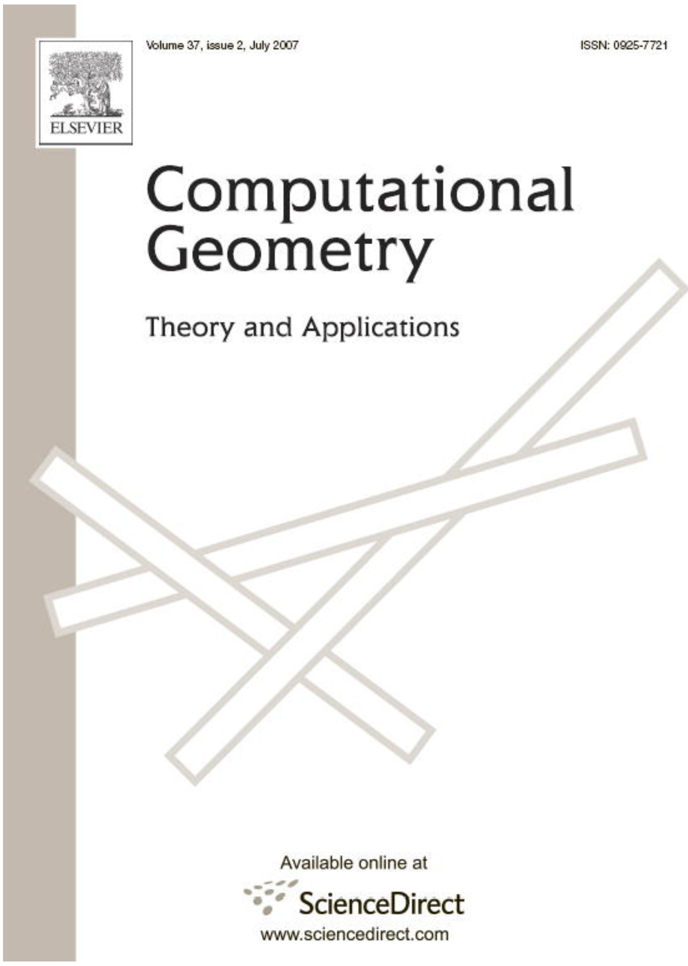
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# Complexity of the minimum-length corridor problem

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**Abstract**

We study the Minimum-Length Corridor (MLC) problem. Given a rectangular boundary partitioned into rectilinear polygons, the objective is to find a *corridor* of least total length. A corridor is a set of line segments each of which must lie along the line segments that form the rectangular boundary and/or the boundary of the rectilinear polygons. The corridor is a tree, and must include at least one point from the rectangular boundary and at least one point from the boundary of each of the rectilinear polygons. We establish the NP-completeness of the decision version of the MLC problem even when it is restricted to a rectangular boundary partitioned into rectangles.


*Keywords:* NP-completeness; Minimum-length corridor; Minimum wire length; Minimum optical fiber length



## 1. Introduction

In this paper we study the Minimum-Length Corridor (MLC) problem and some of its variants. The MLC problem is stated as follows. Given a pair $(F, P)$ where $F$ is a rectangular[2] boundary partitioned into the set $P$ of rectilinear polygons $P_1, P_2, \ldots, P_r$, find a set $S$ of line segments each of which lies along the line segments that form the rectangular boundary $F$ and/or the boundary of the rectilinear polygons. The line segments in $S$ form a tree, and include at least one point from the rectangular boundary $F$ and at least one point from the boundary of each of the rectilinear polygons. The sum of the length of the line segments in $S$ is called the *edge-length* or simply the *length* of $S$, and is denoted by $L(S)$. The objective of the MLC problem is to construct a minimum edge-length set of line segments $S$ with the above properties.

One may view the pair $(F, P)$ as a floorplan with $r$ rooms, and the set $S$ of line segments as a *corridor* connecting the rooms. A corridor is called a *partial corridor* if there is at least one room that is not reached by the corridor, i.e., at least one room is not exposed to the corridor. Fig. 1 shows a problem instance and two possible corridors are

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[2] Throughout this paper we assume that all the rectangles and rectilinear polygons have their boundaries orthogonal to the $x$ or $y$ axis, i.e., they are all orthogonal rectangles or orthogonal rectilinear polygons.

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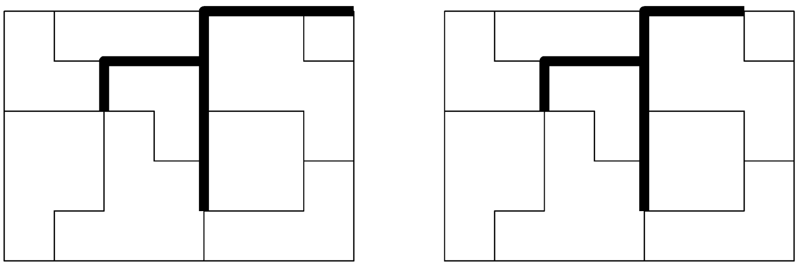


(a) Corridor (b) Optimal corridor

Fig. 1. An instance of the MLC problem.

represented by thick lines. An *access point* of a corridor (or partial corridor) is any point shared with the rectangular boundary $F$. Any corridor may be used to connect all the rooms to the outside of $F$ through an access point.

This problem and its variants have applications when laying optical fiber for data communication or wires for electrical connection in floorplans. In these applications, the rectangular boundary $F$ corresponds to the floorplan; the partition $P$ of polygons corresponds to the configuration of the floorplan; each rectilinear polygon corresponds to an individual room in the floorplan; and a corridor for an MLC problem instance corresponds to the placement of the optical fiber or wires that provides data communication or power to all the rooms in the floorplan. In all our applications we are interested in a minimum edge-length corridor which corresponds to the minimum length optical fiber or wire needed to provide connectivity. Other applications with different objective functions include the laying of water, sewer, and electrical lines on parcels in housing developments, and laying of wires for power or signal communication in circuit layout design.

For the MLC problem instance in Fig. 1, the corridor given by the thick lines in Fig. 1(a) is not an optimal corridor simply because one can obtain a shorter corridor by deleting part of the top line segment. However, the corridor in Fig. 1(b) is an optimal one. A restricted version of the MLC problem is when all the rooms are rectangles. This problem is called the MLC-R problem. In this paper we show that even the decision version of the MLC-R problem is NP-complete.

The MLC problem was initially defined by Naoki Katoh [3] and subsequently Eppstein [4] introduced the MLC-R problem. Experimental evaluations of several heuristics for the MLC problem are discussed in [8]. The question as to whether or not the decision version of these problems are NP-complete is raised in the above three references. In Section 2 we discuss related problems and in Section 3 we establish some preliminary results, and present our notation. In Section 4 we define the general approach and architecture of our polynomial time reductions. In Sections 5 and 6 we show that two restricted versions of the MLC and MLC-R problems are NP-complete. Finally in Section 7 we discuss our results and point out some open problems.

## 2. Related problems

Another restricted version of the MLC problem is when all the possible corridors must include the top-right corner of the rectangular boundary $F$ as an access point. In this case we refer to the problem as the *top-right access* point version of the problem or simply the *TRA-MLC* and *TRA-MLC-R* problems. An optimal solution to the TRA-MLC problem instance defined above is given in Fig. 1(a). Note that the set of thick line segments shown in Fig. 1(b) is not a corridor (feasible solution) for the TRA-MLC problem. Fig. 2 summarizes the relationship between the variants of the MLC problem defined so far and its generalization to graphs (N-MLC), which is formally defined below. In this figure a thick arrow represents polynomial time reducibility and a thin arrow represents problem restriction. A polynomial time reduction from TRA-MLC-R to MLC-R is given in Theorem 3.1. The same reduction shows that TRA-MLC $\alpha$ MLC.

A more general version of the problems allows a set or forest of partial corridors rather than just a corridor, with the partial corridors connecting all the rooms to access points on $F$. We call this problem the $\mathrm{MLC}_f$ problem. The *multiple access* point ($\mathrm{MA\text{-}MLC}_f$) version restricts the partial corridors to be rooted at a given set of access points on $F$. When the top-right corner of $F$ is the only access point, the $\mathrm{MA\text{-}MLC}_f$ problem corresponds to the TRA-

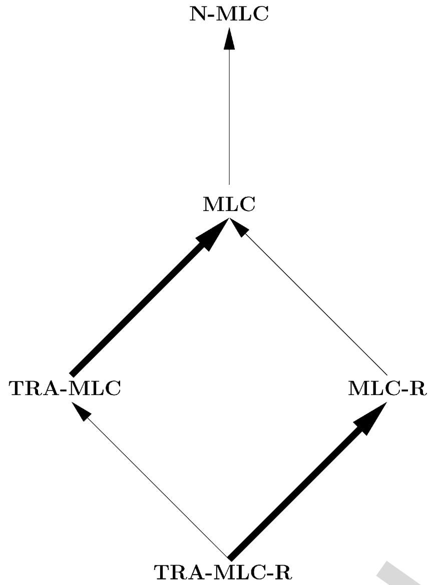



Fig. 2. Relationship between variants of the MLC problem.

MLC problem. From our results it follows that the MA-MLC$_f$ problem with one access point is NP-complete. Later on we establish that the decision version of the MA-MLC$_f$ (when there are two or more access points) and MLC$_f$ problems are NP-complete. Our problems may also be generalized to ones where $F$ is a rectilinear polygon. These more general versions of the MLC problem are NP-complete because they include the MLC problem which we show to be NP-complete.

We may further generalize the MLC problem to graphs. In this case we are given a connected undirected edge-weighted graph and the objective function is to find a tree with least total edge-weight such that every cycle in the graph has at least one of its vertices in the tree. We call this problem the *network MLC* (N-MLC) problem. In graph theoretic terms the set of vertices that break all the cycles in a graph is called a *feedback node set* (FNS). One may redefine the N-MLC problem by requiring that the vertices in the corridor form a feedback node set for the graph. Thus, this problem can also be referred to as the *tree feedback node set* (TFNS) problem, and is formally defined as follows:

*Input*: A connected undirected edge-weighted graph $G = (V, E, w)$, where $w : E \to \mathbb{R}^+$ is an edge-weight function.
*Output*: A tree $T = (V', E')$, where $E' \subseteq E$, $V' \subseteq V$, and $V'$ is a feedback node set (i.e., every cycle in $G$ includes at least one vertex in $V'$) and the total edge-weight $\sum_{e\in E'} w(e)$ is minimized.

The TFNS problem has not been defined elsewhere, but a similar problem, the *tree vertex cover* (TVC) problem is discussed in [1]. The difference between the TFNS and TVC problems is that instead of the set $V'$ being a feedback node set for $G$, it must be a vertex cover for $G$.

The *Group Steiner Tree* (GST) problem may be viewed as a generalization of the MLC problem. Reich and Widmayer [11] introduced the GST problem, motivated by applications in VLSI design. The GST problem is defined by Reich and Widmayer as follows.

*Input*: A connected undirected edge-weighted graph $G = (V, E, w)$, where $w : E \to \mathbb{R}^+$ is an edge-weight function; a non-empty subset $S$, $S \subseteq V$, of *terminals*; and a partition $\{S_1, S_2, \ldots, S_k\}$ of $S$.
*Output*: A tree $T(S) = (V', E')$, where $E' \subseteq E$ and $V' \subseteq V$, such that at least one terminal from each set $S_i$ is in the tree $T(S)$ and the total edge-length $\sum_{e\in E'} w(e)$ is minimized.

Approximation algorithms to the GST problem are given in [2,6,7]. The graph Steiner tree (ST) problem is a special case of the GST problem where each set $S_i$ is a single vertex. Karp [9] proved that the decision version of the ST problem is NP-complete. Since the GST problem includes the ST problem, the decision version of the GST problem is also NP-complete. There is a simple and straight forward reduction from the MLC problem to the GST problem, which can be used to show that any constant ratio approximation algorithm for the GST problem is a constant ratio approximation algorithm for the MLC problem. However there is no known constant ratio approximation algorithm for the GST problem. Other authors [12–14] defined a more general version of the GST problem where $\{S_1, S_2, \ldots, S_k\}$ of $S$ is not a partition, but each $S_i \subseteq S$, i.e, a vertex may be in more than one set $S_i$. This version of the GST problem is called the *Tree Errand Cover* (TEC) problem and it was studied by Slavik [13,14]. Safra and Schwartz [12] established inapproximability results for the 2D version of the GST problem when each set is connected and the sets are allowed to intersect, but again, these results do not seem to carry over to the MLC problem.

As we have seen, our problems are restricted versions of more general ones reported in the literature. But previous results for those problems do not establish NP-completeness results, inapproximability results, nor constant ratio approximation algorithms for our problems.

## 3. Preliminaries and definitions

In the decision version of our minimum edge-length corridor problems we are given an additional input value $B$ and the question is to decide whether or not there is a corridor with length at most $B$. Hereafter when we refer to any of our problems we mean the *decision version* of our problems.

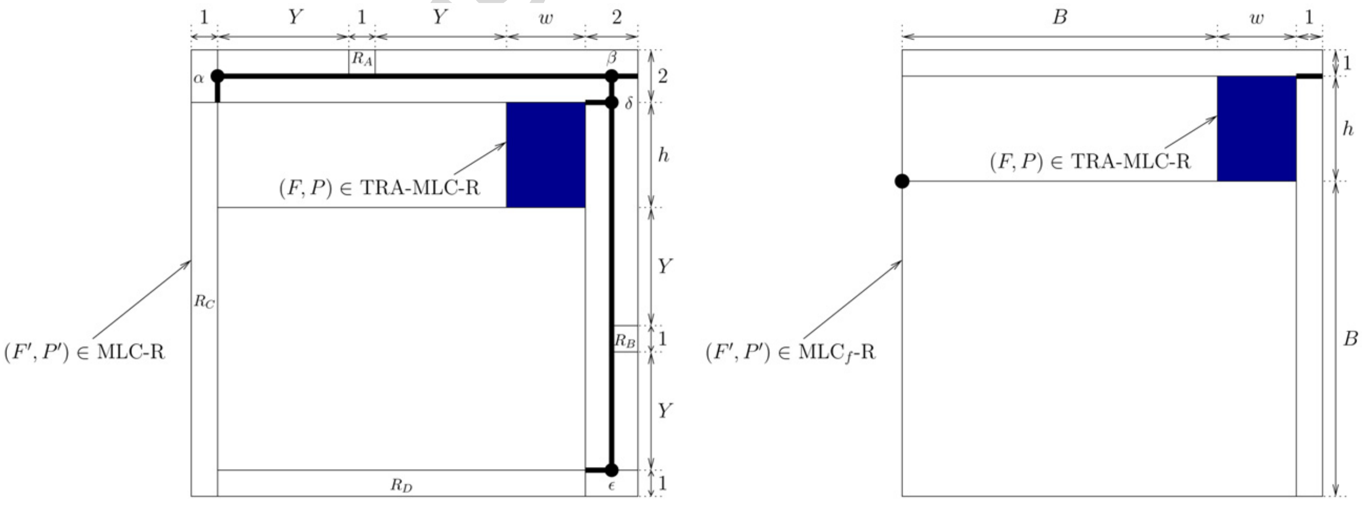
In Theorem 3.1 we show that our NP-completeness result for the TRA-MLC-R problem extends to the MLC-R problem.

**Theorem 3.1.** *TRA-MLC-R $\alpha$ MLC-R.*

**Proof.** Consider any instance $(F, P)$ of the TRA-MLC-R problem and embed it in the rectangle $F'$ to create an instance $(F', P')$ of the MLC-R problem as shown in Fig. 3(a). We claim that the instance of the MLC-R problem has a corridor of length at most $B' = B + 4Y + h + w + 8$ if, and only if, the instance of the TRA-MLC-R problem has a corridor of length at most $B$, where $Y = B + h + w + 9$, and $B$, $w$, and $h$ are greater than 2.

If the instance of the TRA-MLC-R problem has a corridor of edge-length $B$, then the addition of the thick line segment in Fig. 3(a) shows that there is a corridor for the MLC-R problem with edge-length $B + 4Y + h + w + 8$.

