AN APPROXIMATION ALGORITHM FOR ROUTING TWO -TERMINAL NETS AROUND TWO RECTANGLES

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ABSTRACT

The problem of connecting a set of 2m terminals that lie on the sides of two equal-width rectangles to minimize the total area is discussed. We present an O(m log m) approximation algorithm to solve this problem. Our algorithm generates a solution with area $\leq 2 * OPT$, where OPT is the area of an optimal solution. The nets are routed according to the following greedy strategy: the wire connecting the two points in a net is one whose path crosses the least number of corners of the rectangles. For some nets there is more than one path that crosses the least number of corners of two rectangles. These nets are connected by wires whose paths blend with the paths for other nets.

I. INTRODUCTION.

Let T (top) and B (bottom) be two rectangles with equal width (possibly different heights) and placed on the same plane with the same orientation. The left side of T and B have been placed along the same vertical line and rectangle T is above retangle B. The distance between these two rectangles is not fixed and will be decided by our routing algorithm. This distance must be at least λ units. Let S be a set of 2m terminals that lie on the sides of T and B, and let N_1 , N_2 , ..., N_m be any partition of set S such that the size of each subset is exactly two. Each subset N_i is called a net. All the points in each net have to be made electrically common by interconnecting them with wires. The wires follow a path consisting of a finite number of horizontal and vertical line segments. These segments are assigned to two different layers. All the horizontal segments are assigned to one layer and all the vertical segments are assigned to the other layer. Line segments on different layers can be connected directly at any given point z by a wire perpendicular to the layers if both line segments cross point z in their respective layers. Every pair of distinct and parallel line segments must be at least λ units apart and every line segment must be at least λ units from each side of rectangles T and B, except in the region where the path joins a point in S to connect it. Also no path is allowed inside of T and B on any of the layers. We are assuming that no two terminals in the middle channel (the region between the bottom side of rectangle T and the top side of rectangle B) are placed along the same vertical line. This assumption, which is not too restrictive, will simplify our routing algorithm. Later on we explain how this assumption can be eliminated.

Problem 2-R2M (routing two terminal nets around two rectangles) consists of specifying the paths for all the wires in such a way that the total area is minimized. That is, to place T and B together with all the wires (that must satisfy the restrictions imposed above) inside a rectangle (with the same orientation as T) of least possible area. This problem has applications in the layout of integrated circuits [11].

The R1M problem is defined similarly, except that all terminals are located on the sides of one rectangles and the size of each net can be arbitrary. Hashimoto and Stevens [8] present an O(m log m) algorithm to solve the R1M problem for the case when all the points in S lie on one side of a rectangle. An Ω(m log m) lower bound to solve this problem was established in [7]. Algorithms to solve the R1M problem for the case that all nets are restricted to be of size two (called 2-R1M problem) appear in [9] and [:]. The one in [3] is optimal with respect to the time complexity bound. Several approximation algorithms for the R1M problem have been given by Gonzalez and Lee ([4] , [5] and [6]). The time complexity of all these algorithms is O (m*(n + log m)) and the best of these algorithms generates a solution with area $\leq$ 1.6 * OPT, where OPT is the area of an optimal solution, m is the number of terminals and n is the number of nets. If more than two layers are allowed and wire overlap is permitted, the R1M problem becomes an NP-hard problem [14], even when the size of all nets is two. Baker [1] presents an O(m log m) algorithm for the 2-R2M. This algorithm generates a solution within 1.9 of optimal. For this case the optimality is measured with respect to the perimeter of the smallest enclosing rectangle. In this paper we present an approximation algorithm for the 2-R2M problem that generates solutions within a factor of 2 of the optimal solution. Our algorithm is simpler than the one appears in [1]. Our lower bound is obtained by similar technique. However, our analysis is much simpler than the one appears in [1]. One of the reason is our assumption for the terminals located in the middle channel.

Let 2_S-*R2M* (2_G-*R2M*) denote a 2-R2M problem in which for each global net there exists exactly one (more than one) connecting path that crosses the least number of corners. In Section II we introduce our notation and present some basic results. In order to simplify the exposition of our results, we begin by presenting approximation algorithms for restricted versions of the 2-R2M problem. In section III we present an approximation algorithm for the 2_S-R2M problem. An approximation algorithm for the 2_G-R2M is presented in the section IV and in section V we indicate how to combine these results to obtain our approximation algorithm.

