r and s), we know that

$$\frac{A(MV)}{A(D)} \leq \left(1 + \frac{p}{q}\right) \cdot \left(1 + \frac{r}{s}\right),$$

where $p = \frac{1}{2}(t_l t_r + b_l b_r) + 1$; $q = \frac{1}{2}(t_l t_r + b_l b_r) + 1$; r=0; and $s = 2(t_l t_r + b_l b_r) + 2$. Since the remaining part of the proof for case 1 is simple, it is omitted.

Case 2: $H_{MV}(LL) = H_{MV}(BL)$ and $H_{MV}(RR) = H_{MV}(TR)$.

The proof for this case is symmetric to the one for case 1.

Case 3: $H_{MV}(LL) = H_{MV}(TL)$ and $H_{MV}(RR) = H_{MV}(TR)$.

From Lemmas 11 (p and q) and 12 (r and s), we know that

$$\frac{A(MV)}{A(D)} \leq \left(1 + \frac{p}{q}\right) \cdot \left(1 + \frac{r}{s}\right),$$

where $p = t_l t_r + 1$; $q = t_l t_r + 1$; $r = 0$; and $s = 2(t_l t_r + b_l b_r) + 2$. Since the remaining part of the proof for case 3 is simple, it is be omitted.

Case 4: $H_{MV}(LL) = H_{MV}(BL)$ and $H_{MV}(RR) = H_{MV}(BR)$.

The proof for this case is symmetric to the one for case 3. This completes the proof of the theorem. □

4.2. Assignment for the Remaining Nets, $H = T_t T_b \cup B_t B_b \cup T_t B_b \cup T_t B_t \cup T_b B_b$, and Our Analysis for the Assignment Constructed for the $2_C R2M$ Problem

Remember that L is the set of local nets and that the set of nets C is partitioned into sets $V = T_l T_r \cup B_l B_r$ and $H = T_t T_b \cup B_t B_b \cup T_t B_b \cup T_t B_t \cup T_b B_b$. Remember that ML is the assignment constructed by our algorithm for the set of nets in L, MV represents the assignment constructed by our algorithm for the set of nets $L \cup V$, and $ML \subseteq MV$.

All remaining unrouted nets have all their terminals on the top and bottom sides of the rectangles. Because the lower bound for the width of the rectangles is the number of terminals divided by four (rather than two for the total height of the rectangles), the lower bound is not large enough to establish an approximation bound of two when the remaining nets are routed as in the previous subsection. To achieve the approximation bound of two, the remaining nets are connected in several ways and a set of layouts is generated. Our algorithm selects the best of these layouts as its output.

Let us now construct the assignment MH for $L \cup V \cup H$ that includes the partial assignment MV. Our algorithm constructs a set of assignments and then selects one with least area, i.e., least $A(\cdot)$, as MH. Let a (b) be the number of nets in H with a terminal located on the top (bottom) side of rectangle T (B). We only deal with the case when

$$H_{MV}(TL) + H_{MV}(TR) + a \geq H_{MV}(BL) + H_{MV}(BR) + b \,,$$

since the other case can be treated similarly. We construct assignment I, for $0 \leq I \leq a$, as follows. Let $tl = I$ and $tr = a - tl$. The nets in H with a terminal located on the top side of rectangle T are routed as follows on the top rectangle. The leftmost tl nets (those nets in H whose terminal located on the top side of rectangle T is among the (tl)th closest to the top left corner of T when considering only nets in H) are connected by wires that cross the left side of T and the remaining tr nets are connected by wires that cross the right side of T. The nets with a terminal located on the bottom side of rectangle B, are routed on the bottom rectangle by following a similar procedure. In this case we use bl and br, which are determined by procedure FIND defined below. The idea behind the procedure is to find the values for bl and br such that the assignment has minimum width and, as a secondary objective bl and br have values as close to each other as possible. Note that a net could be routed through the left side of rectangle T and through the right side of rectangle B. In this case one needs to add a horizontal wire on the middle channel to join the previously introduced wires. For simplicity of presentation, in what follows we assume that the value of b is even. When the value of b is odd, we need to construct two assignments (since there are two assignments in which bl and br differ by one) whenever we have to deal with balanced cases (e.g., Fig. 9(a)), and then we select the best of these two assignments. Let us now define procedure FIND which determines the values for bl and br from tl and tr.