Suppose now that there is a corridor with length at most $B + 4Y + h + w + 8$ for the instance of the MLC-R problem. We show that the TRA-MLC-R instance has a corridor of length at most $B$. Consider any corridor $S$ for the MLC-R problem. If corridor $S$ includes any of the line segments on the boundary of $F'$, these line segments can be moved to their closest parallel line segments inside $F'$. We then delete superfluous line segments (if any). The edge-length




(a) TRA-MLC-R $\alpha$ MLC-R

(b) TRA-MLC-R $\alpha$ MLC$_f$-R

Fig. 3. Polynomial time reductions.

of the resulting corridor $S$ is not longer than before. Suppose now that $S$ does not include the line segment $(\alpha, \beta)$. Then the corridor must include at least one of the horizontal line segments below $(\alpha, \beta)$ to reach rectangle $R_C$, and must include a portion of line segment $(\alpha, \beta)$ of length at least $Y$ to reach rectangle $R_A$. These segments must have total length at least $3Y$. Now to join $R_D$ to the corridor on the top side of $F'$ we need at least one of the three vertical line segments with length at least $2Y$. But then the total length of line segments identified so far is at least $5Y$ which exceeds $B + 4Y + h + w + 8$. So it must be that the line segment $(\alpha, \beta)$ must be part of the corridor $S$. Similarly, we can argue that $(\delta, \epsilon)$ must also be part of corridor $S$. The segments $(\alpha, \beta)$ and $(\delta, \epsilon)$ have total length $4Y + h + w + 3$. Since the length of the corridor is at most $B + 4Y + h + w + 8$, then the remaining part of the corridor has length at most $B + 5$. One can show that the only possible corridor must have the thick line segments given in Fig. 3(a) plus the line segments inside $F$ of length at most $B$ to connect all the rectangles inside $F'$. Therefore, the instance of the MLC-R problem has a corridor of length at most $B + 4Y + h + w + 8$ if, and only if, the instance of the TRA-MLC-R problem has a corridor of length at most $B$, where $Y = B + h + w + 9$, and $B$, $w$, and $h$ are greater than 2. This concludes the proof of the theorem. □

Now consider any instance $(F, P)$ of the TRA-MLC-R problem and embed it in rectangle $F'$ to create an instance $(F', P')$ of the MLC$_f$-R problem as shown in Fig. 3(b). Clearly, the instance of the MLC$_f$-R problem has a solution with length at most $B' = B + 1$ if, and only if, the instance of the TRA-MLC-R problem has a corridor with length at most $B$, when $B > 2$. Notice that the solution consists of two trees, one with only one point, so its length is zero, and the other has length $B + 1$. It is simple to show that the same transformation holds from the TRA-MLC-R to the MA-MLC$_f$-R problem when there are two or more access points. These observations establish Theorem 3.2.

**Theorem 3.2.** *TRA-MLC-R α MLC$_f$-R, and TRA-MLC-R α MA-MLC$_f$-R with k access points, for $k \geqslant 2$.*

**Proof.** By the above discussion. □

In Section 5 we show that the TRA-MLC problem is NP-complete and in Section 6 we show that even the TRA-MLC-R is NP-complete. Clearly, the result in Section 6 implies the result in Section 5. However, we begin by establishing that the TRA-MLC problem is NP-complete because that reduction is simpler to understand. Then we show how to modify the reduction to a more complex one to establish that the TRA-MLC-R problem is NP-complete. By Theorems 3.1 and 3.2; the problem restriction implied by the definitions; the fact that the MLC, MLC-R, MLC$_f$, MLC$_f$-R, MA-MLC$_f$ and MA-MLC$_f$-R problems are in NP; and our NP-completeness result for the TRA-MLC-R given in Section 6; it then follows that the MLC, MLC-R, MLC$_f$, MLC$_f$-R, MA-MLC$_f$ and MA-MLC$_f$-R problems are also NP-complete. The following theorem formalizes these results.

**Theorem 3.3.** *The MLC, MLC-R, MLC$_f$, MLC$_f$-R, MA-MLC$_f$ and MA-MLC$_f$-R problems are NP-complete.*

**Proof.** By the above discussion. □

To establish our NP-completeness results we reduce Planar 3-SAT (P3SAT) given in a canonical planar embedding (defined below) to our problem. Problem P3SAT is 3-SAT restricted to a formula whose graph representation is planar. P3SAT was shown to be NP-complete by Lichtenstein [10]. Planar 3-SAT is formally defined as follows:

*Input*: Given $I = (X, C)$, where $X$ is a set of Boolean variables $\{x_1, x_2, \ldots, x_n\}$ and $C$ is a non-empty set of clauses $\{C_1, C_2, \ldots, C_m\}$ over $X$ in conjunctive normal form (CNF); every clause has at least two and at most three literals; and the graph $G_I = (V, E)$ for $(X, C)$ is planar, where

$$V = \{C_j \mid 1 \leqslant j \leqslant m\} \cup \{x_i \mid 1 \leqslant i \leqslant n\} \quad \text{and}$$
$$E = \bigl\{\{C_j, x_i\} \mid x_i \in C_j \text{ or } \bar{x}_i \in C_j\bigr\} \cup \bigl\{\{x_i, x_{i+1}\} \mid 1 \leqslant i < n\bigr\} \cup \{x_1, x_n\}.$$

*Question*: Is there a satisfying truth assignment for $C$?

Fig. 4 depicts a planar embedding $D(G_I)$ for the graph $G_I$ associated to the instance $I = (X, C)$, where $X = \{x_1, x_2, \ldots, x_9\}$ and $C = (C_1, C_2, \ldots, C_{14}) = (\{x_1, \bar{x}_2\}, \{x_1, \bar{x}_4, x_5\}, \{\bar{x}_1, x_2\}, \{\bar{x}_1, \bar{x}_2, x_4\}, \{x_2, \bar{x}_4\}, \{\bar{x}_2, x_3, x_4\},$

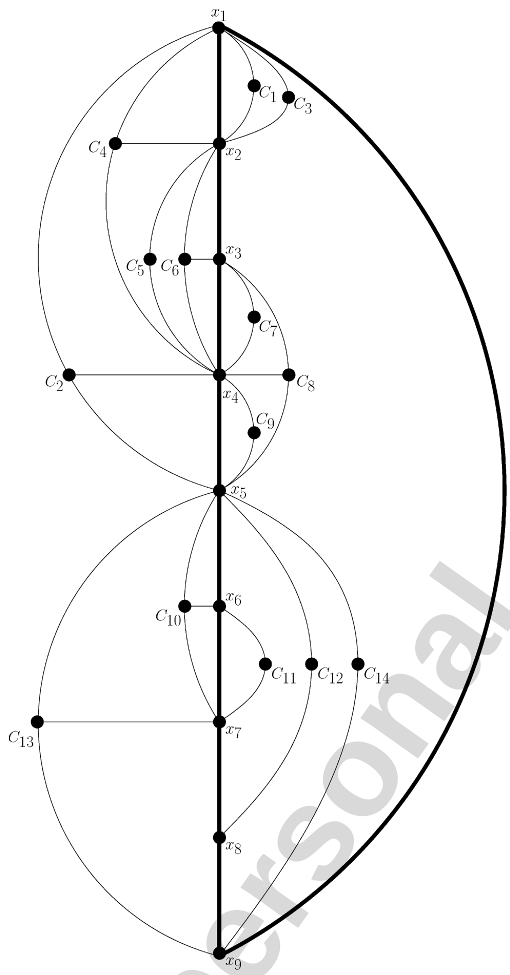



Fig. 4. A canonical planar embedding $D(G_I)$ of the graph $G_I$ representing the instance $I = (X, C)$.

$\{x_3, \bar{x}_4\}$, $\{\bar{x}_3, x_4, x_5\}$, $\{\bar{x}_4, \bar{x}_5\}$, $\{\bar{x}_5, x_6, \bar{x}_7\}$, $\{\bar{x}_6, x_7\}$, $\{\bar{x}_5, x_8\}$, $\{x_5, \bar{x}_7, \bar{x}_9\}$, $\{x_5, x_9\}$).[3] For a planar embedding $D(G_I)$, the set $X$ of vertices are vertically aligned (in some order).

A planar embedding $D(G_I)$ is called a *canonical planar embedding* (drawing) if in addition to the above vertical alignment property, the *ring* formed by the set $X$ of vertices (thick line cycle in Fig. 4) is such that all the clause vertices *outside* the ring are drawn to the left of the vertices in $X$, and the ones *inside* the ring are drawn to the right of the vertices in $X$. A canonical planar embedding (drawing) can be generated by making simple additions to several existing polynomial time algorithms including the algorithm (specified through the polynomial time reduction from 3SAT to P3SAT) given by Lichtenstein [10]. Any canonical planar embedding can be easily represented in $O(n + m)$ space. In what follows we assume without loss of generality that the Boolean variables in every instance of P3SAT has been reordered so that there is a canonical planar embedding with the variables $x_1, x_2, \ldots, x_n$ appearing from top to bottom in that order.

For any instance $I = (X, C)$ of P3SAT given in a canonical planar embedding, let $N_2$ be the set of clauses with exactly two literals, i.e., $N_2 = \{c \mid c \in C$ and $c$ has two literals$\}$. Similarly we define the set of clauses $N_3 = \{c \mid c \in C$ and $c$ has three literals$\}$. Given any canonical planar embedding $D(G_I)$ for the graph $G_I$ we define the following terms. The set $C$ of clauses is partitioned into two sets: the set $L$ of clauses whose clause vertex is located outside the ring formed by the set $X$ of vertices, and the set $R$ of clauses whose clause vertex is located inside the ring. For every clause $c \in N_2$ we define as its *ring* the set of vertices $c, x_{i_1}, x_{i_1+1}, \ldots, x_{i_2-1}, x_{i_2}$, where $i_1 < i_2$ are the indices of the

[3] This is equivalent to $C = ((x_1 \vee \bar{x}_2) \wedge (x_1 \vee \bar{x}_4 \vee x_5) \wedge (\bar{x}_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_4) \wedge (x_2 \vee \bar{x}_4) \wedge (\bar{x}_2 \vee x_3 \vee x_4) \wedge (x_3 \vee \bar{x}_4) \wedge (\bar{x}_3 \vee x_4 \vee x_5) \wedge (\bar{x}_4 \vee \bar{x}_5) \wedge (\bar{x}_5 \vee x_6 \vee \bar{x}_7) \wedge (\bar{x}_6 \vee x_7) \wedge (\bar{x}_5 \vee x_8) \wedge (x_5 \vee \bar{x}_7 \vee \bar{x}_9) \wedge (x_5 \vee x_9))$.

literals in $c$. Similarly, for every clause $c \in N_3$ we define as its *rings* the set of vertices $c, x_{i_1}, x_{i_1+1}, \ldots, x_{i_2-1}, x_{i_2}$ and $c, x_{i_2}, x_{i_2+1}, \ldots, x_{i_3-1}, x_{i_3}$, where $i_1 < i_2 < i_3$ are the indices of the literals in $c$.

We refer to the clauses whose clause vertex is not inside the ring of another clause as *peripheral* clauses. The *depth* of a peripheral clause is defined as the maximum number of clause vertices inside any of its rings plus one. The *depth of the left side* of the canonical planar embedding $D(G_I)$ is defined as the maximum depth of the peripheral clauses whose clause $c \in L$. Similarly, the *depth of the right side* of the canonical planar embedding $D(G_I)$ is defined as the maximum depth of the peripheral clauses whose clause $c \in R$. Let $l$ and $r$ be the depth of the left and right side of the canonical planar embedding $D(G_I)$, respectively. The canonical planar embedding $D(G_I)$ of Fig. 4 has the peripheral clauses $C_2, C_3, C_8, C_{13}, C_{14}$, and its depth on the left and right side is $l = 4$ and $r = 3$, respectively.

## 4. Architecture of the reductions

Let us now define the general approach for our polynomial time transformations from P3SAT given in a canonical planar embedding to the decision version of the TRA-MLC and TRA-MLC-R problems. For any instance $I = (X, C) \in$ P3SAT given in a canonical planar embedding $D(G_I)$, our polynomial time transformation constructs the instance $f(I)$ of the TRA-MLC or TRA-MLC-R problem inside a rectangular boundary $F$.

From the planar embedding $D(G_I)$ we define the instance $f(I)$ of our problem. This instance consists of four different types of components: (*truth*) *setting, clause-checking, top-frame* and *setting-terminator*. Each setting component corresponds to a Boolean variable and its size, which will be defined formally later on, is related to the number of occurrences of the Boolean variable or its complement in the clauses. As we establish later on, every corridor with length at most $B$ for $f(I)$ corresponds to the assignment of the value of true or false for each variable in instance $I$. These assignments can be easily identified by the way the corridor visits the rectilinear polygons in the setting components. Furthermore, the values assigned to the variables in each of these assignments satisfy all the clauses in $C$. To ensure this last property we have the clause-checking components. There is a clause-checking component for each clause $c \in C$ on the left side of the setting components when $c \in L$, and on the right side when $c \in R$ in $D(G_I)$. A clause-checking component will be reached by a feasible corridor only when at least one of the literals in the clause it represents has the value of true. The top-frame component is used to allow for the setting of the value of the topmost Boolean variable in $D(G_I)$, and the setting-terminator component allows for adjacent variables to be assigned values independent from each other. Fig. 5 shows the overall architecture and all the components. In Sections 5 and 6 we discuss each of the components separately and then explain their interactions.

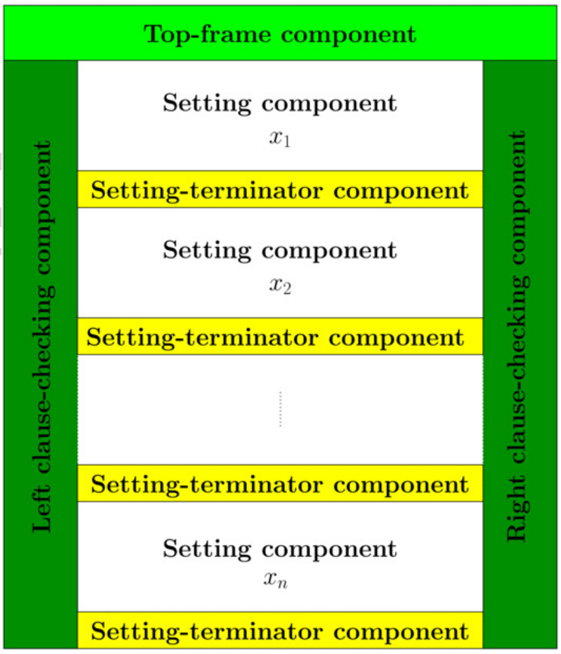



Fig. 5. Overall architecture.

For the reduction in Section 5, the width of the rectangles circumscribing the left and right clause-checking components is given by $\alpha_L = 2l+1$ and $\alpha_R = 2r+1$, respectively. The values for the reduction in Section 6 are $\beta_L = 4(l+1)$, and $\beta_R = 4(r+1)$, respectively. Remember that we define the depth $l$ and $r$ for the left and right side of the canonical planar embedding $D(G_I)$ in the previous section.

The horizontal dimension of the clause-checking components depends on the indentation of the clause in $D(G_I)$. The *indentation* function, $I : C \to Z^+$, is defined recursively as follows. For $c \in L$, if $c$ is a peripheral clause in $D(G_I)$, its indentation $I(c)$ is the depth of the left side. Otherwise, for a non-peripheral clause $c$, its indentation $I(c)$ is defined as the minimum indentation of a clause whose ring includes the clause vertex for clause $c$ minus 1. The indentation $I(c)$ for $c \in R$ is defined similarly. The indentation value for every clause in $D(G_I)$ given in Fig. 4 is given in Table 1. We also define the parameter $\gamma = \max\{\beta_L, \beta_R\}$ which will be used to define the dimensions of the components in the construction given in Section 6. Note that $\gamma \geqslant 4(I(c)+1)$ for any $c \in C$; and $\gamma > 0$ because the set $C$ of clauses is non-empty.

For every $x_i \in X$, we define $\lambda_i$ and $\rho_i$ as the number of edges of the form $\{x_i, c\}$ for any $c \in C$ in the canonical planar embedding $D(G_I)$ leaving the vertex $x_i$ from the left side (when $c \in L$) and right side (when $c \in R$), respectively. The size $s_i$ of the setting component for variable $x_i$ is $\max\{\lambda_i, \rho_i\}$. Table 2 gives the values for $\lambda_i$ and $\rho_i$ for Fig. 4.

For every edge in $D(G_I)$ of the form $\{c, x_i\}$ such that $c \in L$ we define its *relative order* as $y$ if there are exactly $y-1$ edges of the form $\{c', x_i\}$ with $c' \in L$ that appear above edge $\{c, x_i\}$ in $D(G_I)$. Similarly we define the *relative order* for the edges of the form $\{c, x_i\}$ for $c \in R$. Table 3 gives the relative order for every edge of the form $\{c, x_i\}$ in the canonical planar embedding given in Fig. 4.