II. NOTATION AND BASIC RESULTS.

We begin by defining the 2-R2M problem and introducing notation similar to the one in [2]. Let T and B be two rectangular components located on the same plane with size h_T by w_T (height by width) and h_B by w_B, respectively. We assume that $w_T = w_B$. Rectangle T is above rectangle B and the distance between these two rectangles is not fixed, but must be at least λ units. The left side of T and B have been placed along the same vertical line. Since T and B have the same width, the right side of T and the right side of B are also located along the same vertical line. There are 2m terminals (T_1 , T_2 , ... , T_{2m}) on the sides of T and B. The set of terminals is partitioned into m subsets denoted by N_1 , N_2 , ... , N_m and each subset contains exactly two terminals. Each subset N_i is called a *net*. It is assumed that every pair of terminals is at least λ units apart and every terminal is located at least λ units from each of the corners of T and B. All the terminals in each net must be made electrically common by connecting them with wires. The path followed by these wires consists of a finite number of horizontal and vertical line segments. Each of these line segments must lie on the same plane as T and B, be parallel to a side of T and B, and be on the outside of T and B. Perpendicular line segments can intersect at any point, but parallel line segments must be at least λ units apart. Also, all line segments must be at least λ units away from every side of rectangles T and B except in the vicinity where a line segment connects a terminal. The 2-R2M problem consists of specifying paths for all the interconnections subject to the rule mentioned above in such a way that the total area is minimized, i.e., place the components T and B together with all the wires inside a rectangle (with the same orientation as T and B) of least possible area.

Label the sides of the components (in the obvious way) left, top, right and bottom. Starting in the bottom-left corner of T, traverse the sides of rectangle T clockwise. The ith corner visited is labeled S_{i-1} and the ith terminal visited is terminal T_i. Assume that terminal T_{m_1} is the last terminal visited by this procedure. Using a similar procedure traverse the sides of rectangle B, the ith corner of B visited is labeled R_{i-1} and the ith terminal (that lies on the sides of B) visited is terminal T_{m_1+i}. The *close interval [x ,y]* consists of all the points on the sides of rectangle X ,Y $\in$ {T , B} that are visited while traversing the sides of rectangle X in the clockwise direction starting at point x and ending at point y. Note that the interval [x , x] consists of a single point. Parentheses are used instead of brackets for open intervals. We use [S_0 , S_1], [S_1 , S_2], [S_2 , S_3] and [S_3 , S_0] ([R_0 , R_1], [R_1 , R_2], [R_2 , R_3] and [R_3 , R_0]); T^l, T^t, T^r and T^b; (B^l, B^t, B^r and B^b) to represent the left, top, right and bottom sides of T (B), respectively. The function C(j) indicates the index of the terminal to which terminal T_j has to be connected.

In order to simplify our notation we introduce the following lemma which can be proved by simple interchange arguments.

Lemma 2.1: Let W be any layout for an instance of the 2-R2M problem. Let T_q be any terminal located on the bottom (top) side of T (B). If T_q and $T_{C(q)}$ are located on different rectangles and T_q is connected to $T_{C(q)}$ by a path that crosses the top (bottom) side of rectangle T (B), then there exists another layout W' such that T_q is connected to $T_{C(q)}$ by a path that does not cross top (bottom) side of rectangle T (B), and the area of W' is not larger than the area of W.
Proof: For brevity the proof is omitted. □

By lemma 2.1 whenever we refer to an optimal layout, we may assume it cannot be transformed by applying the interchange argument given in lemma 2.1. In the layout that our algorithm generates we cannot apply the transformation implied by lemma 2.1. Note that this assumption will not make the problem easier, it will just allow us to use a simpler notation.