PROCEDURE FIND (tl,tr,bl,br);
/* Given tl and tr, compute the values for bl and br. */
/* Remember that we are assuming that

$$H_{MV}(TL) + H_{MV}(TR) + a \geq H_{MV}(BL) + H_{MV}(BR) + b\ ,$$

and that the value of b is even. As pointed out earlier, the other cases can be treated similarly. */
$a_1 \leftarrow tl$; $a_2 \leftarrow tr$;
case 1: $H_{MV}(LL) = H_{MV}(TL)$ and $H_{MV}(RR) = H_{MV}(TR)$.
$b_2 \leftarrow 2\min\{a_1 + H_{MV}(TL) - H_{MV}(BL), H_{MV}(TR) + a_2 - H_{MV}(BR)\}$;
if $b \leq b_2$ **then** $bl \leftarrow br \leftarrow b/2$ /* Fig. 9(a) */
else $b_1 \leftarrow b - b_2$
if $a_1 + H_{MV}(TL) = b_2/2 + H_{MV}(BL)$
then $bl \leftarrow b_2/2$; $br \leftarrow b_1 + b_2/2$ /* Fig.9(b) */
else $bl \leftarrow b_1 + b_2/2$; $br \leftarrow b_2/2$ /* Fig. 9(c) */

case 2: $H_{MV}(LL) = H_{MV}(BL)$ and $H_{MV}(RR) = H_{MV}(BR)$.
/* The code for cases 2-4 is omitted since it is similar to the one for case 1. */

case 3: $H_{MV}(LL) = H_{MV}(BL)$ and $H_{MV}(RR) = H_{MV}(TR)$.

case 4: $H_{MV}(LL) = H_{MV}(TL)$ and $H_{MV}(RR) = H_{MV}(BR)$.

END-OF-PROCEDURE FIND

Let D be an optimal assignment for $L \cup V \cup H$ such that $ML \subseteq D$. Let $D(H^-)$ be assignment D after eliminating the set of nets H. Let $D(MV)$ be assignment D after routing all nets in $L \cup V$ as in MV. In what follows we use the notation $LB(f,s)$ to denote a lower bound for function f computed using only the set of nets s. Let w represent the width of the rectangles and h represent the sum of the height of each rectangle.

From Theorem 2, we know that

$$\left(1 + \frac{\Delta w'}{w'}\right) \cdot \left(1 + \frac{\Delta h'}{h'}\right) = \left(\frac{w' + \Delta w'}{w'}\right) \cdot \left(\frac{h' + \Delta h'}{h'}\right) \leq 2\ ,\ \text{where}\ ,$$

$$\begin{aligned}
\Delta w' &= (H_{MV}(LL) + H_{MV}(RR)) - (H_{D(H^-)}(LL) + H_{D(H^-)}(RR))\ ,\\
w' &= LB(H_D(LL) + H_D(RR) + w, L \cup V)\ ,\\
\Delta h' &= H_{MV}(TMB) - H_{D(H^-)}(TMB)\ ,\ \text{and}\\
h' &= LB(H_D(TMB) + h, L \cup V)\ .
\end{aligned}$$