Table 1
Indentation for the canonical planar embedding given in Fig. 4

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ | $C_8$ | $C_9$ | $C_{10}$ | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{14}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $I(c)$ | 2 | 4 | 3 | 3 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 4 | 3 |

Table 2
The values of $\lambda_i$ and $\rho_i$ for the canonical planar embedding given in Fig. 4

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ |
|---|---|---|---|---|---|---|---|---|---|
| $\lambda_i$ | 2 | 3 | 1 | 4 | 3 | 1 | 2 | 0 | 1 |
| $\rho_i$ | 2 | 2 | 2 | 3 | 4 | 1 | 1 | 1 | 1 |

Table 3
The relative order for every edge of the form $\{c, x_i\}$ in the canonical planar embedding given in Fig. 4

| Edge (left side) | Relative order | Edge (right side) | Relative order |
|---|---|---|---|
| $\{C_2, x_1\}$ | 1 | $\{C_3, x_1\}$ | 1 |
| $\{C_4, x_1\}$ | 2 | $\{C_1, x_1\}$ | 2 |
| $\{C_4, x_2\}$ | 1 | $\{C_1, x_2\}$ | 1 |
| $\{C_5, x_2\}$ | 2 | $\{C_3, x_2\}$ | 2 |
| $\{C_6, x_2\}$ | 3 | $\{C_8, x_3\}$ | 1 |
| $\{C_6, x_3\}$ | 1 | $\{C_7, x_3\}$ | 2 |
| $\{C_6, x_4\}$ | 1 | $\{C_7, x_4\}$ | 1 |
| $\{C_5, x_4\}$ | 2 | $\{C_8, x_4\}$ | 2 |
| $\{C_4, x_4\}$ | 3 | $\{C_9, x_4\}$ | 3 |
| $\{C_2, x_4\}$ | 4 | $\{C_9, x_5\}$ | 1 |
| $\{C_2, x_5\}$ | 1 | $\{C_8, x_5\}$ | 2 |
| $\{C_{13}, x_5\}$ | 2 | $\{C_{14}, x_5\}$ | 3 |
| $\{C_{10}, x_5\}$ | 3 | $\{C_{12}, x_5\}$ | 4 |
| $\{C_{10}, x_6\}$ | 1 | $\{C_{11}, x_6\}$ | 1 |
| $\{C_{10}, x_7\}$ | 1 | $\{C_{11}, x_7\}$ | 1 |
| $\{C_{13}, x_7\}$ | 2 | $\{C_{12}, x_8\}$ | 1 |
| $\{C_{13}, x_9\}$ | 1 | $\{C_{14}, x_9\}$ | 1 |

Table 4
The values of $t(c)$, $m(c)$, and $b(c)$ for the canonical planar embedding given in Fig. 4

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ | $C_8$ | $C_9$ | $C_{10}$ | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{14}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t(c)$ | 2 | 1 | 1 | 2 | 2 | 3 | 2 | 1 | 3 | 3 | 1 | 4 | 2 | 3 |
| $m(c)$ | – | 4 | – | 1 | – | 1 | – | 2 | – | 1 | – | – | 2 | – |
| $b(c)$ | 1 | 1 | 2 | 3 | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |

For each clause $c$ such that $c \in L$ and $c \in N_2$ we define $t(c)$ as the relative order of the edge $\{c, x_{i_1}\}$, and $b(c)$ as the relative order of the edge $\{c, x_{i_2}\}$, where $i_1 < i_2$ are the indices of the literals in $c$. For each clause $c$ such that $c \in L$ and $c \in N_3$, we define $t(c)$ as the relative order of the edge $\{c, x_{i_1}\}$, $m(c)$ as the relative order of the edge $\{c, x_{i_2}\}$ and $b(c)$ as the relative order of the edge $\{c, x_{i_3}\}$, where $i_1 < i_2 < i_3$ are the indices of the literals in $c$. We define similarly $t(c)$, $m(c)$ and $b(c)$ for all the clauses $c \in R$. Table 4 gives the $t(c)$, $m(c)$, and $b(c)$ values for all the clauses for the canonical planar embedding given in Fig. 4. The values of $t(c)$, $m(c)$ and $b(c)$ are used to define how the clause-checking components join to the setting components in our construction.

## 5. TRA-MLC problem

Let us now define our polynomial time transformation from P3SAT given in a canonical planar embedding to the decision version of the TRA-MLC problem. For any instance $I = (X, C) \in$ P3SAT given in a canonical planar embedding $D(G_I)$, our polynomial time transformation constructs the instance $f(I)$ of the TRA-MLC problem. The overall architecture of the reduction and all the components of our construction are given in Fig. 5. It is important to point out that the setting-terminator component is not actually needed in this reduction. However, we added it in order to use the same architecture for the reduction given in the next section. In what follows we discuss each of the components separately and then explain their interactions.

*Setting component.* Each *setting* component is associated with a Boolean variable and the way the corridor visits its rectilinear polygons identifies the assignment of a value to the variable. For Boolean variable $x_i$, a setting component consists of $s_i$ *basic setting* components stacked on top of each other with a pair of rectilinear polygons joining them. The basic setting component consists of *variable-repository rectangles*, *horizontal-fixing rectangles*, and *vertical-fixing octagons*.

A basic setting component, $s$, has four variable-repository rectangles (light gray colored), two vertical-fixing octagons (dark gray colored), and two horizontal-fixing rectangles (darker gray colored) (see Fig. 6). A basic setting component has also five rectilinear polygons (white colored) with eight or twelve corners. The variable-repository region represents the literals $x_i$ and $\bar{x}_i$. The top-left variable-repository rectangle corresponds to $x_i$, and the top-right variable-repository rectangle corresponds to $\bar{x}_i$. As we proceed downwards the literals are assigned to the variable-repository rectangles in an alternating way. Fig. 6 shows the length of each line segment of the basic setting component $s$. Let $h_b$ and $w_b$ be the height and width of $s$, respectively. Let $h_r$ and $w_r$ be the height and width of the variable-repository rectangle, respectively. The height $h_r$ of the variable-repository rectangle is equal to $5w_r$, and the height $h_b$ of the basic setting component is equal to $14w_r$. Every horizontal line segment that joins two variable-repository rectangles on opposite sides of basic setting component $s$ has length $d$ which is set to $2h_b + 4w_r$. Note that the scale of Fig. 6 is not proportional to the length of its segments. The specific values in our reduction for $h_b$ and $w_r$ are 14 and 1, respectively.

Suppose that there is a partial corridor *PC* that ends both on the top-left and top-right corners of the basic setting component $s$ and does not include any other point of $s$. Remember that every partial corridor for the TRA-MLC problem instance includes the top-right corner of the rectangular boundary $F$.

Consider all the possible sets of line segments that extend the partial corridor *PC* to reach all the rectilinear polygons inside the basic setting component $s$. Additionally, each of those sets of line segments inside $s$ reaches its bottom-left and bottom-right corners in order to provide connection to the other basic setting component that will be placed under $s$. Lets refer to a subset of those resulting partial corridors as the collection $\mathcal{S}$. Each partial corridor $S \in \mathcal{S}$ consists of *PC* plus a set of line segments connecting all the rectilinear polygons inside $s$ and its bottom-left and bottom-right corners. As we prove below, every partial corridor $S \in \mathcal{S}$ consists of *PC* plus either one of the two sets of line segments of the general form given in Fig. 7.

Formally, the collection $\mathcal{S}$ consists of all partial corridors $S$ that satisfy the following properties:

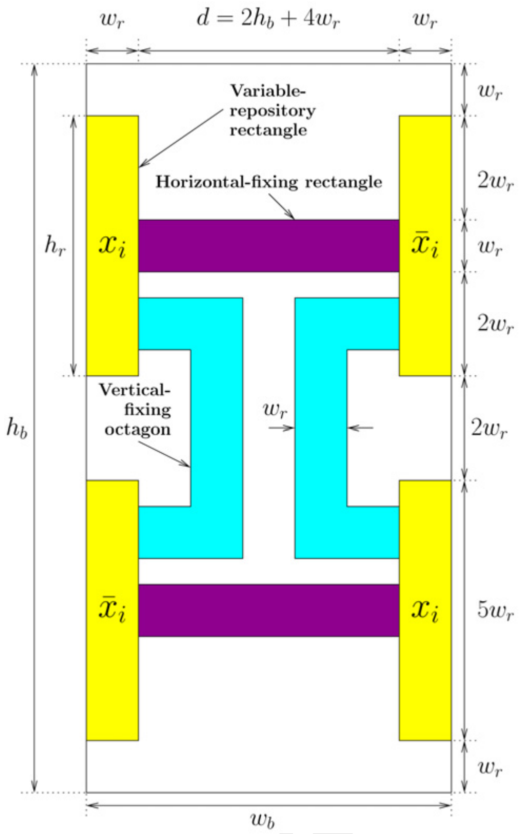
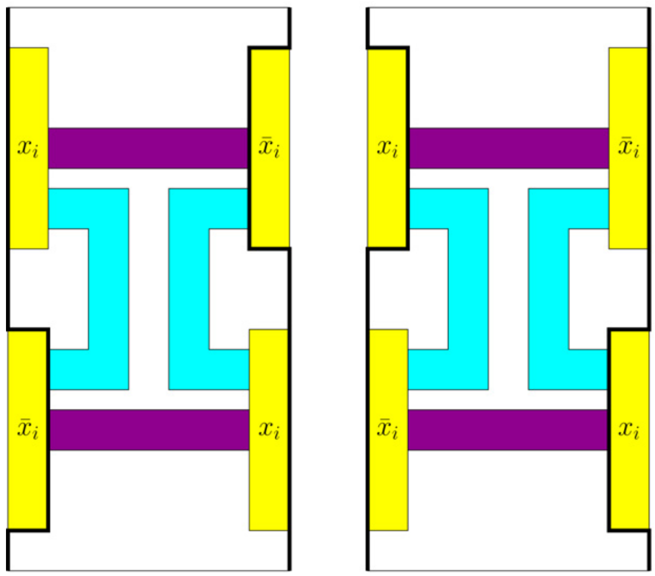



Fig. 6. Basic setting component.

1. Partial corridor $S$ includes $PC$ as well as the bottom-left and bottom-right corners of the basic setting component $s$, and at least one point from each of the rectilinear polygons inside $s$.
2. All line segments in $S \setminus PC$ must be inside or on the boundary of $s$.
3. $L(S \setminus PC) \leqslant 2h_b + 4w_r$.

Since $d = 2h_b + 4w_r$ it must be that the line segments of every set $S \setminus PC$, where partial corridor $S$ belongs to $\mathcal{S}$, consists of two disjoint sets of line segments. One set joins the top-left to the bottom-left corner of $s$ and the other set joins the two remaining corners of $s$. Suppose that for some $S \in \mathcal{S}$ these sets are simply two line segments both of which start at the top and end at the bottom of $s$, one goes through the left and the other goes through the right side of $s$. Their total length is $2h_b$. However, these extensions to the partial corridor $PC$ do not reach the horizontal-fixing rectangles nor the vertical-fixing octagons of $s$. One way to reach the horizontal-fixing rectangles is by adding line segments of least possible length. But then, the length of the resulting partial corridor minus $L(PC)$ is greater than $2h_b + 4w_r$. So the only way such rectangles and octagons could be reached, under the line segment length constraint, is when the partial corridor is routed through the interior side of the variable-repository rectangle. Each time this takes place we need to add horizontal line segments with length equal to $2w_r$. Therefore, the only feasible partial corridors with edge-length $2h_b + 4w_r$ plus $L(PC)$ are the ones where the above detour takes place only once for each horizontal-fixing rectangle at the same level. Since there are two levels, we have several sets of line segments each with total length $2h_b + 4w_r$. But a feasible set of corridors must also reach the vertical-fixing octagons. It is easy to prove that this can only be accomplished by the two paths of line segments (dark thick lines) given in Fig. 7. In both cases the partial corridor $S$ has edge-length equal to $2h_b + 4w_r$ plus $L(PC)$.

The partial corridor given in Fig. 7(a) corresponds to the variable $x_i$ being assigned to the value of true and Fig. 7(b) corresponds to the value of false. It is important to remember that when the corridor goes through the exterior vertical edge of the variable-repository rectangle that represents $x_i$, and through the interior vertical edge of the variable-repository rectangle for $\bar{x}_i$, then the variable $x_i$ has the value of true. Otherwise, when the corridor goes through the interior vertical edge of $x_i$ and through the exterior vertical edge of $\bar{x}_i$, the value for $x_i$ is false.




(a) $x_i = true$ (b) $x_i = false$

Fig. 7. Sets $S \setminus PC$ of line segments for a basic setting component corresponding to setting $x_i$ to the values of true and false.

From the above discussion it is simple to see that the horizontal-fixing rectangle ensures that one of the left and right variable-repositories will have the corridor running through its internal side, whereas the vertical-fixing octagon guarantees an alternating detour behavior of the corridor going through the repositories on each of the sides. The following lemma characterizes the set of partial corridors for the basic setting component.

**Lemma 5.1.** *The set $\mathcal{S}$ is not empty and every partial corridor $S \in \mathcal{S}$ will traverse the basic setting component $s$ by either visiting the exterior vertical edge of the variable-repository rectangle that represents $x_i$, and traverse $s$ by visiting the interior vertical edge of the variable-repository rectangle for $\bar{x}_i$; or vice-versa. Furthermore, $L(S \setminus PC) = 2h_b + 4w_r$.*

**Proof.** By the above discussion. □

The setting component associated to the Boolean variable $x_i$ consists of $s_i$ basic setting components stacked on top of each other as shown in Fig. 8. Between every pair of adjacent basic setting components there are two vertical-fixing octagons joining the variable-repository rectangles on each side of the setting component. These are introduced to ensure consistency for the value of $x_i$ along the setting component. Remember that the size, $s_i$, of the setting component for $x_i$ was defined in Section 4 as $\max\{\lambda_i, \rho_i\}$. The whole sequence of variable-repository rectangles contains the variable $x_i$ and its complement $\bar{x}_i$, alternating along each side. The height of the setting component for variable $x_i$ is $h_b \cdot s_i$, and the total length of line segments needed to extend the corridor $PC$ all the way to the bottom-left and bottom-right sides of the setting component is $(2h_b + 4w_r)s_i$. The set of line segments, extending the partial corridor $PC$, of the form given in Fig. 8(a) corresponds to the value of $x_i$ equal to true and the one in Fig. 8(b) corresponds to the value of false. We establish these claims in Lemma 5.2.

Let $\mathcal{Q}$ be the obvious generalization of $\mathcal{S}$ for the basic setting component to a setting component. In this case every partial corridor $S \in \mathcal{Q}$ has length at most $(2h_b + 4w_r)s_i$ plus $L(PC)$.

**Lemma 5.2.** *The set $\mathcal{Q}$ is not empty and every set of line segments $S \setminus PC$ for $S \in \mathcal{Q}$ must be of the form given in Fig. 8. Furthermore, $L(S \setminus PC) = (2h_b + 4w_r)s_i$.*

**Proof.** By Lemma 5.1 and the above discussion. □

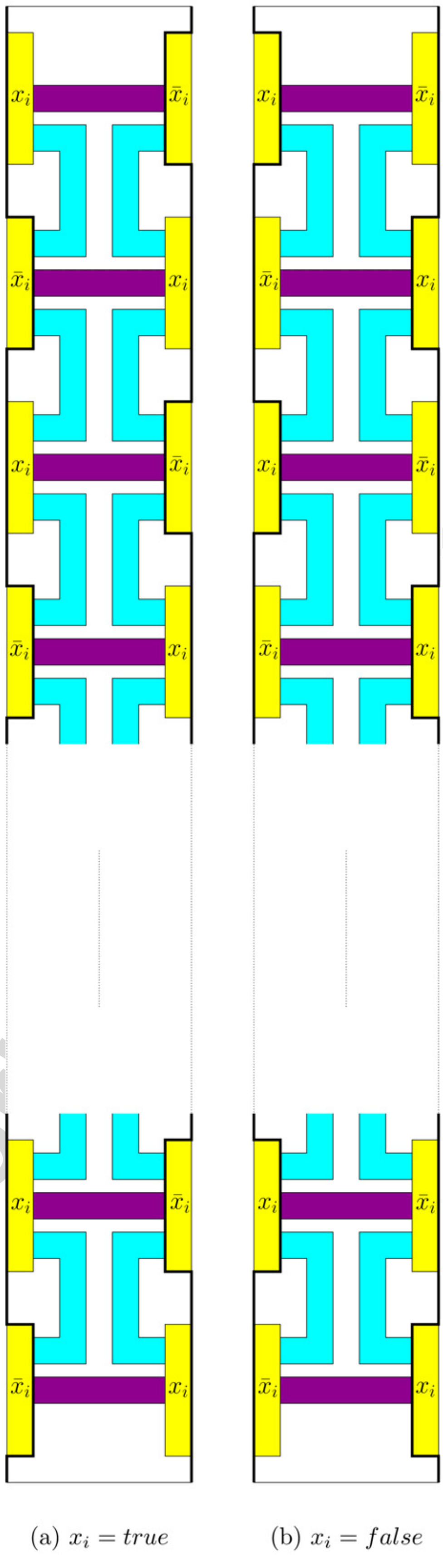



Fig. 8. Sets of line segments for a setting component corresponding to setting $x_i$ to the values of true and false.