Set D = { $(d_1,x(a_1),y(a_1))$, $(d_2,x(a_2),y(a_2))$, ... , $(d_m,x(a_m),y(a_m))$ }, where $1 \leq a_i \leq 2m$ and the functions x and y are defined from {1,2,...,2m} to {C,R}, is said to be an *assignment* if $|\{ a_1,C(a_1), a_2,C(a_2), \ldots , a_m,C(a_m) \}| = 2m$. Any subset of an assignment is said to be a *partial assignment*. An assignment D indicates the direction of the path connecting the two points in each net. If $(i,x(i),y(i)) \in D$, then the path connecting this net is as follows. If T_i and $T_{C(i)}$ are located on the same rectangle then $x(i) \neq y(i)$ and the path connecting this nets starts at terminal T_i ($T_{C(i)}$) if $x(i) = C$ ($x(i) \neq C$) moving perpendicular to the side that contains this terminal and then it continues in the clockwise direction (with respect to the rectangle) until it reaches point $T_{C(i)}$ (T_i). This second terminal is joined to this path by a line segment perpendicular to the side where it is located. On the other hand if T_i and $T_{C(i)}$ are located on different rectangles, assume that T_i is located on rectangle T, the path connecting this net consists of three segments. The first segment starts at terminal T_i moving perpendicular to the side that contains this terminal and then it continues in the clockwise direction, if $x(i) = C$ otherwise counterclockwise direction, until it reaches either the point to be connected , or the bottom-left or bottom-right corner of rectangle T. The second segment starts at terminal $T_{C(i)}$ moving perpendicular to the side which contains this terminal and then it continues in the clockwise direction, if $y(i) = C$ otherwise counterclockwise direction, until it reaches either the the point to be connected, or the top-left or top-right corner of rectangle B. The third segment will join the end points of the previous two segments. Note that this segment could be a horizontal line segment through the middle channel. Note that a path that connects a net which could be changed by applying the transformation implied by lemma 2.1 cannot be represented by an assignment. Also, it is simple to see that doglegs can not be represented by an assignment.

For any (k, x(k) , y(k)) $\in$ D, we say that the path connecting T_k , $T_{C(k)}$ given by D *crosses point z* (z is on the sides of T and B but not a corner point) if the connecting path intersects a line segment perpendicular to the side where z is located that starts at point z and ends at the point where it intersects the rectangle where z is not located (note that if it never intersects the rectangle, it is a half line). Introduce all horizontal and vertical half lines that intersect exactly one corner point. Introduce all vertical line segments that intersect exactly one corner of each of the rectangles. It is simple to verify that the above procedures introduce exactly two line segments (one vertical and one horizontal) that intersect each of the corner points of the rectangles. We called these segments extension lines. A connecting path *vertically (horizontally) crosses corner z* if the connecting path crosses the horizontal (vertical) extension associated with corner z.

For any assignment (or partial assignment) D we define the *height function* H_D for [x , y] representing a side in one of the rectangles. as follows:

$H_D(x , y) = \max\{$ number of the paths given by D that cross point z $| z \in (x,y)\}$

Similarly, for any assignment D we define height function VE_D and HO_D for corner point z:

$ve_D(z) =$ number of the paths given by D that vertically cross point z.

$ho_D(z) =$ number of the paths given by D that horizontally cross point z.

The *height of assignment D for side X of rectangle T (B) ($X \in \{top , left , right, bottom\}$)* refers to the value of $H_D(x , y)$, where [x , y] is the interval that represents side X of rectangle T (B). For an assignment D, we define $H_D(R_0 , S_1)$ and $H_D(S_2 , R_3)$ as follows:

$$H_D(R_0 , S_1) = \max(H_D(S_0 , S_1) , H_D(R_0 , R_1)), \text{ and}$$
$$H_D(S_2 , R_3) = \max(H_D(S_2 , S_3) , H_D(R_2 , R_3)).$$

The next two lemmas establish that the 2-R2M problem reduces to the problem of finding an assignment D with least

$$(h_T+h_B + (H_D(S_1 , S_2) + H_D(R_1 , R_2) + H_D(R_3 , R_0)) * \lambda) * (w_T+ H_D(R_0 , S_1) + H_D(S_2 , R_3)) * \lambda)$$

and then in O(n log n) time one can construct a layout of area h_Q by w_Q for it.

Lemma 2.2: For every assignment D, there is a rectangle Q of size h_Q by w_Q, where

$$h_Q = h_T + h_B + (H_D(S_1 , S_2) + H_D(R_1 , R_2) + H_D(R_3 , R_0)) * \lambda, \text{ and}$$
$$w_Q = w_T+(H_D(R_0 , S_1) + H_D(S_2 , R_3)) * \lambda.$$

with the property that rectangle T and B together with the interconnecting paths defined by assignment D can be made to fit inside Q.

Proof: The proof is a direct generalization of the proof for the R1M problem that appears in [10]. One additional vertical track might be required when there is no nets with exactly one terminal located in the middle channel [13]. □

Lemma 2.3: For any assignment D a layout with the area given by lemma 2.2 can be obtained in O(n log n) time.