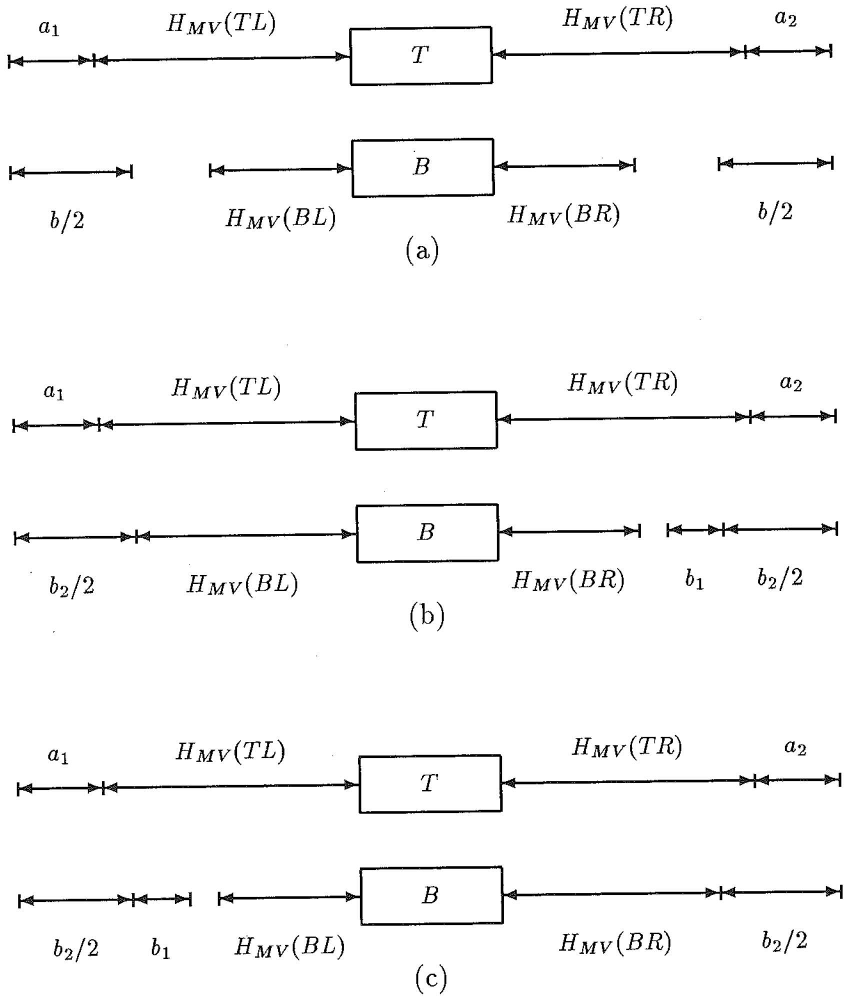


Fig. 9. Examples for case 1.

For the assignment I that corresponds to $D(MV)$, i.e., $H_{D(MV)}(TL) = H_I(TL)$, we know that

$$\left(1+\frac{(H_I(LL)+H_I(RR))-(H_D(LL)+H_D(RR))}{LB(H_D(LL)+H_D(RR)+w, L\cup V\cup H)}\right)\left(1+\frac{H_I(TMB)-H_D(TMB)}{LB(H_D(TMB)+h, L\cup V\cup H)}\right)$$

Replacing the above bound, we know the above equation equals XY, where

$$X = 1 + \frac{\Delta w' + (H_I(LL) + H_I(RR)) - (H_{D(MV)}(LL) + H_{D(MV)}(RR))}{w' + LB(H_D(LL) + H_D(RR) + w, H)} \tag{1}$$

$$Y = 1 + \frac{\Delta h' + H_I(TMB) - H_{D(MV)}(TMB)}{h' + LB(H_D(TMB) + h, H)} \tag{2}$$

Therefore, it is equal to $(1 + \frac{\Delta w' + \Delta w''}{w' + w''})(1 + \frac{\Delta h' + \Delta h''}{h' + h''})$, where

$$\begin{aligned} \Delta w'' &= (H_I(LL) + H_I(RR)) - (H_{D(MV)}(LL) + H_{D(MV)}(RR)) , \\ w'' &= LB(H_D(LL) + H_D(RR) + w, H) , \\ \Delta h'' &= H_I(TMB) - H_{D(MV)}(TMB) , \text{ and} \\ h'' &= LB(H_D(TMB) + h, H) . \end{aligned}$$

Let us now establish Lemma 13 where we prove conditions under which the above approximation bound is two. Later on we show that these conditions hold.

Lemma 13. *Let* $w' \geq q$, $\Delta w' \leq p$, $h' \geq s$, $\Delta h' \leq r$, $w'' \geq x$, $\Delta w'' \leq -y$, $h'' \geq x$ *and* $\Delta h'' \leq x + y$. *Assume that* x, q, $s > 0$; y, $p \geq 0$; $r \leq 0$; $((p+q)(r+s))/qs \leq 2$; $p + q \leq s$; $2p \leq s$ *and* $0 \leq y \leq x$. *Then*

$$\left(1 + \frac{\Delta w' + \Delta w''}{w' + w''}\right)\left(1 + \frac{\Delta h' + \Delta h''}{h' + h''}\right) \leq 2 .$$