*Clause-checking component.* The clause-checking component, for each clause $c \in C$, consists of a rectilinear polygon which joins to certain variable-repository rectangles of the literals in $c$ on the left side if $c \in L$ or on the right side when $c \in R$. These variable-repository rectangles are said to be associated with the clause-checking component. Each of these variable-repository rectangles will not be the associated with another clause-checking component. We will elaborate on this property later on.

Fig. 9(a) shows the clause-checking component for $c = \{y_{i_1}, y_{i_2}\} \in N_2$, $c \in L$, and $y_{i_1}$ ($y_{i_2}$, resp.) is either $x_{i_1}$ or $\bar{x}_{i_1}$ ($x_{i_2}$ or $\bar{x}_{i_2}$, resp.). Fig. 9(b) shows the clause-checking component for $c = \{y_{i_1}, y_{i_2}, y_{i_3}\} \in N_3$, $c \in L$, and $y_{i_3}$ is setting similarly as $y_{i_1}$ and $y_{i_2}$ above. The clause-checking components on the right hand side are symmetric. On the vertical axis the clause-checking component for $c \in N_2$ spans from the middle of the variable-repository rectangle for literal $y_{i_1}$ on the basic setting component number $t(c)$ (from top) for variable $x_{i_1}$, to the middle of the variable-repository rectangle for literal $y_{i_2}$ on the basic setting component number $b(c)$ (from top) for variable $x_{i_2}$. Remember that $t(c)$ and $b(c)$ for each clause $c \in C$ were defined in Section 4. The clause-checking component for $c \in N_3$ spans from the middle of the variable-repository rectangle for literal $y_{i_1}$ on the basic setting component number $t(c)$ (from top) for variable $x_{i_1}$, to the middle of the variable-repository rectangle for literal $y_{i_3}$ on the basic setting component number $b(c)$ (from top) for variable $x_{i_3}$. The middle variable-repository rectangle that is used for literal $y_{i_2}$ is the basic setting component number $m(c)$ (from top) for variable $x_{i_2}$. Remember that $m(c)$ was also defined in Section 4. For example, the clause $C_4 = \{\bar{x}_1, \bar{x}_2, x_4\}$ for the problem instance whose canonical planar embedding is given in Fig. 4 has its clause-checking component ending at the variable-repository rectangle corresponding to the literal $\bar{x}_1$ for the second basic setting component of the first variable (because its $t(c)$ value is 2), the variable-repository rectangle corresponding to the literal $\bar{x}_2$ for the first basic setting component of the second variable (because its $m(c)$ value is 1), and the variable-repository rectangle corresponding to the literal $x_4$ for the third basic setting component of the fourth variable (because its $b(c)$ value is 3). Fig. 9 gives the precise dimensions of the clause-checking component. Note its dependence on $I(c)$. This guarantees that every pair of line segments from different clause-checking components are at least one unit apart. The implication of this construction is that the existing partial corridor through the setting component will reach the clause-checking component without the need to add any new line segments when at least one of the literals satisfies the corresponding clause. On the other hand, one needs additional line segments so that the partial corridor reaches the corresponding clause-checking component when the values assigned to the variables do not satisfy the corresponding clause. By setting the value of $B$ appropriately none of these additional segments can be included in a feasible corridor. We summarize these claims in the following lemma.

**Lemma 5.3.** *Let $S \in \mathcal{Q}$ be a partial corridor that reaches all the setting components and it is of the form given by Lemma* 5.2. *All the clause-checking components will be exposed to the corridor $S$ if, and only if, all the clauses $c \in C$ are satisfied by the corresponding values for the variables assigned by $S$.*

**Proof.** By the above discussion. □

*Setting-terminator component.* As we mentioned above, the setting-terminator component is not needed for this transformation. However, we added it to use the same overall architecture in both reductions. Thus, the setting-terminator has the function of ending the setting component for each variable. This allows for the possibility of adjacent variables to have different values. Fig. 10(b) shows the setting-terminator and the corridor through it. The component has height $h_t$ and width $w_b$.

*Top-frame component.* The top-frame component is used to distribute the corridor from the top-right corner of $F$ to the two exterior vertical sides of the setting components. Fig. 10(a) shows the top-frame component. It has width equal to $\alpha_L + w_b + \alpha_R$, and its height is $h_f + w_r$. Remember that in Section 4, the width of the left and right clause-checking components were defined as $\alpha_L = 2l + 1$ and $\alpha_R = 2r + 1$, respectively. A feasible partial corridor with length $\alpha_R + w_r + w_b + 2h_f$ that ends at the two points that place on the top-left and top-right corners of the topmost setting component is also given in Fig. 10(a).

Finally, the value of $B$ is set to

$$\sum_{i=1}^{n}\bigl((2h_b + 4w_r)s_i\bigr) + 2(n-1)h_t + \alpha_R + w_r + w_b + 2h_f.$$

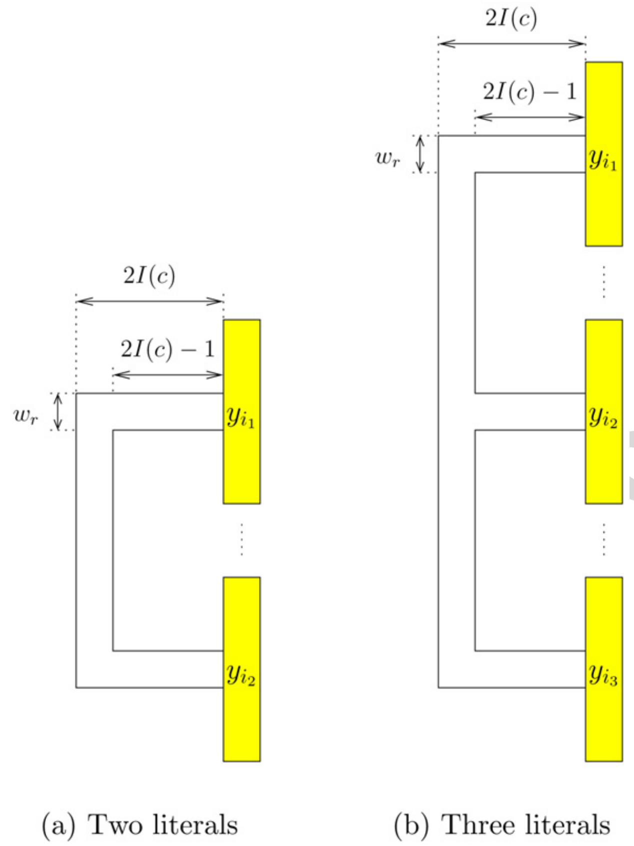



Fig. 9. Clause-checking component.

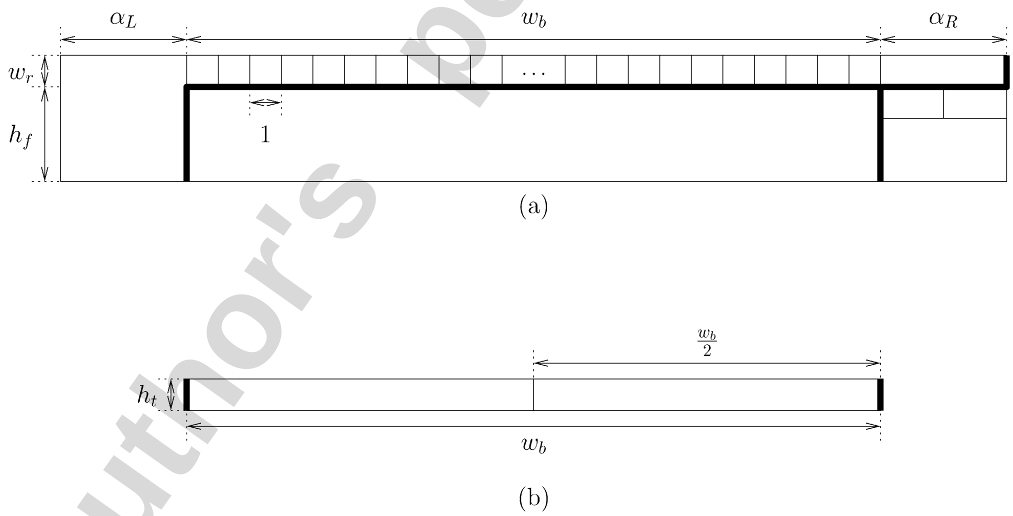



Fig. 10. (a) Top-frame component. (b) Setting-terminator component.

*Applying the reduction.* Before we illustrate the whole process we should point out that our reduction results in an instance $f(I)$ with large height, whose drawing would be hard to read in this media. Thus, instead of drawing in this paper such an instance $f(I)$ we draw an *equivalent compressed* version of the instance which we call $f_c(I)$. The equivalent compressed instance has $s_i' \leqslant s_i$ basic setting-components for each Boolean variable $x_i$. We accomplish the compression process by moving up as much as possible and resizing the clause-checking components so that their ends are aligned with equivalent variable-repository rectangles. Of course one needs to change the value of $B$ to one

Table 5
Original size $s_i$ and compressed size $s'_i$ of the setting component for each variable $x_i$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|
| $s_i$ | 2 | 3 | 2 | 4 | 2 |
| $s'_i$ | 1 | 2 | 1 | 2 | 1 |

we call $B_c$. The formula depends on $s'_i$ rather than on $s_i$. We will use the compressed version in Figs. 11 and 12. We illustrate in Example 1 all the components assembled together following to the above overall architecture (Fig. 5).

**Example 1.** This instance is a shorter version of the instance $I = (X, C)$ given in Section 3, whose canonical planar embedding $D(G_I)$ is in Fig. 4. This shorter version includes only the edges and vertices induced by the clauses $C_1, C_2, \dots, C_9$ and the vertices $x_1, x_2, \dots, x_5$; and the edge $\{x_1, x_5\}$. The resulting instance is a yes-instance. The size $s_i$ of each setting component is given in Table 5. Thus the instance $f(I)$ is formed by 13 setting components. However, one can resize almost all of the clause checking components. The resizing of the clause checking components reduces the number of setting components to 7. The compressed value of the size of the setting components $s'_i$ is given also in Table 5. We will use this equivalent compressed version $f_c(I)$ in all our figures. Fig. 11 gives the resulting compressed instance of the TRA-MLC problem. In this figure we show two sets of line segments (medium thick lines) with length $B_c$. The first one is a partial corridor (i.e., the black filled octagon is not exposed), but the second one is a corridor (i.e., all the polygons are exposed). The partial corridor corresponds to the truth assignment with all the variables having the value of true, which does not satisfy the clause $C_9$, and the corridor corresponds to the truth assignment with all the variables having the value of false, which satisfies all the clauses.

**Example 2.** Consider the following instance $I = (X, C = (\{x_1, \bar{x}_2\}, \{x_1, \bar{x}_3\}, \{\bar{x}_1, x_2, x_3\}, \{\bar{x}_1, \bar{x}_2\}, \{x_2, x_3\}, \{x_2, \bar{x}_3\}))$, and a canonical planar embedding $D(G_I)$ with the clauses $C_2, C_4$ and $C_5$ on the left side and $C_1, C_3$ and $C_6$ on the right side. It is simple to show that this instance is a no-instance.

Applying our reduction and then compressing it results in the instance of the TRA-MLC problem that is given in Fig. 12. In this figure we show 8 partial corridors (medium thick lines) with length $B_c$ of the form given in Fig. 8 for each variable. In the bottom side of each sub-figure we give the values for $x_1, x_2$ and $x_3$ corresponding to the corridor. Each of these partial corridors corresponds to a truth assignment for the variables in the instance $I$. For each of the truth assignments, the black filled dodecagon and octagons are not exposed to the partial corridor. This corresponds to a clause not being satisfied by the corresponding truth assignment. For example the clause-checking component, corresponding to $\{x_2, \bar{x}_3\}$ in the assignment $x_1 = \text{true}$, $x_2 = \text{false}$, $x_3 = \text{true}$, is the only one which is not reached by the corridor in the sub-figure labeled TFT in Fig. 12 and therefore instance $I$ is not satisfied by that truth assignment.

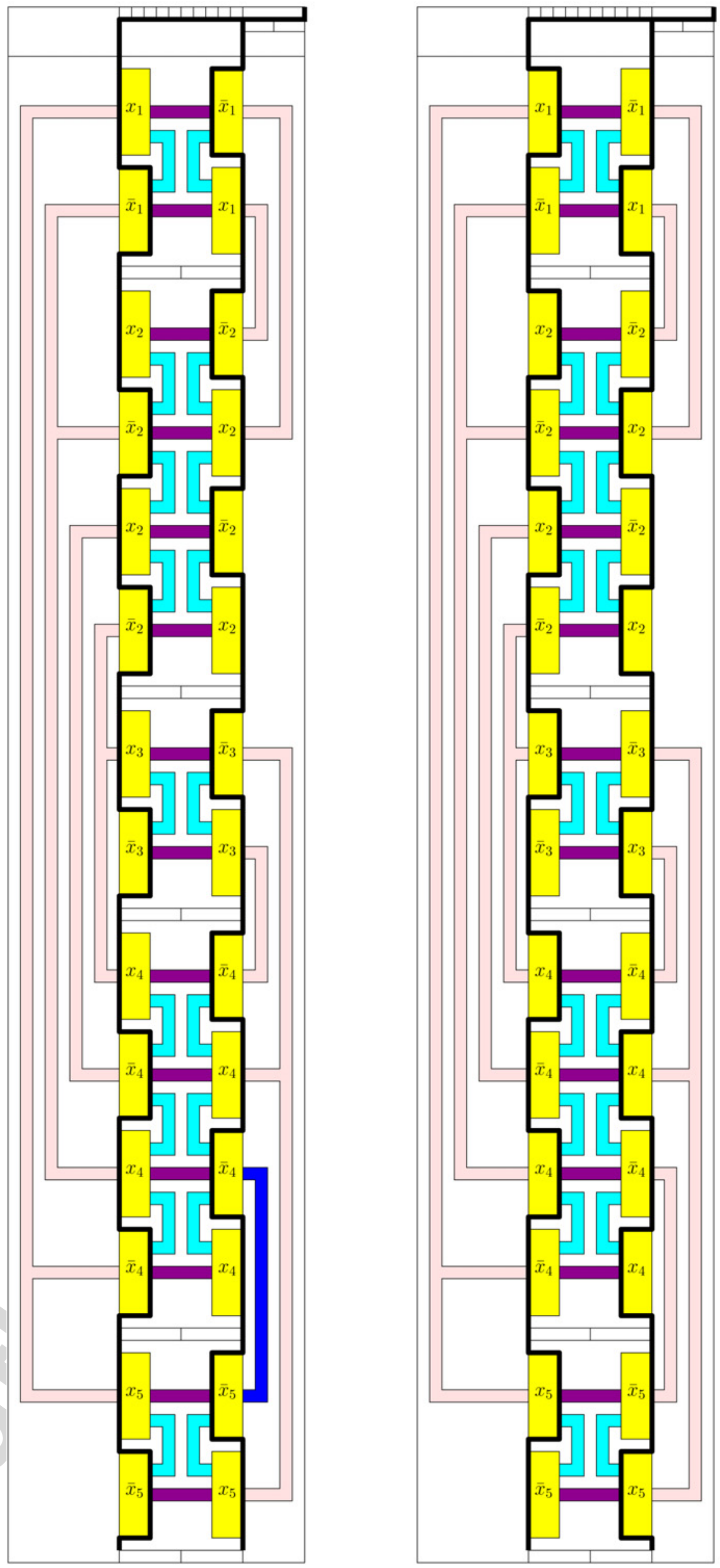
Let us now use our reduction to show that the TRA-MLC problem is NP-complete.