Proof: The proof of this lemma is a straight forward generalization of the proof for the R1M problem that appears in [10]. The algorithm that constructs the final layout uses as a subalgorithm the procedure given in [7], [8] and [13]. □

Net N_i is said to be a *local net* if its two terminals are located on the same side of one rectangle or if one terminal is located on the bottom side of T and the other is located on the top side of B. Otherwise, net N_i is said to be *global*. Let $X^j Y^k$ represent the set of nets in which one of its terminals is located on side j of rectangle X and the other is located on side k of rectangle Y, where X,Y $\in$ {T , B} and j , k $\in$ {l , t, r, b}. Let $|X^j Y^k|$ represent the number of nets in this set. For an assignment D we define the *function A(D)* as

$$(h_T+h_B + (H_D(S_1, S_2) + H_D(R_1, R_2) + H_D(R_3, R_0)) * \lambda) * (w_T+ H_D(R_0, S_1) + H_D(S_2, R_3)) * \lambda).$$

i.e., the total area required for a layout of T and B and all interconnections given by D.

Definition 2.1: D'

Let D' be the partial assignment in which all the local nets are connected by paths that do not cross any of the corners of T and B.

Lemma 2.4: Every assignment D can be transformed to an assignment M such that D' $\subseteq$ M and A(M) $\leq$ A(D).
Proof: The proof follows the same lines as the one for the 2-R1M problem that appears in [10]. □

It should be clear that it is only required to specify the paths connecting all the global nets since we know that the local nets can be routed optimally by routing them as indicated in assignment D'. Also, once we have an assignment the proof of lemma 2.3 (a constructive proof) can be used to find a layout of optimal area for it.

III. APPROXIMATION ALGORITHM FOR THE 2_S-R2M PROBLEM.

In this section we present our approximation algorithm for the 2_S-R2M problem. Let M_S be the set of global nets for which the number of distinct connecting paths that cross the least number of corners is exactly one. Clearly,

$$M_S = \{ T^fT^g , B^fB^g \mid f,g \text{ are adjacent sides on the same rectangle}\} \cup \{T^jB^k , T^kB^j \mid j \in \{l , t, b, r\} \text{and } k \in \{l , r\} \}.$$

For the set of nets M_S, we construct assignment D_S as follows:

D_S = { all nets in M_S are connected by paths that cross the least number of corners of T and B}.

Lemma 3.1: Let D be an optimal assignment such that D' $\subseteq$ D. Let M be D except that all nets in T^fT^g and B^fB^g, where f and g are adjacent sides on the same rectangles, are assigned as in our algorithm. Then

$$H_M(S_1,S_2)+H_M(R_1,R_2)+H_M(R_3,R_0) \leq H_D(S_1,S_2)+H_D(R_1,R_2)+H_D(R_3,R_0), \text{ and}$$

$$H_M(R_0,S_1)+H_M(S_2,R_3) \leq H_D(R_0,S_1)+H_D(S_2,R_3)+ \max(\sum_{\substack{f,g \\ \text{adjacent}}} | Y^fY^g |, \sum_{\substack{f,g \\ \text{adjacent}}} (| Z^fZ^g |).$$

where Y^fY^g (Z^fZ^g) is the subset of nets in T^fT^g (B^fB^g) that are connected differently in D and M.
Proof: For brevity the proof is omitted. □

set	contribution to our lower bound for $w_T/(\lambda)$ + $H_D(R_0,S_1)+H_D(S_2,R_3)$	contribution to our lower bound for $(h_T+h_B)/(\lambda)$ + $H_D(S_1,S_2)+H_D(R_1,R_2)+H_D(R_3,R_0)$	
T^fT^f	$0.5*(\|T^fT^g\|+\|Y^fY^g\|)$	$\|T^fT^g\|+\|Y^fY^g\|$	f,g adjacent
B^fB^f	$0.5*(\|B^fB^g\|+\|Z^fZ^g\|)$	$\|B^fB^g\|+\|Z^fZ^g\|$	f,g adjacent
T^jB^k	$0.5*(\|T^jB^k\|+\|Y^jZ^k\|)$	$2*\|T^jB^k\|+\|Y^jZ^k\|$	$j,k \in\{l,r\}$
T^tB^k	$\|T^tB^k\|$	$2*\|T^tB^k\|$	$k\in\{l,r\}$
T^bB^k	$0.5*(\|T^bB^k\|+\|Y^bZ^k\|)$	$\|T^tB^k\|+\|Y^bZ^k\|$	$k\in\{l,r\}$
T^kB^t	$0.5*(\|T^kB^t\|+\|Y^kZ^t\|)$	$\|T^kB^t\|+\|Y^kZ^t\|$	$k\in\{l,r\}$
T^kB^b	$\|T^kB^b\|$	$2*\|T^kB^b\|$	$k\in\{l,r\}$

Table 3.1: Lower bounds for the 2_S-R2M problem.