Proof. Substituting the bounds for w', $\Delta w'$, h', $\Delta h'$, w'', $\Delta w''$, h'' and $\Delta h''$, we know that

$$\left(1 + \frac{\Delta w' + \Delta w''}{w' + w''}\right) \cdot \left(1 + \frac{\Delta h' + \Delta h''}{h' + h''}\right) \leq \left(1 + \frac{p - y}{q + x}\right) \cdot \left(1 + \frac{r + x + y}{s + x}\right)$$

which is equal to

$$\frac{(p+q)(r+s) + (p+q)(2x+y) + (r+s)(x-y) + (x-y)(2x+y)}{qs + qx + sx + x^2} .$$

Substituting $(p+q)(r+s) \leq 2qs$ and expanding terms, we know the expression is

$$\leq \frac{2qs + 2px + 2qx + (p+q)y + r(x-y) + sx - sy + 2x^2 - xy - y^2}{qs + qx + sx + x^2} .$$

Substituting $p + q \leq s$ and $2p \leq s$, and rearranging terms,

$$\leq \frac{2qs + 2sx + 2qx + r(x-y) + 2x^2 - xy - y^2}{qs + qx + sx + x^2} .$$

Eliminating all the non-positive terms (remember that $r \leq 0$ and $x \geq y$),

$$\leq \frac{2qs + 2sx + 2qx + 2x^2}{qs + qx + sx + x^2} = 2 \ .$$

This completes the proof of the lemma. □

By definition p, q, $s \geq 0$ and $r \leq 0$ and in what follows we define $x \geq y \geq 0$. If q or s is zero, then it must be that $p = q = r = s = 0$ and the layout constructed in the first stage is optimal. If $x = 0$, then since $x \geq y \geq 0$ we know that the layout constructed in the second stage is optimal. If both of these layouts are optimal, then the approximation bound holds. If only one of the layouts is optimal, then a proof similar to the one in Lemma 13 can be used to show that the approximation bound is at most two. The remaining case is when x, p, q, $s > 0$ and $r \leq 0$. From Theorem 2 we know that $((p+q) \cdot (r+s))/qs \leq 2$. The assumptions that $p + q \leq s$ and $2p \leq s$ can be easily verified in each of the four cases in the proof of Theorem 2.[‡] In the proof of Theorem 3 we define x and y in such a way that $x \geq y \geq 0$. Therefore, all the assumptions in the statement of Lemma 13 hold.

Let D be an optimal assignment for $L \cup V \cup H$ and let $D(MV)$ be as defined before. Let assignment I be one of the preliminary assignments constructed by our algorithm that corresponds to $D(MV)$ (i.e., $H_{D(MV)}(TL) = H_I(TL)$). Let s be the number of wires (from the nets in H) crossing the left side of T and let t be the number of wires (from the nets in H) crossing the right side of T in assignment I. Clearly, in assignment $D(MV)$ there are s wires (from the nets in H) crossing the left side of T and t wires (from the nets in H) crossing the right side of T. We use the function $sp(s,t)$ to denote the maximum increase of $H_{D(MV)}(TMB)$ when we transform assignment MV so that the $s+t$ wires identified above are routed as in assignment I. Clearly, the maximum number of pair of wires that need to be interchanged is $g \leq \min\{s,t\}$. By construction, each of these g pair of wires do not cross on the top side of T in assignment I, but they cross on assignment $D(MV)$. Therefore, each time that we interchange a pair of these wires we increase the vertical height of the assignment by at most two. Hence, the maximum increase of $H_{D(MV)}(TMB)$ is at most $s+t$. We say that when we transform an assignment, $D(MV)$, with s wires that cross the left side of rectangle B (from the nets in H) and t wires that cross the right side of rectangle B (from the nets in H), to assignment I with u wires crossing the left side of rectangle B (from the nets in H) and v wires crossing the right side of rectangle B (from the nets in H), the maximum increase of $H_{D(MV)}(TMB)$ is given by the function

[‡]Note that $p \leq q$ in all cases in the proof of Theorem 2. This together with the fact that $p+q \leq s$ is enough to prove that $2p \leq s$. We did not used this approach because when we reapply these arguments in Theorem 6, the bound $p \leq q$ does not hold.

$$2\max\{\min\{t,u\},\min\{v,s\}\}\ .$$

The justification for this formula is a straight forward generalization of the previous case.