**Theorem 5.1.** *The TRA-MLC problem is NP-complete.*

**Proof.** The TRA-MLC can be solved by a nondeterministic polynomial time Turing machine. Given a set of line segments one can verify in polynomial time whether or not the segments form a corridor and its length is at most $B$. Therefore, TRA-MLC is in NP.

Now we show that the problem transformation defined above is a valid transformation. Since the proof that the transformation takes polynomial time with respect to the instance $I = (X, C) \in$ P3SAT problem given in a canonical planar embedding $D(G_I)$ is simple, we omit it. In what follows we show that no matter what instance $I$ given in any canonical representation $D(G_I)$ we start from, the instance $f(I) \in$ TRA-MLC problem has a corridor with length at most $B$ iff the instance $I \in$ P3SAT used to construct $f(I)$ is satisfiable.

We now show that if the instance $I$ is satisfiable then the instance $f(I)$ has a feasible corridor with length at most $B$. Let $A$ be a truth assignment for instance $I$ that satisfies all the clauses in $C$. The corridor that we construct corresponds to the assignment $A$ and starts on the top-frame component as in Fig. 10(a). The corridor will follow the routes given in Fig. 8(a) or (b), depending on whether or not the variable $x_i$ has the value true in $A$ on each setting component, with length equal to $(2h_b + 4w_r)s_i$. The corridor in the setting-terminator component consists of the



(a) Partial corridor produced by the assignment TTTTT

(b) Corridor produced by the assignment FFFFF

Fig. 11. Instance $f_c(I)$ constructed from Example 1.

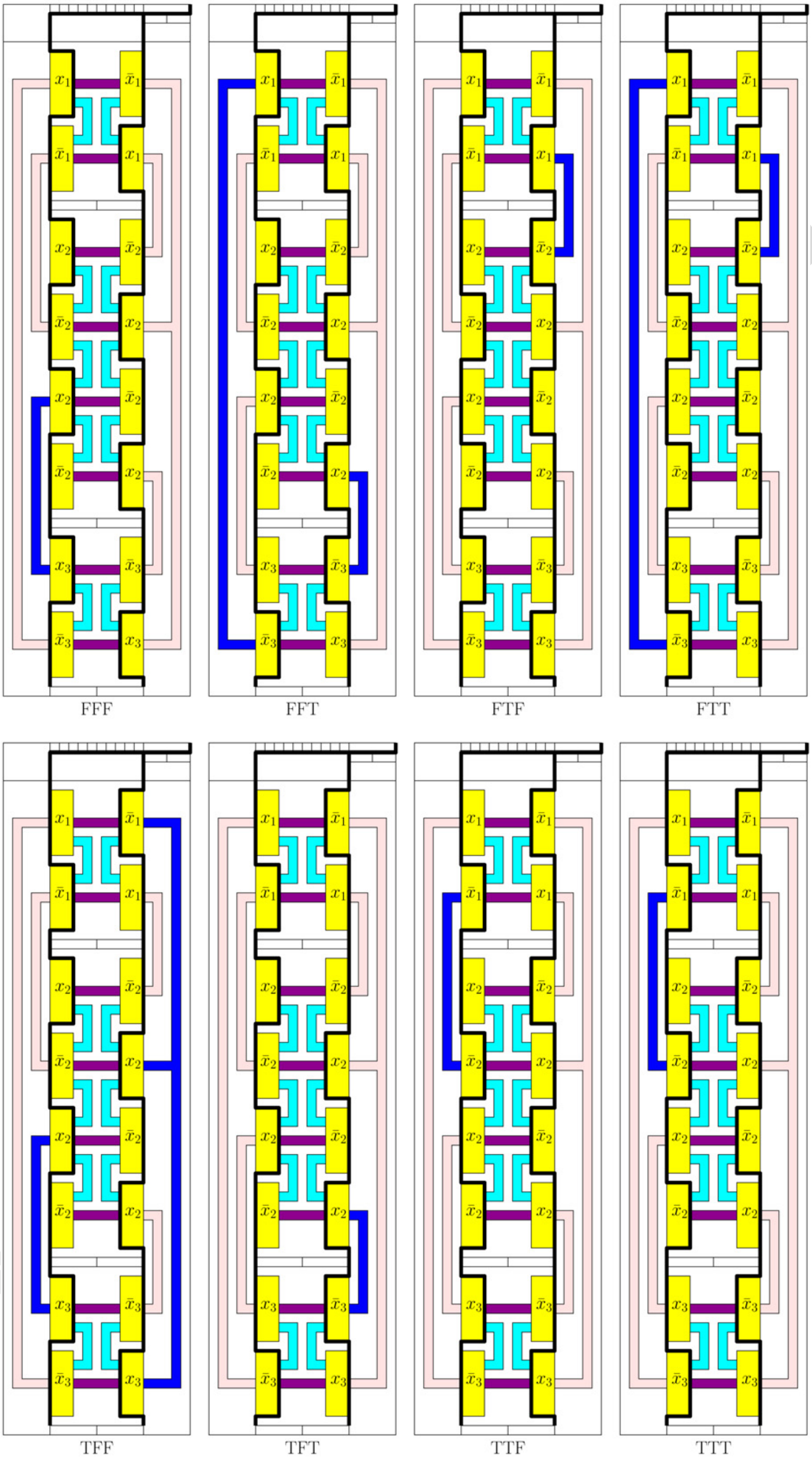



Fig. 12. Partial corridors for instance $f_c(I)$ constructed from Example 2.

segments on its left and right sides. Clearly the total length of the corridor is exactly $B$. By construction and the fact that assignment $A$ satisfies all the clauses, all the polygons, including the ones for the clause-checking components, are exposed to the corridor. Therefore the instance $f(I)$ has a feasible corridor.

We establish now that if the instance $f(I)$ has a feasible corridor then the instance $I$ is satisfiable. Let $S$ be a corridor of length at most $B$ that starts at the top-right corner of the rectangle $F$. By using a proof similar to Lemma 5.2 we can show that a feasible corridor cannot zigzag from the left edge to the right edge of the setting components, and one can establish that a feasible corridor cannot traverse the edges of the clause-checking components that do not coincide with the repository rectangles. So it must be that the corridor $S$ traverses each setting component as shown in Fig. 8(a) or (b), and the transition to the next setting component is by using the left and right sides of the setting-terminator component. The way the corridor visits the setting components (shown in Fig. 8(a) or (b)) corresponds to the values of true or false that we assign to the Boolean variables $X$. Since the length of the segments of the corridor we have identified is equal to $B$, it then follows that no other segments can be in $S$. Since all the clause-checking components are exposed to the corridor $S$, it follows from our construction rules, that the values assigned to the variables satisfy all the clauses. Therefore instance $I$ is satisfiable. □

## 6. TRA-MLC-R problem

In this section we establish that the TRA-MLC-R problem is NP-complete, i.e., the TRA-MLC problem when the rectangular boundary $F$ is partitioned into rectangles. The reduction in the previous section does not apply here because some of the rectilinear polygons are not rectangles. For example, the white basic setting component polygons in Fig. 6 have 8 or 12 corners each, and the vertical-fixing octagons have 8 corners. Rectilinear polygons that lie between the clause-checking components in Figs. 11 and 12 may have more than 12 corners.

Our reduction in this section follows the same approach as the one in the previous section, however it is much more complex. As we said before, the reduction in this section implies the result in the previous section. However, once you understand the reduction in the previous section, the one in this section is easier to follow.

The overall architecture of our construction is the same as the one given in Fig. 5. We have the same type of components with the same functionality as in the previous reduction, but their internal composition is different. Also, the corridor must include several "short" line segments, to join the clause-checking components to the corridor along the variable-repository regions. This is different from the reduction in the previous section where additional segments to expose the clause-checking component were not required.

As in the previous section, for any instance $I = (X, C) \in$ P3SAT given in a canonical planar embedding $D(G_I)$, our polynomial time transformation constructs the instance $f(I) \in$ TRA-MLC-R. In what follows we discuss separately each of the components in our reduction and then explain their interaction.

*Setting component.* Each setting component is associated with a Boolean variable and the way the corridor visits its rectangles identifies the assignment of a value to the variable. For Boolean variable $x_i$, a setting component consists of $s_i$ *basic setting* components stacked on top of each other. The basic setting component is formed by: *variable-repository regions*, *horizontal-fixing regions*, and *vertical-fixing regions.*

A *basic* setting component, $s$, has four variable-repository regions (light gray colored), two vertical-fixing regions (dark gray colored), and two horizontal-fixing regions (white) with five, eighteen, and five rectangles each one, respectively (Fig. 13). The rectangles $r_1$, $r_2$, $r_3$, $l_1$, $l_2$, and $l_3$ as well as the special points $a_1(z)$, $a_2(z)$, $b_1(z)$, $b_2(z)$, $c_1(z)$, $c_2(z)$, $d(z)$, $e(z)$, $f_1(z)$, $f_2(z)$, $g_1(z)$, $g_2(z)$, $h(z)$, $i_1(z)$, and $i_2(z)$, for $z \in \{L, R\}$, are identified in Fig. 13. These names will be used later on. The rectangles inside the horizontal-fixing regions are referred to as *middle* rectangles. The remaining rectangles are referred to as *left* or *right* rectangles depending on the side of the component where they reside. The rectangle that includes the center point of a horizontal-fixing region is called the *h-central* rectangle.

As in our previous reduction the variable-repository region represents the literals $x_i$ and $\bar{x}_i$. The top-left variable-repository region corresponds to $x_i$, and the top-right variable-repository region corresponds to $\bar{x}_i$. As we proceed downwards the literals are assigned to the variable-repository regions in an alternating way.

Let $h_b$ and $w_b$ be the height and width of $s$, respectively. The value of $h_b$ is equal to $240\gamma$ and $w_b$ is equal to $6\gamma + w_I$, where $w_I$ has the value $988\gamma$. Remember that $\gamma$ is defined in Section 4. The height $h_r$ and width $w_r$ of the variable-repository region is $60\gamma$ and $3\gamma$, respectively. Fig. 14 shows the length of each line segment of the basic setting component $s$. The dimensions for the vertical-fixing and horizontal-fixing regions are also given in Fig. 14. The rectangles of size $18\gamma$ by $\gamma$ in the vertical-fixing regions are called the *v-central rectangles*.

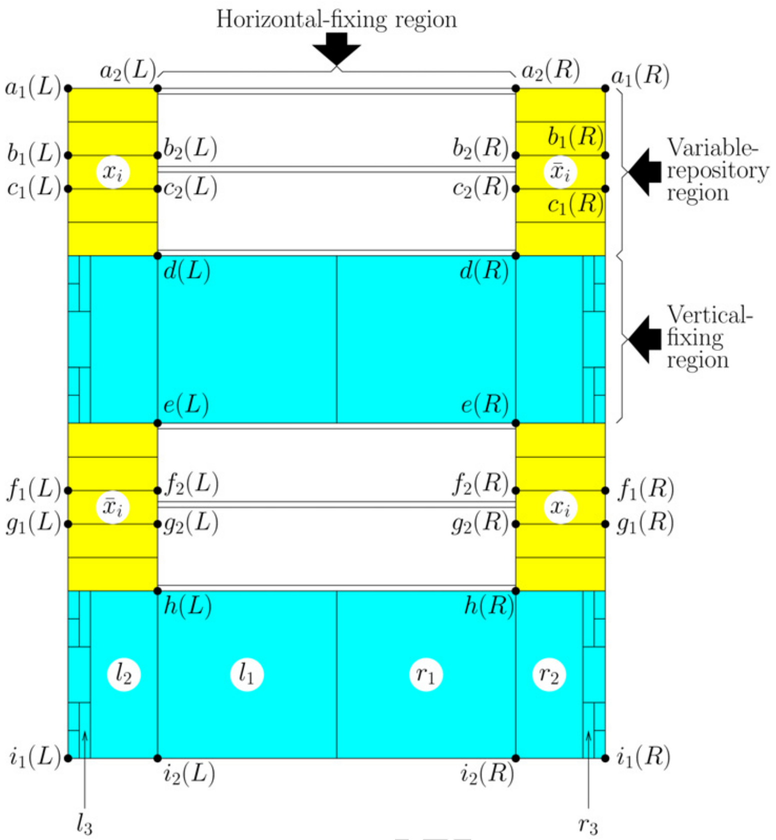



Fig. 13. Basic setting component architecture.



Suppose that there is a partial corridor $PC$ that ends both on the top-left and top-right corners of the basic setting component $s$ and does not include any other point of $s$. As in the previous section, remember that every partial corridor includes the top-right corner of the rectangular boundary $F$.

Consider all the possible sets of line segments that extend the partial corridor $PC$ to reach all the rectangles inside the basic setting component $s$ except for either some of the rectangles $r_1$, $r_2$, and $r_3$, or some of the rectangles $l_1$, $l_2$, and $l_3$. Additionally, each of those sets of line segments inside $s$ reaches its bottom-left and bottom-right corners in order to extend the partial corridor to the other basic setting component placed below $s$. The collection $\mathcal{S}$, formally defined below, includes a special subset of the resulting partial corridors. As we prove below, every partial corridor $S \in \mathcal{S}$ consists of $PC$ plus either one of the two sets of line segments of the general form given in Fig. 15.

Formally, the collection $\mathcal{S}$ consists of all partial corridors $S$ that satisfy the following properties:

1. Partial corridor $S$ includes $PC$ as well as the bottom-left and bottom-right corners of the basic setting component $s$, and at least one point from each of the rectangles inside $s$ except for either some of the rectangles $r_1$, $r_2$, and $r_3$, or some of the rectangles $l_1$, $l_2$, and $l_3$. For any given partial corridor $S$ the exception only applies to one of the two sets of rectangles.
2. All segments in $S \setminus PC$ must be inside or on the boundary of $s$.
3. $L(S \setminus PC) \leqslant 494\gamma$.

The *exterior edge* of the variable-repository region $r$ is the intersection of the boundary of $s$ and the vertical boundary edges of the variable-repository region $r$. We say that a set $S' = S \setminus PC$ of line segments for a partial corridor $S \in \mathcal{S}$ is *completely visible* from the outside of the basic setting component $s$ at the variable-repository region $r$ if $S'$ includes the exterior edge of $r$. When it includes at most two points (the top and bottom point of the exterior edge of $r$) we say that $S'$ is *hidden* from the outside of $s$ at the variable-repository region $r$. We say it is *partially hidden* from the outside of $s$ at the variable-repository region $r$ if it includes only a non-empty portion of the exterior edge of $r$. Note that a set $S'$ that is hidden might also be partially hidden. In the following lemma we show that every