Lemma 3.2: Let D be an optimal assignment such that D' $\subseteq$ D. Let M be D except that all nets in $T^jB^k \cup T^kB^j$, where $j \in \{l, t, b, r\}$, $k \in \{l, r\}$ are assigned as in our algorithm. Then

$$H_M(S_1,S_2)+H_M(R_1,R_2)+H_M(R_3,R_0) \leq H_D(S_1,S_2)+H_D(R_1,R_2)+H_D(R_3,R_0) \text{ , and}$$

$$H_M(R_0,S_1)+H_M(S_2,R_3) \leq H_D(R_0,S_1)+H_D(S_2,R_3)+ \sum_{\substack{j \in \{l,t,b,r\} \\ k \in \{l,r\}}} |Y^jZ^k| + \sum_{\substack{j \in \{t,b\} \\ k \in \{l,r\}}} |Y^kZ^j| .$$

where Y^jZ^k (Y^kZ^j) is the subset of T^jB^k (T^kB^j) that are connected differently in D and M.
Proof: For brevity the proof is omitted. □

Before proving our main result in this section we establish a lower bound on the area required by an optimal solution. Note that from lemma 3.1 and 3.2 we know that our algorithm generates a solution with minimum height for the 2_S-R2M problem. The lower bounds are given in table 3.1.

Lemma 3.3: Let D be an optimal assignment such that D' $\subseteq$ D. Assignment D and rectangles T and B satisfy the lower bound given in table 3.1, where Y^fY^g (Z^fZ^g) is the subset of nets in T^fT^g (B^fB^g) and Y^jZ^k (Y^kZ^j) is the subset of nets in T^jB^k (T^kB^j), that are connected differently in D and M.
Proof: For brevity the proof is omitted. □

Theorem 3.1: For the 2_S-R2M problem, let D be an optimal assignment such that D' $\subseteq$ D and let M be the assignment generated by our algorithm. Then $A(M) \leq 2 * A(D)$.
Proof: For brevity the proof is omitted. □

IV. APPROXIMATION ALGORITHM FOR THE 2_G-R2M PROBLEM.

In this section we present an approximation algorithm for the 2_G-R2M problem. Let M_G be the set of global nets for which the number of distinct connecting paths that cross the least number of corners is more than one. Clearly,

$$M_G = \{ T^fT^g , B^fB^g \mid f,g \text{ are two opposite sides of one rectangle}\} \cup \{T^tB^k , T^kB^b \mid k \in \{t , b\} \}.$$

In what follows we only explain how the assignment $D_G(T^tB^b)$ for all nets in T^tB^b is constructed, since all assignments $D_G(X^jY^k)$, where $X^jY^k \in \{ T^lT^r, B^lB^r, T^tT^b, B^tB^b, T^tB^t, T^bB^b, T^tB^b \}$, are constructed by similar procedures. For any permutation, π, of the nets in set T^tB^b we define an assignment ASG(T^tB^b , π) as follows: the first net in π is connected by a path that begins on side T^t , crosses the left side of rectangle T and B and ends at side B^b; and the path connecting the kth net ($1 \le k \le |T^tB^b|$) in π begins on side B^b (T^t), crosses the right (left) side of T and B and ends at side T^t (B^b), when k is even (odd). We claim that there is a permutation, π, of the nets in set T^tB^b such that there is a layout for assignment ASG(T^tB^b , π) with the property that for any k ($1 \le k \le |T^tB^b|$) the path connecting the kth net in π can share the same track on the side where the path connecting the (k-1)st net ends. In this case we say that π is a *valid permutation* for the set of nets T^tB^b.