Theorem 3. *For the $2_C - R2M$ problem, let D be an optimal assignment such that $ML \subseteq D$ and let MH be the assignment generated by our algorithm. Then, $A(MH) \leq 2A(D)$.*

Proof. Let I be the assignment with $H_I(TL) = H_{D(MV)}(TL)$. In what follows we show that $A(I) \leq 2A(D)$. Since $A(MH) \leq A(I)$, we know that $A(MH) \leq 2A(D)$. Assume without loss of generality that $H_{MV}(TL) + H_{MV}(TR) + a \geq H_{MV}(BL) + H_{MV}(BR) + b$, and b is even. The proof for the other cases is similar. Clearly, each net in H is routed in D by a wire that crosses at least two corners. Therefore, $h'' \geq a+b$. This bound together with the fact that each net in H has at least two pins on a horizontal size of T and/or B, we know that $w'' \geq a+b$. Also, since

$$\begin{aligned}H_I(LL) + H_I(RR) = H_I(TL) + H_I(TR) &= H_{D(MV)}(TL) + H_{D(MV)}(TR)\\ &\leq H_{D(MV)}(LL) + H_{D(MV)}(RR)\ ,\end{aligned}$$

we know that $\Delta w'' \leq 0$. In what follows we establish a bound on $\Delta h''$ and, when needed, a sharper bound for $\Delta w''$. There are four cases depending on the values of $H_{MV}(TL)$, $H_{MV}(TR)$, $H_{MV}(BL)$, and $H_{MV}(BR)$.

Case 1: $H_{MV}(LL) = H_{MV}(TL)$ and $H_{MV}(RR) = H_{MV}(TR)$.

There are three subcases depending on the values of a, b, $H_{MV}(TL)$, $H_{MV}(TR)$, $H_{MV}(BL)$, and $H_{MV}(BR)$ (see Fig. 9).

Subcase 1.1: Assignment I is of the form given by Fig. 9(a).

The optimal assignment D after transforming it to $D(MV)$ is given by Fig. 10. From the above discussion we know that transforming assignment $D(MV)$ to assignment I increases $H_{D(MV)}(TMB)$ by at most

$$\Delta h'' \leq sp(a_1,a_2) + 2\max\{\min\{b-b',b/2\},\min\{b/2,b'\}\} \leq a+b\ .$$

From above, we know that $\Delta w'' \leq 0$, $w'' \geq a+b$ and $h'' \geq a+b$. Setting $x = a+b$, and $y = 0$, and applying Lemma 13, we know that $A(I) \leq 2A(D)$ for this subcase.

Subcase 1.2: Assignment I is of the form given by Fig. 9(b).

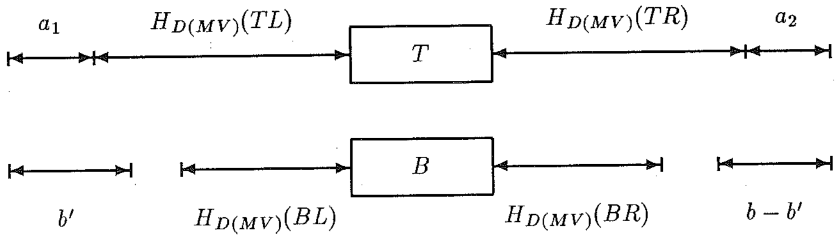


Fig. 10. Assignment $D(MV)$ for subcase 1.1.

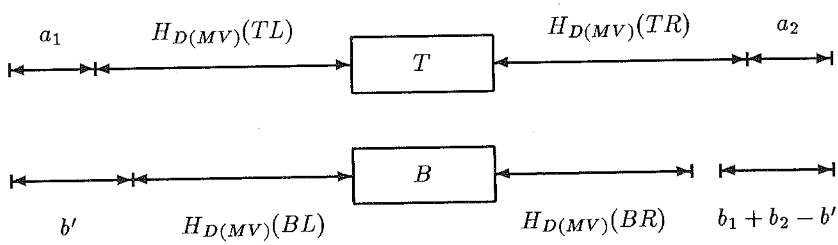


Fig. 11. Assignment $D(MV)$ for subcase 1.2.