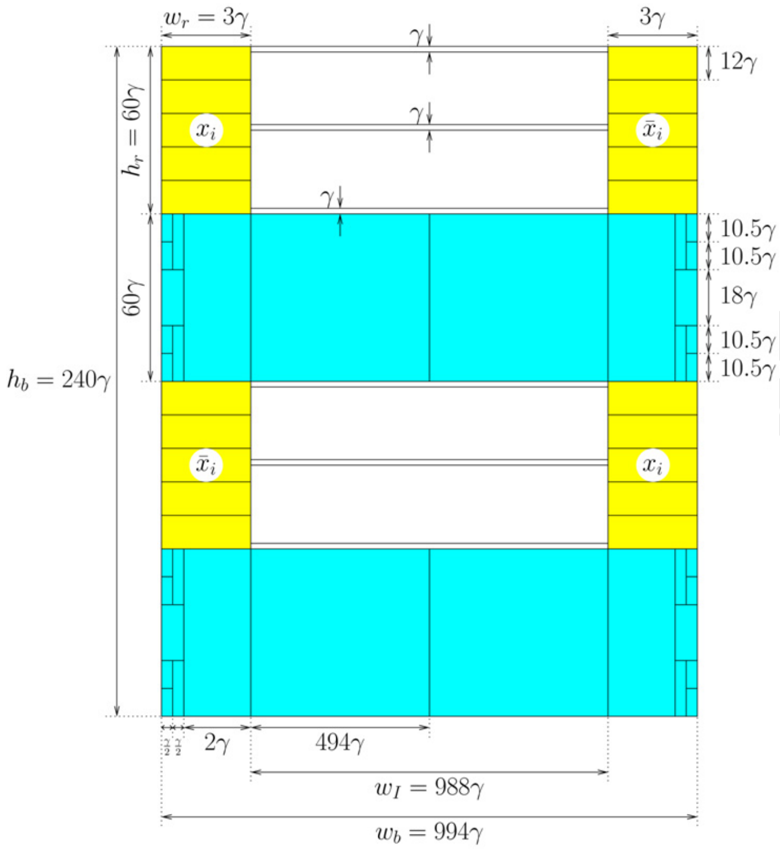
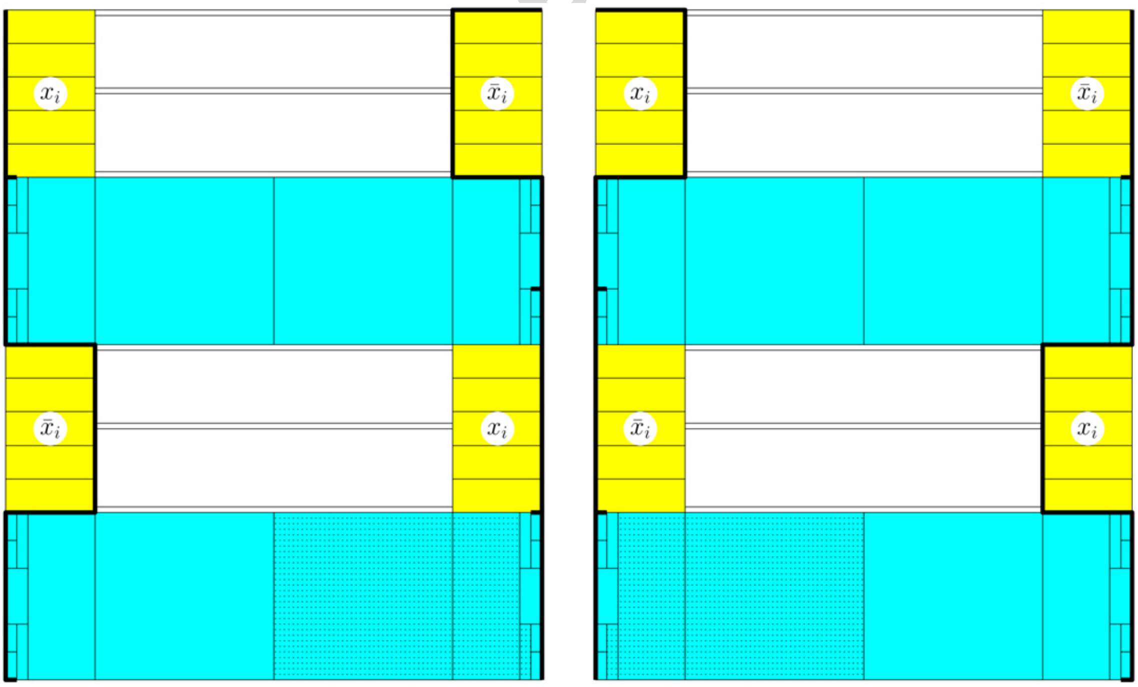



Fig. 14. Measurements for the basic setting component.




(a) $x_i = true$ (note that $r_1$, $r_2$, and $r_3$ are not reached)

(b) $x_i = false$ (note that $l_1$, $l_2$, and $l_3$ are not reached)

Fig. 15. Sets $S \setminus PC$ of line segments for a basic setting component corresponding to setting $x_i$ to the values of true and false.

set $S \setminus PC$ of line segments is either hidden at the $x_i$ variable-repository regions and completely visible at the $\bar{x}_i$ variable-repository regions or vice versa. In this sense all the sets $S \setminus PC$ of line segments, for $S \in \mathcal{S}$, are equivalent to the ones (thick line segments) given in Fig. 15. It is important to note that when the set $S \setminus PC$ of line segments is completely visible from the outside of the basic setting component at a variable-repository region that represents $x_i$, then the variable $x_i$ has the value of true, and when the set $S \setminus PC$ of line segments is hidden from the outside of the basic setting component at a variable-repository region that represents $x_i$, the variable $x_i$ has the value of false.

The following lemma characterizes the set of line segments $S \setminus PC$ for $S \in \mathcal{S}$.

**Lemma 6.1.** *The set $\mathcal{S}$ is not empty an every set $S \setminus PC$ of line segments for the partial corridor $S \in \mathcal{S}$ is either hidden at both of the $x_i$ variable-repository regions and completely visible at both of the $\bar{x}_i$ variable-repository regions, or vice versa. Furthermore, $L(S \setminus PC) = 494\gamma$.*

**Proof.** Let $S$ be any set in $\mathcal{S}$. By definition $L(S \setminus PC) \leqslant 494\gamma$. We will show that $S$ satisfies the conditions of the lemma. Since $w_I = 988\gamma$ it must be that the set $S \setminus PC$ of line segments consists of two disjoint sets of line segments which we will refer to as the set $S(L)$ of left line segments (along the rectangles labeled left) and the set $S(R)$ of right line segments (along the rectangles labeled right). The set $S(L)$ includes the top-left ($a_1(L)$) and the bottom-left ($i_1(L)$) corners of the basic setting component and $S(R)$ includes the top-right ($a_1(R)$) and bottom-right ($i_1(R)$) corners of the basic setting component. Furthermore, each of these two sets of line segments do not reach any rectangles on the opposite side of the basic setting component.

Clearly, there must be one path from $a_1(L)$ to $i_1(L)$ in $S(L)$, and one path from $a_1(R)$ to $i_1(R)$ in $S(R)$. We refer to these paths as $p(L)$ and $p(R)$, respectively. The vertical distance from the top-left to the bottom-left and from the top-right to the bottom-right corners of the basic setting component is $240\gamma$. Therefore the total edge-length of the vertical line segments in $p(L)$ and $p(R)$ is at least $480\gamma$.

The sets of line segments $S(L)$ and $S(R)$ fall into two types depending on whether all of the rectangles $l_1$, $l_2$ and $l_3$, or $r_1$, $r_2$ and $r_3$ are exposed to the corridor.

Case 1: All the rectangles $l_1$, $l_2$ and $l_3$ are exposed to the corridor.

Since all the vertical fixing rectangles are exposed to the partial corridor $S$ it must be that $d(L)$ or $e(L)$, and $h(L)$ or $i_2(L)$ must be part of $S(L)$. If any of these four points are in $p(L)$, then $p(L)$ has horizontal line segments with length at least $6\gamma$. Otherwise, the paths from any of these four points to $p(L)$ have horizontal line segments with length at least $6\gamma$, or the total length of the vertical line segments in $S(L) \setminus p(L)$ is at least $60\gamma$. In the latter case the total length of the segments in $S$ exceeds $494\gamma$. In all the remaining cases we know that the length of the horizontal line segments in $S(L)$ is at least $6\gamma$.

Since all the rectangles in the top vertical-fixing region are exposed to $S$, it must be that $d(R)$ or $e(R)$ must be part of $S(R)$. Therefore, the length of the horizontal line segments in $S(R)$ must be at least $3\gamma$.

The horizontal and vertical line segments so far identified in $S(L)$ and $S(R)$ have total length at least $489\gamma$. The remaining line segments in $S(L)$ and $S(R)$ must have length at most $5\gamma$.

We claim that the line segments $(d(L), e(L))$, $(h(L), i_2(L))$, $(d(R), e(R))$, and $(h(R), i_2(R))$ may not be part of the corridor $S$. The reason is that if any such segments were present then one would need vertical line segments with length at least $21\gamma$ to reach the v-central rectangle of the corresponding vertical-fixing region. But then there would be line segments with length at least $21\gamma$ not included with the segments with length at least $489\gamma$ previously identified and the $494\gamma$ bound would be exceeded.

The set $S(L)$ must include exactly one of the segments $(b_1(L), c_1(L))$ or $(b_2(L), c_2(L))$, and exactly one of the segments $(f_1(L), g_1(L))$ or $(f_2(L), g_2(L))$. Note that if either line segments $(b_1(L), c_1(L))$ and $(b_2(L), c_2(L))$, or $(f_1(L), g_1(L))$ and $(f_2(L), g_2(L))$ are part of $S(L)$ then there would be line segments with length at least $12\gamma$ that are not included in the previous count and the $494\gamma$ bound would be exceeded. The same argument can be used for $S(R)$.

We now show that if $(b_2(L), c_2(L))$ and $(f_2(L), g_2(L))$ are both in $p(L)$ then the total length of the segments in $S(L)$ is greater than $494\gamma$. The reason for this is that the path from $a_1(L)$ to $b_2(L)$ must contain horizontal line segments with length at least $3\gamma$. The same holds for the path from $g_2(L)$ to $i_1(L)$. As we established above the segment $(d(L), e(L))$ is not part of $S(L)$. Therefore, the vertical line segments in $S(L) - p(L)$ and the horizontal line segments in $S(L)$ located between line segments $(c_1(L), c_2(L))$ and $(f_1(L), f_2(L))$ (including both lines) must have length at least $6\gamma$. Therefore we have identified line segments in $S(L)$ with length at least $12\gamma$ which together with the horizontal segments in $S(R)$ with length at least $3\gamma$ and the $480\gamma$ vertical segments in $p(L)$ and $p(R)$ exceeds $494\gamma$. So $S(L)$ must contain at most one of $(b_2(L), c_2(L))$ and $(f_2(L), g_2(L))$. The same argument can be used to show that $S(R)$ must contain at most one of $(b_2(R), c_2(R))$ and $(f_2(R), g_2(R))$.

Lets now consider the h-central rectangles. It cannot be that $(b_1(L), c_1(L))$ is in $p(L)$ and $(b_1(R), c_1(R))$ is in $p(R)$ because then there is at least one vertical line segments with length $5.5\gamma$ that is not in $p(L)$ or $p(R)$ joining the topmost h-central rectangle to at least one of the points $b_1(L)$, $c_1(L)$, $b_1(R)$, or $c_1(R)$. This segment plus the previously identified ones have length greater than $494\gamma$. So it cannot be that $(b_1(L), c_1(L))$ is in $p(L)$ and $(b_1(R), c_1(R))$ is in $p(R)$. The same arguments can be used to show that it cannot be that $(f_1(L), g_1(L))$ is in $p(L)$ and $(f_1(R), g_1(R))$ is in $p(R)$.

Suppose now that $(b_2(L), c_2(L))$ and $(f_1(L), g_1(L))$ is in $p(L)$ and $(b_1(R), c_1(R))$ and $(f_2(R), g_2(R))$ is in $p(R)$. Since $(f_2(R), g_2(R))$ is in $p(R)$ we know that the horizontal line segments in $p(R)$ must have length at least $6\gamma$. Since $(b_2(L), c_2(L))$ is in $p(L)$ the horizontal line segments in the path $p(L)$ from $a_1(L)$ to $g_1(L)$ must have length at least $6\gamma$. The horizontal line segments below the line $(f_1(L), f_2(L))$ must be at least $3\gamma$ since one of the vertices $h(L)$ and $i_2(L)$ must be part of $S(L)$. Therefore we have identified segments with length at least $15\gamma$ which together with the vertical segments identified in $p(L)$ and $p(R)$ will exceed the $494\gamma$.

It is easy to show that the only remaining possibility is when $(b_1(L), c_1(L))$ and $(f_2(L), g_2(L))$ is in $p(L)$ and $(b_2(R), c_2(R))$ and $(f_1(R), g_1(R))$ is in $p(R)$. Since line segment $(f_2(L), g_2(L))$ is in $p(L)$ there must be at least $6\gamma$ horizontal segments in $p(L)$, and since line segment $(b_2(R), c_2(R))$ is in $p(R)$ there must be at least $6\gamma$ horizontal segments in $p(R)$. In order for all the rectangles in the horizontal-fixing region to be exposed to $S$, the rectangle in the upper horizontal-fixing region must be exposed to $S(R)$ and the lower rectangle in the horizontal-fixing region must be exposed to $S(L)$. To expose all the rectangles in the vertical-fixing region one needs four vertical segments each with length $0.5\gamma$. The total length of the segments is equal to $494\gamma$. So there are no other segments in $S$ and the corridor is visible at the $x_i$ variable-repository rectangles and hidden at the $\bar{x}_i$ variable-repository rectangles. Fig. 15(a)) gives one such corridor.

Case 2: Some of the rectangles $l_1$, $l_2$ and $l_3$ might not be exposed to the corridor.

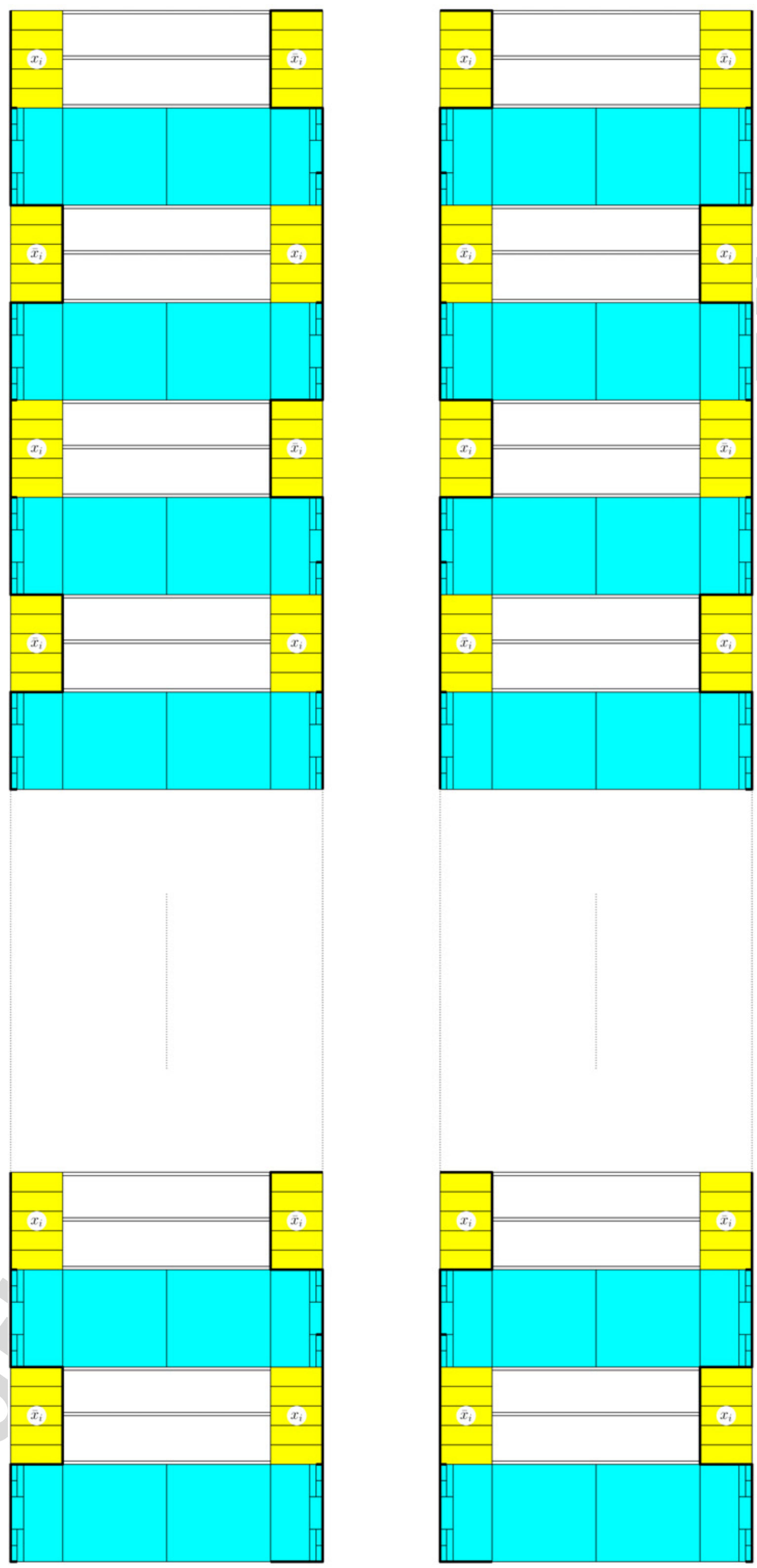
A proof similar to the one for Case 1 can be used to show that in this case the total length of the segments in $S(L)$ and $S(R)$ is equal to $494\gamma$, and the corridor is hidden at the $x_i$ variable-repository rectangles and visible at the $\bar{x}_i$ variable-repository rectangles. Fig. 15(b)) gives one such corridor. This completes the proof of the lemma. □

From Lemma 6.1, the only two possible types of partial corridors in $\mathcal{S}$ have length equal to $494\gamma$ plus $L(PC)$, and are completely visible at both of the variable-repository regions for $x_i$ and hidden at both of the variable-repository regions for $\bar{x}_i$, or vice versa. The line segments given in Fig. 15(a) correspond to the variable $x_i$ being assigned the value of true and the ones in Fig. 15(b) correspond to the variable $x_i$ being assigned the value of false. As in the previous reduction, it is simple to see that the rectangles of the horizontal-fixing region ensure that one of the left and right variable-repository regions will have the corridor running through its internal boundary edges, while the rectangles of the vertical-fixing region guarantee an alternating detour behavior of the corridor going through the variable-repository region on each of the sides.