Claim: There is a valid permutation for the set of nets X^jY^k, where $X^jY^k \in \{ T^lT^r, B^lB^r, T^tT^b, B^tB^b, T^tB^t, T^bB^b, T^tB^b \}$.
Proof: For brevity the proof is omitted. □

set	contribution to our lower bound for $w_T/(\lambda)$ + $H_D(R_0,S_1)+H_D(S_2,R_3)$	contribution to our lower bound for $(h_T+h_B)/(\lambda)$ + $H_D(S_1,S_2)+H_D(R_1,R_2)+H_D(R_3,R_0)$
$T^lT^r \cup B^lB^r$	$0.5*(\lvert T^lT^r\rvert+\lvert B^lB^r\rvert)$	$2 *(\lvert T^lT^r\rvert+\lvert B^lB^r\rvert)$
$T^tT^b \cup B^tB^b$	$(\lvert T^tT^b\rvert+\lvert B^tB^b\rvert)$	$(\lvert T^tT^b\rvert+\lvert B^tB^b\rvert)$
$T^tB^t \cup T^bB^b$	$(\lvert T^tT^b\rvert+\lvert B^tB^b\rvert)$	$(\lvert T^tT^t\rvert+\lvert B^bB^b\rvert)$
T^tB^b	$1.5 *\lvert T^tB^b\rvert$	$\lvert T^tB^b\rvert$

Table 4.1: Lower bounds for the 2_G-R2M problem.

Lemma 4.1: Let D be an optimal assignment such that D' $\subseteq$ D. Let M be D except that all nets in X^jY^k, where $X^jY^k \in \{ T^lT^r, B^lB^r, T^tT^b, B^tB^b, T^tB^t, T^bB^b, T^tB^b \}$, are assigned as in our algorithm. Then

$H_M(S_1,S_2)+H_M(R_1,R_2)+H_M(R_3,R_0) \le H_D(S_1,S_2)+H_D(R_1,R_2)+H_D(R_3,R_0) + |X^jY^k|$,
where $X^jY^k \in \{ T^tT^b, B^tB^b, T^tB^t, T^bB^b, T^tB^b \}$,

$H_M(S_0,S_1)+H_M(S_2,S_3) \le H_D(S_0,S_1)+H_D(S_2,S_3)+ x *|X^jY^k|$,
where $x = 1$ if $X^jY^k = T^lT^r$, $x = 0.5$ if $X^jY^k \in \{ T^tT^b, T^tB^b, T^tB^t \}$, and

$H_M(R_0,R_1)+H_M(R_2,R_3) \leq H_D(R_0,R_1)+H_D(R_2,R_3)+ x *|X^jY^k|$,
where $x = 1$ if $X^jY^k = B^lB^r$, $x = 0.5$ if $X^jY^k \in \{ B^tB^b, T^tB^b, T^bB^b \}$.

Proof: For brevity the proof is omitted. □

Before proving our main result in this section we establish a lower bound on the area required by an optimal solution. Our lower bound is given in table 4.1.

Lemma 4.2: Let D be an optimal assignment such that D' ⊆ D. Assignment D and rectangles T and B satisfy the lower bound given in table 4.1.
Proof: The proof for these bounds is similar to the one in lemma 3.3. □

Theorem 4.1: For the 2_G-R2M problem, let D be an optimal assignment such that D' ⊆ D and let M be the assignment generated by our algorithm. Then A(M) ≤ 2 * A(D).
Proof: For brevity the proof is omitted. □

V. APPROXIMATION ALGORITHM FOR THE 2-R2M PROBLEM.

In this section we show that our algorithm takes O(m log m) time and generates a solution with area ≤ 2 * OPT, where OPT is the area of an optimal solution.

algorithm for the 2-R2M problem
Construct assignment D';
Construct assignment D_S;
Construct assignment $D_G(T^lT^r)$,$D_G(B^lB^r)$,$D_G(T^tT^b)$,$D_G(B^tB^b)$,$D_G(T^tB^t)$,$D_G(T^bB^b)$,$D_G(T^tB^b)$.
Let P be the assignment generated by the above steps.
Construct and output a layout with area A(P) for P.

end of algorithm

Theorem 5.1: The time complexity of our algorithm is O(m log m).
Proof: For brevity the proof is omitted. □

Theorem 5.2: Let D be an optimal assignment such that D' ⊆ D and let P be the solution generated by our algorithm. Then, A(P) ≤ 2 * A(D).
Proof: For brevity the proof is omitted. □

VI. DISCUSSION.

We have shown that there is an efficient approximation algorithm that generates a solution within a factor of 2 of the optimal solution value for the 2-R2M problem. The algorithm takes O(m log m) time and the constant associated with this bound is small. In section I we assumed that no two terminals in the middle channel were placed along the same vertical line. If this is not the case, we may use four layers to route the set of nets T^bB^t by applying the optimal algorithm given in [12]. This procedure will not increase the time complexity of our algorithm. It is possible to obtain a better solution (not a better approximation bound) by using an optimal algorithm for the R1M problem to route the set of nets in M_G. However, for brevity this method will not be discussed.

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