The optimal assignment D after transforming it to $D(MV)$ is given by Fig. 11. From the above discussion we know that transforming assignment $D(MV)$ to assignment I increases $H_{D(MV)}(TMB)$ by at most

$$\Delta h'' \leq sp(a_1, a_2) + 2\max\{\min\{b', b_1 + b_2/2\}, \min\{b_1 + b_2 - b', b_2/2\}\} .$$

There are three subcases depending on the value for b'.

Subcase 1.2.1: $b' \leq b_2/2$.

Clearly, $\Delta h'' \leq a + b_2$ and $\Delta w'' \leq 0$. The proof now proceeds as the one for subcase 1.1.

Subcase 1.2.2: $b_2/2 \leq b' \leq b_1 + b_2/2$.

Let $z = b' - b_2/2$. Clearly, $z \leq b_1$, $\Delta h'' \leq a + b_2 + 2z$ and since

$$b' + H_{D(MV)}(BL) - a_1 - H_{D(MV)}(TL) = z$$

(remember that $b_2/2 + H_I(BL) = a_1 + H_I(TL)$), we know that $\Delta w'' \leq -z$. From above we know that $w'' \geq a + b \geq a + b_2 + z$, and $h'' \geq a + b \geq a + b_2 + z$. Setting $x = a + b_2 + z$, and $y = z$, and applying Lemma 13, we know that $A(I) \leq 2A(D)$ for this subcase.

Subcase 1.2.3: $b' \geq b_1 + b_2/2$.

Clearly $\Delta h'' \leq a+b_2+2b_1$ and since $b'+H_{D(MV)}(BL)-a_1-H_{D(MV)}(TL) \geq b_1$, we know that $\Delta w'' \leq -b_1$. From above we know that $w'' \geq a+b \geq a+b_2+b_1$, and $h'' \geq a+b \geq a+b_2+b_1$. Setting $x = a+b_2+b_1$, and $y = b_1$, and applying Lemma 13, we know that $A(I) \leq 2A(D)$ for this subcase.

Subcase 1.3: Assignment I is of the form given by Fig. 9(c).

The proof for this case is omitted since it is similar to the one for subcase 1.2.

The proof for remaining cases is omitted since it is similar to the one for case 1. This completes the proof of the lemma. □

The main problem with the above algorithm is that it is unlikely that it can be implemented to take $O(n)$ time. The reason is that one could generate $\Omega(n)$ layouts each requiring $\Omega(n)$ time. However, an $O(n)$ algorithm that generates solutions within a factor of two of the optimal solution exists. The problem with this new algorithm is that the proof that it generates solutions within two of optimal involves many cases. For brevity we do not include the algorithm nor its proof, however, we discuss the basic idea behind it. An interested reader can derive the algorithm and the proof for the approximation bound from our description. Let us assume that $H_{MV}(TL) + H_{MV}(TR) + a \geq H_{MV}(BL) + H_{MV}(BR) + b$ and that b is even. We also assume that "a" is even (note that when this is not the case, more layouts need to be constructed). Instead of generating "a" layouts, we only generate a constant number of layouts and then select the best of these layouts as our solution. First we set $tl = tr = a/2$. Let us suppose case 1 in procedure FIND holds. We use case 1 of procedure FIND to generate one of the three layouts (see Figs. 9(a)–(c)). If we generate the layout in Fig. 8(a), no other layout needs to be constructed. Let us now consider the case in Fig. 9(b) (the case in Fig. 9(c) is treated similarly to the case in Fig. 9(b)). In this case we generate another layout (see Fig. 12).

The proof for the approximation bound of two is similar to the one in Theorem 3. The main difference is that we need to compare the optimal area layout against one of the two layouts generated. In the proof of Theorem 3 when b' is small, we compare it against the assignment given in Fig. 12(a), but when b' is large it gets compared against the assignments given in Figs. 12(b) or (c). We state the following theorem without proving it.