The setting component associated to the variable $x_i$ consists of $s_i$ basic setting components stacked on top of each other as shown in Fig. 16. Remember that the size, $s_i$, of the setting component is defined in Section 4 as $\max\{\lambda_i, \rho_i\}$.

As in the case of Lemma 5.2 one can establish that the line segments in a partial corridor that include the top-left, top-right, bottom-left and bottom-right of the setting component for $x_i$ have total length at least $494\gamma \cdot s_i + 3\gamma$. Note that the last term is added to reach all the rectangles in the bottommost basic setting component, i.e., to include $l_1$, $l_2$ and $l_3$, or $r_1$, $r_2$ and $r_3$ from the bottom basic setting component. One can also establish that all the feasible sets of line segments are of the form given in Fig. 16, in the sense that either they are completely visible at all the $x_i$ variable-repository regions and hidden at all the $\bar{x}_i$ variable-repository regions, or vice versa. The set of line segments of the form given in Fig. 16(a) corresponds to the value of $x_i$ equal to true and the one in Fig. 16(b) corresponds to the value of false. We establish these claims in Lemma 6.2.

Let $\mathcal{Q}$ be the obvious generalization of $\mathcal{S}$ for the basic setting component to the setting component, except that all the rectangles must be exposed to the partial corridor (including the ones in the bottommost basic-setting component). In this case every partial corridor $S \in \mathcal{Q}$ has length at most $494\gamma \cdot s_i + 3\gamma$ plus $L(PC)$.



(a) $x_i = true$ (b) $x_i = false$

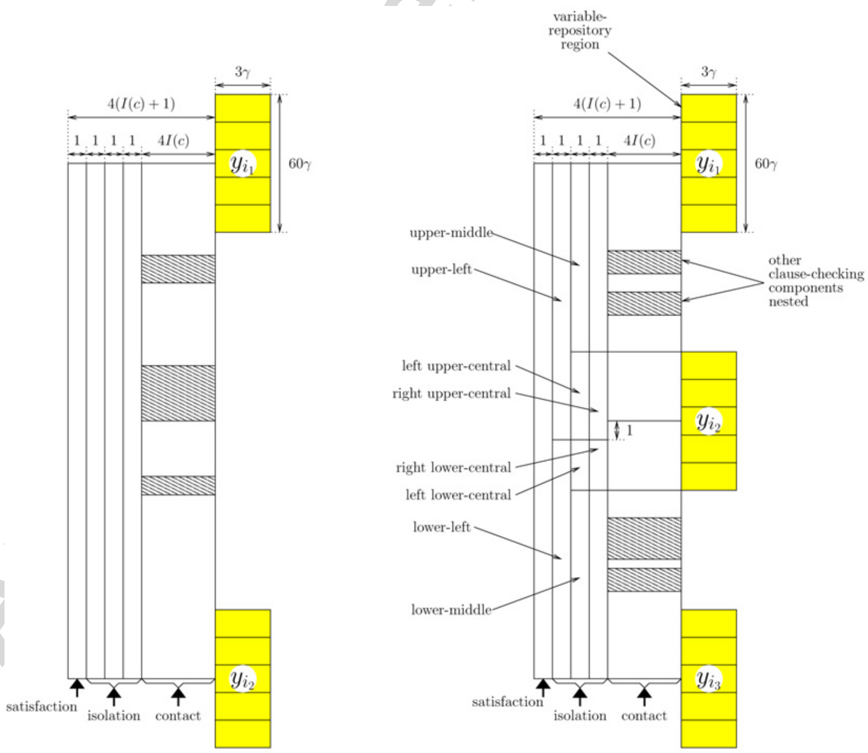
Fig. 16. Sets of line segments for a setting component corresponding to setting $x_i$ to the values of true and false.

**Lemma 6.2.** *The set $\mathcal{Q}$ is not empty and every set of line segments $S \setminus PC$ for the partial corridor $S \in \mathcal{Q}$ must be of the form given in Fig.* 16 *in the sense that every set of line segments $S \setminus PC$ is either completely visible at all the $x_i$ variable-repository regions and hidden at all the $\bar{x}_i$ variable-repository regions, or vice versa. Furthermore, $L(S \setminus PC) = 494\gamma \cdot s_i + 3\gamma$.*

**Proof.** The proof follows from Lemma 6.1 and the fact that the set of line segments from every partial corridor for adjacent basic setting components in the setting component must be of the same form and thus expose the rectangles $r_1$, $r_2$, and $r_3$, or $l_1$, $l_2$, and $l_3$ that were not previously exposed. These corridors have total segment length equal to $494\gamma \cdot s_i$. The extra $3\gamma$ is used to expose the uncovered $r_1$, $r_2$, and $r_3$, or $l_1$, $l_2$, and $l_3$ rectangles in the bottommost basic setting component. □

*Clause-checking component.* The clause-checking component, for each clause $c \in C$, consists of a rectangle that is partitioned into three regions called: *contact*, *isolation*, and *satisfaction* regions. The overall architecture of the clause-checking component is given in Fig. 17. Notice that the architecture is very similar for the clauses with two and three variables. The contact region of the clause-checking component for clause $c$ must be adjacent to certain variable-repository regions corresponding to the literals in the clause. These variable-repository regions are said to be associated with the clause-checking component. These variable-repository regions will not be associated with other clause-checking components. We will elaborate on this property later on.

Fig. 17(a) shows the horizontal dimensions for the regions of the clause-checking component for $c = \{y_{i_1}, y_{i_2}\}$ such that $c \in N_2$, $c \in L$, and $y_{i_1}$ ($y_{i_2}$, resp.) is either $x_{i_1}$ or $\bar{x}_{i_1}$ ($x_{i_2}$ or $\bar{x}_{i_2}$, resp.). Fig. 17(b) shows the regions for the clause-checking component for $c = \{y_{i_1}, y_{i_2}, y_{i_3}\}$ such that $c \in N_3$, $c \in L$, and $y_{i_3}$ is defined similarly as $y_{i_1}$ and $y_{i_2}$ above. The clause-checking components on the right hand side are symmetric. On the vertical axis the three regions of the clause-checking component for $c \in N_2$ span from the middle of the variable-repository region for literal $y_{i_1}$ on the basic setting component number $t(c)$ (from top) for variable $x_{i_1}$, to the middle of the variable-repository region for literal $y_{i_2}$ on the basic setting component number $b(c)$ (from top) for variable $x_{i_2}$. Remember that $t(c)$ and $b(c)$ for




(a) Two literals (b) Three literals

Fig. 17. Clause-checking components.

each clause $c \in C$ is defined in Section 4. For $c \in N_3$ the three regions span from the middle of the variable-repository region for literal $y_{i_1}$ on the basic setting component number $t(c)$ (from top) for variable $x_{i_1}$, to the middle of the variable-repository region for literal $y_{i_3}$ on the basic setting component number $b(c)$ (from top) for variable $x_{i_3}$. The middle variable-repository region that is used for literal $y_{i_2}$ is the basic setting component number $m(c)$ (from top) for variable $x_{i_2}$. Remember that $m(c)$ is also defined in Section 4. The width of the clause-checking component is $4(I(c)+1)$ and the width of the contact region is $4I(c)$. The isolation region is three units wide and the satisfaction region has unit width.

The clause-checking component for clause $c = \{y_{i_1}, y_{i_2}\} \in N_2$ is simple. The satisfaction region is just one rectangle and refer to it as the *satisfaction rectangle*. The isolation region consists of three rectangles. But the contact region may consist of several rectangles. In Fig. 17(a) the shaded regions correspond to clause-checking components nested directly inside the clause-checking component for $c$, i.e., the clauses corresponding to those clause-checking components have indentation value equal to $I(c) - 1$ and their clause nodes are inside the ring for $c$ in $D(G_I)$. In Fig. 17(a) there are three clause-checking components nested directly, so the contact region consists of four rectangles, excluding the three shaded regions.

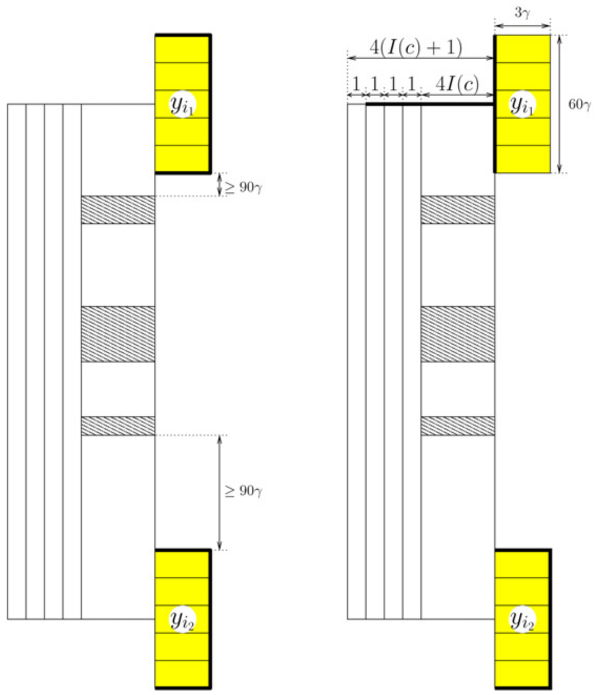
Suppose that every clause-checking component for the clauses in $C$ is connected to the partial corridor $S$ that goes through the setting-components, except for one clause $c \in N_2$. Furthermore, suppose the connection for $c' \in C - \{c\}$ is by line segments with length $4I(c') + 3 < \gamma$ when $c' \in N_2$ and $12I(c') + 6 < 3\gamma$ when $c' \in N_3$.

From the construction one can easily verify that the shortest segments needed to connect the satisfaction rectangle of the clause-checking component for clause $c \in N_2$ to the variable-repository region corresponding to its literals, $y_{i_1}$ and $y_{i_2}$, is by adding a line segment with length $4I(c) + 3$ that goes through the top or bottom boundaries of the contact and isolation regions (see Fig. 18(b)). When the partial corridor is completely visible at either of the variable-repository regions corresponding to the literals in clause $c$, i.e., one of the literals has the value of true, then the total length of the line segments needed to expose the satisfaction rectangle to the corridor is exactly $4I(c) + 3$. When both of the literals have the value of false (see Fig. 18(a)), any possible connection has edge-length greater than $\gamma$ (by definition, we know that $\gamma > 4I(c) + 3$). This is because the isolation region prevents any direct connection to the satisfaction region of the clause checking component for $c$ from partial corridor line segments for clause-checking components that are nested directly inside the one for $c$, or from those going through the variable-repository regions. Also, clause-checking components that have the clause-checking component for $c$ nested directly inside them have all their corridor line segments at a distance greater than $\gamma$ from the satisfaction rectangle for the clause-checking component for $c$. Therefore, when both the literals have the value of false, the line segments needed to expose the satisfaction rectangle have total length greater than $\gamma$. We summarize these observations in the following lemma.

**Lemma 6.3.** *Suppose that there is a partial corridor $S \in \mathcal{Q}$ that reaches all the setting components and it is of the form given by Lemma 6.2. Suppose that the connection from every clause-checking component $c' \in C$, except for the one for $c \in N_2$, joins to the partial corridor $S$ by line segments with length $4I(c') + 3 < \gamma$ for $c' \in N_2$ and $12I(c') + 6 < 3\gamma$ for $c' \in N_3$, originating at the variable-repository regions associated with them. Then by adding line segments with length $4I(c) + 3$ to the partial corridor $S \in \mathcal{Q}$, it is possible to expose all the rectangles in the clause-checking component for clause $c \in N_2$ if, and only if, the partial corridor corresponds to an assignment of values to the variables such that at least one of the literals for clause $c$ has the value true.*

**Proof.** By the above discussion. □

For clause $c = \{y_{i_1}, y_{i_2}, y_{i_3}\} \in N_3$, the construction is more complex. As before, the satisfaction region is just one rectangle, that we refer to as the *satisfaction rectangle*. The isolation region is subdivided into three equal-width regions, and each of these regions consists of several rectangles as shown in Fig. 17(b). The contact region contains at least four rectangles. There are exactly four rectangles when the clause-checking component corresponds to a clause whose rings in $D(G_I)$ do not contain any clause vertices inside them. Fig. 17(b) shows an example where the contact region consists of 8 rectangles, excluding the four shaded regions. The shaded regions correspond to clause-checking components for clauses that are nested directly within clause $c$, i.e., these clauses have indentation value equal to $I(c) - 1$ and their clause vertices are inside the ring for $c$.




(a) $y_{i_1} = y_{i_2} = false$ (b) $y_{i_1} = true,\ y_{i_2} = false$

Fig. 18. Possible sets of line segments for a two-literal clause-checking component.

Suppose that every clause-checking component for the clauses in $C$ is connected to the partial corridor $S$ that goes through the setting-components, except for one clause $c \in N_3$. Furthermore, suppose the connection for $c' \in C - \{c\}$ is by line segments with length $4I(c') + 3 < \gamma$ when $c' \in N_2$ and $12I(c') + 6 < 3\gamma$ when $c' \in N_3$.

From the construction one can easily verify that the only way one may expose the satisfaction rectangle to the corridor that goes through the setting components is by introducing line segments. More specifically, the segments have total length equal to $4I(c) + 3$ if the segments emanate from the top or bottom variable-repository region associated with the clause, and $4I(c) + 4$ when they emanate from the middle variable-repository region. Since we are restricted (at this time) to partial corridors $S \in \mathcal{Q}$, this can only happen when the partial corridor is completely visible at a variable-repository region corresponding to the literals in the clause $c$, i.e., one of its literals has the value true (this assumes that the partial corridor at each setting component is of the form given in Fig. 16). On the other hand, when the three literals have the value of false (Fig. 19(a)), any possible connection to the satisfaction rectangle has edge length greater than $3\gamma$. This is because the isolation region prevents any direct connection to the satisfaction rectangle of the clause-checking component for $c$ from other partial corridor segments for clause-checking components that are nested directly inside the one for $c$, or from those going through the variable-repository regions. Also, the partial corridor segments in a clause-checking component that have the clause-checking component for $c$ nested directly inside it are at a distance greater than $3\gamma$ from the satisfaction rectangle for the clause-checking component for $c$. Therefore, when the three literals in $c$ have the value of false, the line segments needed to expose the satisfaction rectangle will have total length greater than $3\gamma$.

It is simple to see that all the rectangles in the contact region are exposed to the partial corridor $S$. However, we need to show how to expose the other rectangles in the isolation region to the corridor. We now establish that line segments with length at least $12I(c) + 6$ are needed to expose all the rectangles in the clause-checking component. There are two cases depending on which line segments are added to expose the satisfaction rectangle to $S$.

Case 1: The line segments added to expose the satisfaction rectangle join to $S$ in the middle of the top or the bottom variable-repository region associated with the clause $c$.

Assume without loss of generality the connection is through the top variable-repository region. Clearly, the length of the line segment introduced so far is $4I(c) + 3$. Some of the rectangles that need to be exposed to the corridor are the left upper-central, left lower-central, lower-left and lower-middle rectangles. There are three sub-cases depending on the set of line segments introduced to expose the lower-left rectangle.