Theorem 4. *For the $2_C - R2M$ problem, let D be an optimal assignment such that $ML \subseteq D$ and let MH be the assignment generated by our algorithm that generates only a constant number of intermediate rectangles. Then, $A(MH) \leq 2A(D)$.*

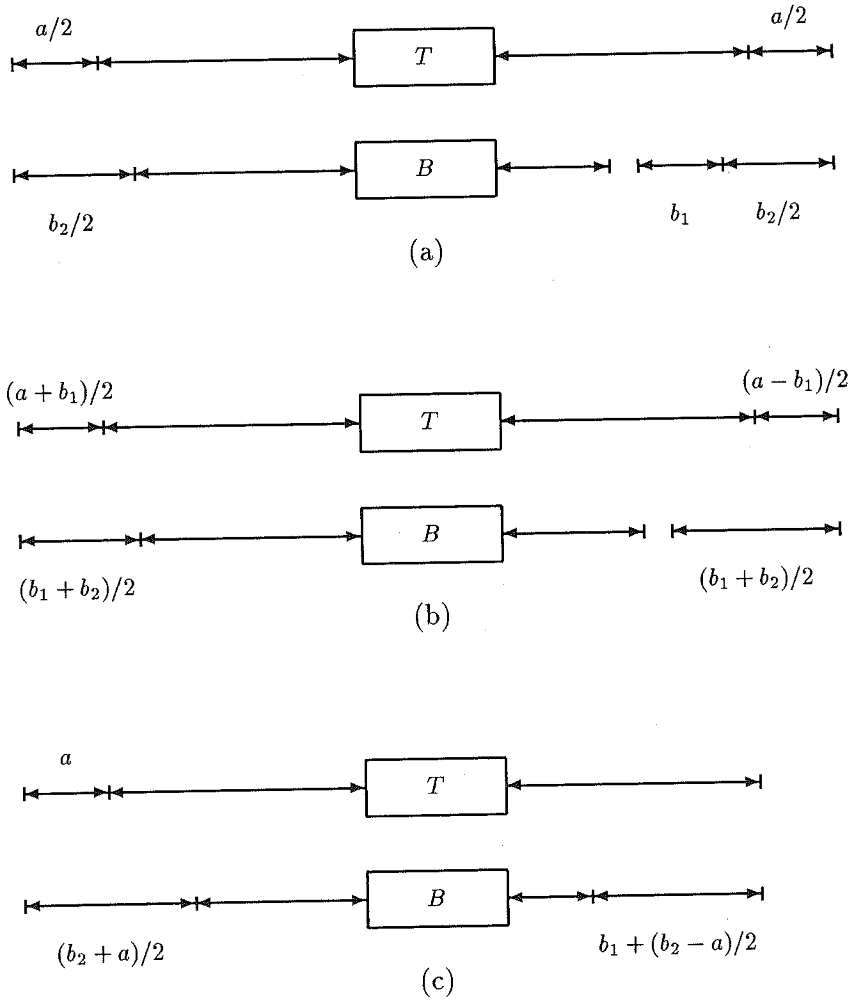


Fig. 12. (a) Assignment Fig. 9(b), (b) assignment constructed when $a \geq b_1$, and (c) assignment constructed when $a < b_1$.

5. Approximation Algorithm for the $2-R2M$ Problem

In this section we show that our algorithm takes $O(n)$ time and generates a layout with area at most $2\,OPT$, where OPT is the area of an optimal layout.

algorithm for the $2 - R2M$ problem
Construct assignments ML, MS, and MH
(the one just before the proof of Theorem 4);
Construct and output a layout with area $A(MH)$ for MH.
end of algorithm

Theorem 5. *The time complexity of our algorithm is $O(n)$.*

Proof. The assignment constructed in Sec. 4.1 can be obtained in $O(n)$ time by a simple recursive procedure that manipulates two priority queues and uses a simple marking scheme.[12] It is simple to verify that the remaining part of the assignment and the final layout can be constructed in $O(n)$ time. □

Theorem 6. *Let D be an optimal assignment such that $ML \subseteq D$ and let Q be the layout generated by our algorithm. Then, $A(Q) \leq 2A(D)$.*

Proof. Arguments similar to those used in Theorems 1 and 4 can be used to prove this theorem. The main difference is that one needs to justify the assumptions in Lemma 13 for this more general case (i.e., arguments similar to those that follow Lemma 13). □

6. Discussion

We have presented an efficient approximation algorithm that generates a layout with area within a factor of two of the area in an optimal layout for the $2 - R2M$ problem. The algorithm takes $O(n \log n)$ ($O(n)$ if the set of terminals is initially sorted) time and the constant associated with this bound is small. It is possible to obtain a better solution (not a necessarily a better approximation bound) by using the optimal algorithms for $R1M$ problem to route the set of nets in C. Our results also hold when the rectangles are placed side by side, and both rectangles have equal height. For brevity this other problem is not discussed in detail. The worst case scenario for our approximation algorithm does not arise all of the time. We suspect that most of the time our solutions are near optimal. When implementing the algorithm it is simple to compute our lower bound for the area of an optimal area layout. This gives a good estimate of how close from optimal is the assignment generated by the algorithm.

In Sec. I we made the middle-channel assumption. We can remove this assumption at the expense of an additional layer when under the knock-knee wiring model. In this case the routing in the middle channel is performed by a well known algorithm.[3] Before one can approximate efficiently the two-layer Manhattan mode $2 - R2M$ problem, we need an efficient approximation algorithm for approximating the area of the channel routing problem under the Manhattan model. It is not clear whether or not such algorithm exits.

With respect to the multiterminal net case ($R2M$ problem) we do not have an efficient approximation algorithm for it. The main problem is finding an approximation algorithm to approximate the area of the channel routing problem. A good heuristic for this case can be based on Gonzalez and Lee's algorithm.[11] Another intersting open problem is when the rectangles have different widths and can slide horizontally.

Acknowledgements

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Appendix A

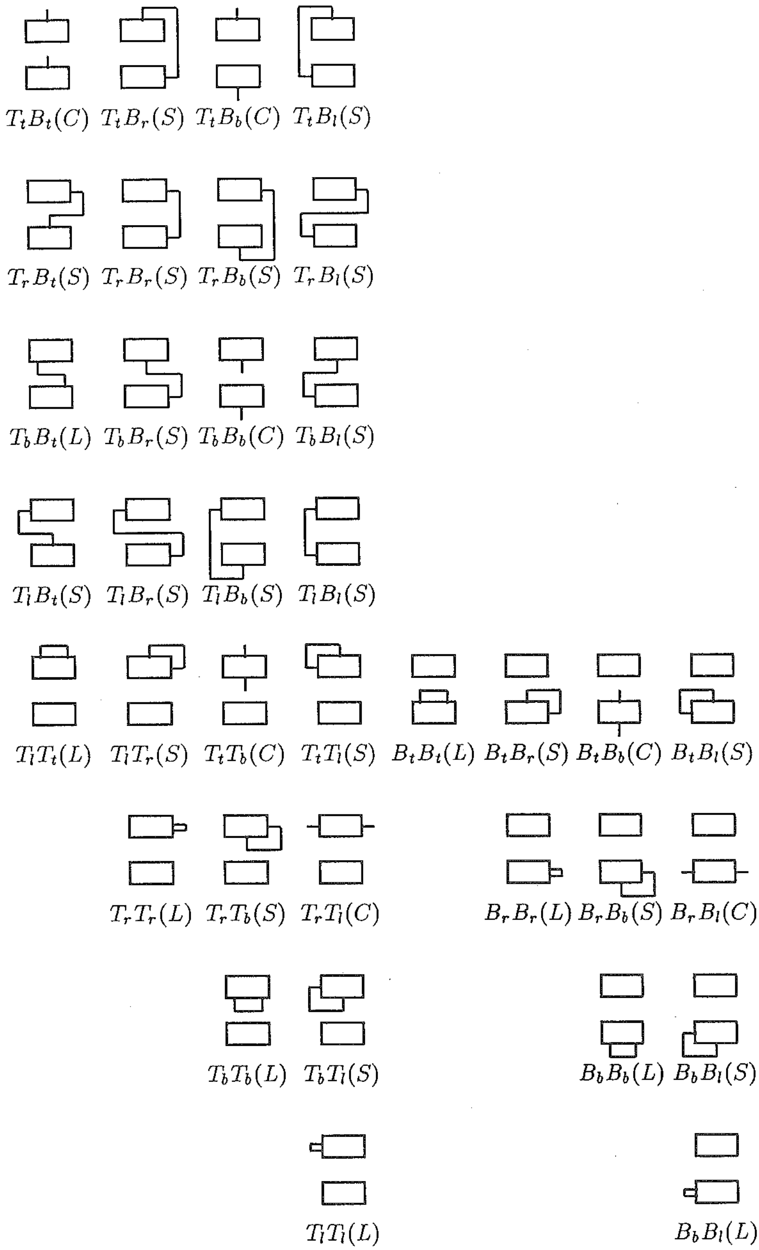

Fig. A.1. Types of nets.