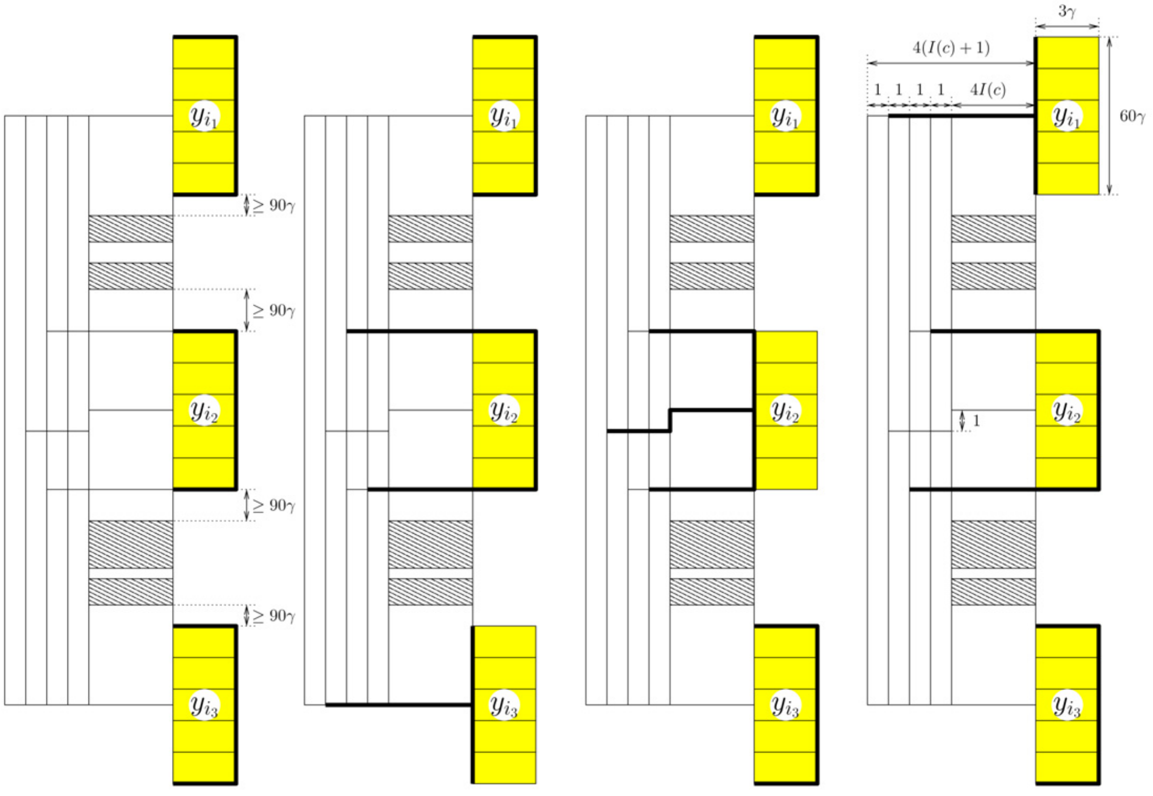
Case 1.1: The lower-left rectangle is exposed to the corridor by introducing two horizontal and a vertical line segment starting in the middle point of the middle variable-repository region associated with the clause $c$.
These three line segments have total length equal to $4I(c) + 3$. The lower-middle rectangle needs to be joined to $S$. This requires line segments with length equal to $4I(c) + 1$. This final segment also exposes the remaining rectangles to $S$ in the clause-checking component. All the segments introduced have total length equal to $12I(c) + 7$.

Case 1.2: The lower-left rectangle is exposed to $S$ by introducing a line segment starting in the bottom point of the middle variable-repository region associated with clause $c$.
This segment has length equal to $4I(c) + 2$. The left upper-central rectangle needs to be joined to $S$. This requires a segment with length equal to $4I(c) + 1$. This final segment also exposes all the rectangles in the clause-checking component and the line segments introduced have total length equal to $12I(c) + 6$. This corresponds to Fig. 19(d) (Fig. 19(b) shows the symmetric case).

Case 1.3: The lower-left rectangle is exposed to $S$ by introducing a line segment starting in the middle point of the bottom variable-repository region associated with the clause $c$.
This segment has length equal to $4I(c) + 2$. We need to expose the left upper-central and left lower-central rectangles. This requires line segments of length at least equal to $4I(c) + 2$. This final segment exposes all the rectangles in the clause-checking component and the line segments introduced have total length equal to $12I(c) + 7$.

Case 2: The line segments added to expose the satisfaction rectangle join to $S$ in the middle of the middle variable-repository region associated with the clause $c$.






(a) $FFF$ (b) $FFT$ (c) $FTF$ (d) $TFF$

Fig. 19. Possible sets of line segments for a three-literal clause-checking component.

Clearly, the total length of the additional segments introduced so far is $4I(c) + 4$. Some of the rectangles that need to be exposed to the corridor are the upper-middle and lower-middle rectangles. Lets consider the possible ways to expose the upper-middle rectangle since the other case (exposing the lower-middle rectangle) is similar. One can expose the upper-middle rectangle through the middle point of the top variable-repository region associated with clause $c$ by introducing line segments with length $4I(c) + 1$; or through the top point of the middle variable-repository region associated with clause $c$ by line segments with length $4I(c) + 1$. So the segments introduced have total length equal to $12I(c) + 6$ in all cases. This corresponds to Fig. 19(c).

Notice that the feasible set of line segments extending the partial corridor $S \in \mathcal{Q}$ with length $12I(c) + 6$ for Cases 1.2 and 2 do not have vertical line segments except for the one with length one parallel to the middle variable-repository region (see Fig. 17(b)). Thus, we can establish that no other partial corridors different to the ones analyzed in Cases 1.2 and 2 above are shorter than $12I(c) + 6$ as they all include at least one vertical line segment with length greater than $3\gamma > 12(I(c) + 6)$ connecting the satisfaction rectangle associated with clause $c$ to the partial corridor connecting clause-checking components for clauses nested inside one of the rings in $D(G_I)$ for clause vertex $c$, or to the partial corridor connecting clause-checking components of clauses that have the clause vertex $c$ nested inside one of their rings in $D(G_I)$.

The above arguments can be used to prove the following lemma.

**Lemma 6.4.** *Suppose that there is a partial corridor $S \in \mathcal{Q}$ that exposes all the setting components and it is of the form given by Lemma 6.2. Suppose that every clause-checking component $c' \in C$, except for the one for $c \in N_3$, is exposed to the partial corridor $S$ by line segments with length $4I(c) + 3 < \gamma$ for $c' \in N_2$ and $12I(c) + 6 < 3\gamma$ for $c' \in N_3$, originating at the variable-repository regions associated with them. Then by adding line segments with length $12I(c) + 6$ to the partial corridor $S \in \mathcal{Q}$, it is possible to expose all the rectangles associated with the clause-checking component for clause $c \in N_3$ if, and only if, the partial corridor $S$ corresponds to an assignment of values to the variables such that at least one of the literals for clause $c$ has the value true.*

**Proof.** By the above discussion. □

*Setting-terminator component.* The setting-terminator component has the function of ending the setting component for each variable. This allows for the possibility of adjacent variables to have different values. Fig. 20(b) shows the setting-terminator component and a possible corridor (thick black lines). The component has height $h_t$ and width $w_b$.

*Top-frame component.* The top-frame component is used to distribute the corridor from the top-right corner of $F$ to the two exterior sides of the setting components. Fig. 20(a) shows the top-frame component. It has width equal to

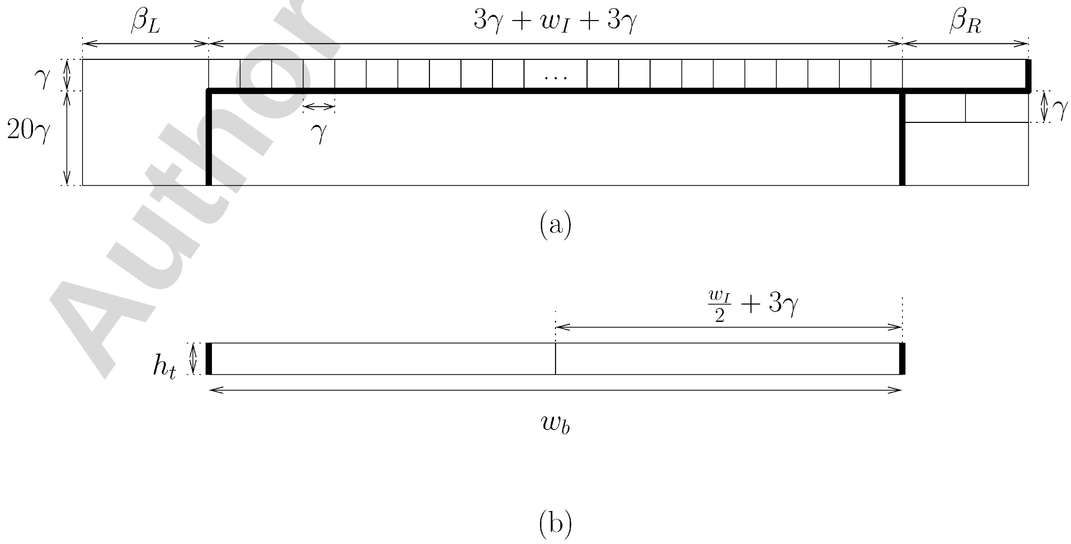



Fig. 20. (a) Top-frame component. (b) Setting-terminator component.

$\beta_L + 6\gamma + w_I + \beta_R$, and its height is $21\gamma$. A feasible partial corridor with length $\beta_R + w_I + 47\gamma$ that ends at the two points that place on the top-left and top-right points of the first setting component is given in Fig. 20(a).

**Lemma 6.5.** *There is a partial corridor with total edge-length at most $\beta_R + w_I + 47\gamma$ that connects the rectangles of the top-frame component and the two points that place on the top-left and top-right points of the topmost setting component to the top-right corner of $F$ (see Fig. 20(a)).*

**Proof.** By the above discussion. □

Finally, the value of $B$ is equal to

$$\beta_R + w_I + 47\gamma + \sum_{i=1}^{n}(494\gamma \cdot s_i + 3\gamma) + 2(n-1)\cdot h_t + \sum_{c\in N_2}\bigl(4I(c)+3\bigr) + \sum_{c\in N_3}\bigl(12I(c)+6\bigr).$$

*Applying the reduction.* Before we illustrate the whole process we should point out that our reduction results in an instance with very large height. So instead of drawing in this paper such an instance which would be hard to read in this media, we draw an *equivalent compressed* version of the instance which we call $f_c(I)$. The compressed instance has size $s'_i \leqslant s_i$ for the setting component for Boolean variable $x_i$. We accomplish the compression process by moving up as much as possible and resizing the clause-checking components so that their ends are aligned with equivalent variable-repository regions. Of course one needs to change the value of $B$ to one we call $B_c$. The formula depends on $s'_i$ rather than on $s_i$. We will use the compressed version in Fig. 21. The assembling of all the components is illustrated using the instance given in Example 1 in Section 5.

This instance is a yes-instance. In Fig. 21(a) we show the corridor corresponding to all variables having the value of false, an assignment that satisfies all the clauses. In Fig. 21(b) and (c) we show two partial corridors. One is for the assignment in which all the variables are false except for $x_3$, and the other one corresponds to all variables having the value true. These two assignments do not satisfy all the clauses, and as we shall prove later on the problem instance does not have a corridor with the desired edge-length. The partial corridor exposes the maximum number of rectangles and have length less than $B_c$. In order to expose the rectangles that have not been reached by the partial corridor, one needs to add line segments, increasing the total length of the corridor to more than $B_c$.

We establish the NP-completeness of the TRA-MLC-R problem in the following theorem.

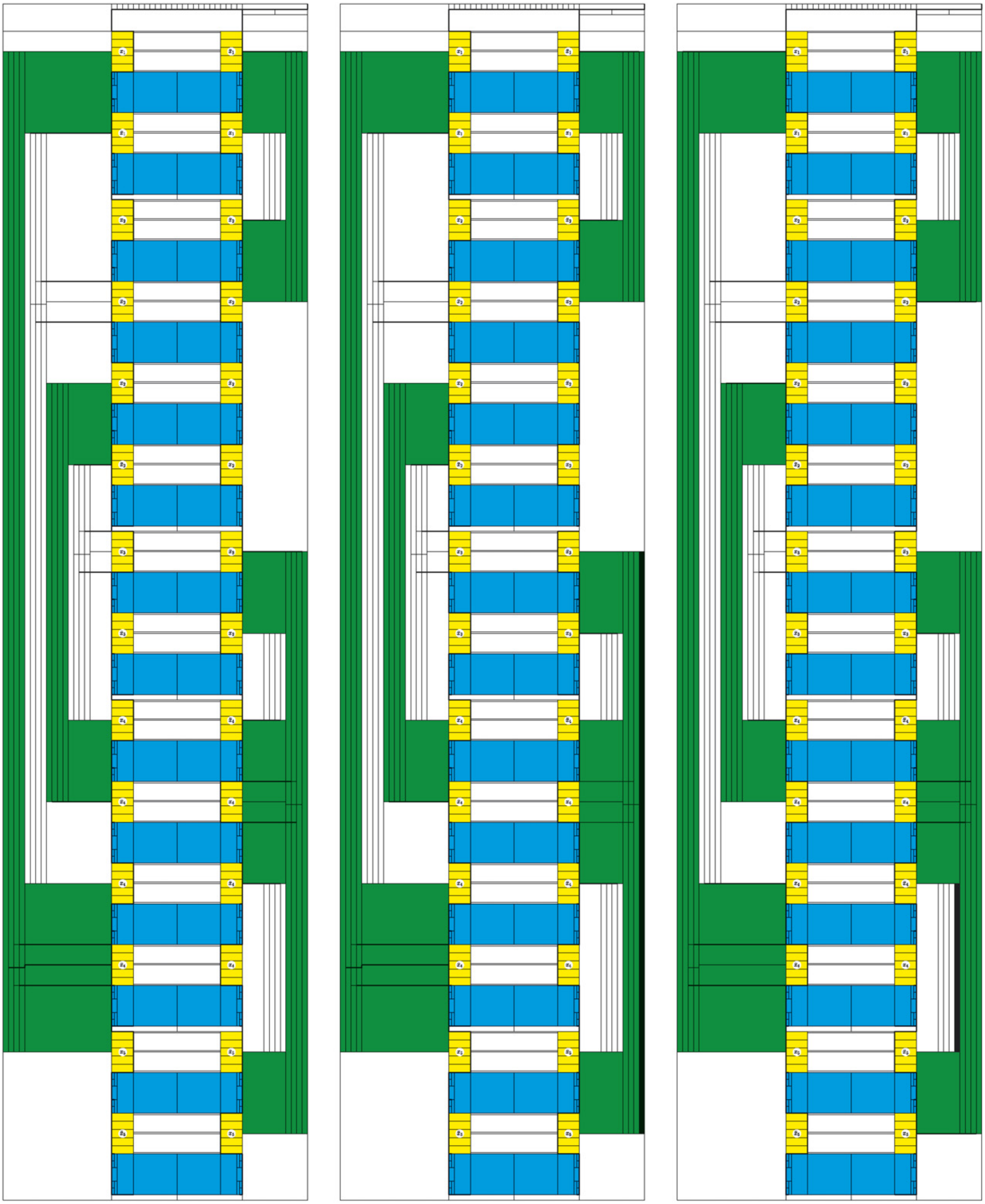
**Theorem 6.1.** *The TRA-MLC-R problem is NP-complete.*

**Proof.** Since the more general problem TRA-MLC is in NP, then so is the TRA-MLC-R problem.

We show now that the problem transformation defined above is a valid transformation. Since the proof that the transformation takes polynomial time with respect to the instance $I = (X, C)$ of the P3SAT problem given in a canonical planar embedding $D(G_I)$ is simple, we omit it. In what follows we show that no matter what instance $I$ we start from, the instance $f(I)$ of the TRA-MLC-R problem has a corridor with length at most $B$ iff the instance $I$ is satisfiable.

We show now that if the instance $I$ is satisfiable then the instance $f(I)$ has a feasible corridor. Let $A$ be a truth assignment for instance $I = (X, C)$ that satisfies all the clauses. The corridor that we construct corresponds to the assignment $A$ and starts with the partial corridor for the top-frame component given in Fig. 20(a). This connects the top-right corner of $F$ with the top-right and top-left points of the topmost setting component. The corridor will follow the routes given in Fig. 16(a) or (b) on each setting component depending on whether or not the Boolean variable $x_i$ has the value of true in $A$. The corridor in the setting-terminator component consists of the segments on its left and right sides. Clearly the total length of the line segments so far introduced is exactly $B - \sum_{c\in N_2}(4I(c)+3) - \sum_{c\in N_3}(12I(c)+6)$. Since $A$ satisfies all the clauses in $C$ we can show by using Lemmas 6.3 and 6.4 that all the rectangles, in the clause-checking components, can be exposed to the corridor by introducing line segments with length $\sum_{c\in N_2}(4I(c)+3) + \sum_{c\in N_3}(12I(c)+6)$. It then follows that if instance $I$ is satisfiable, then $f(I)$ has a corridor with length at most $B$.

We now establish that if the instance $f(I)$ has a feasible corridor, then the instance $I$ is satisfiable. Let $Q$ be a corridor with length at most $B$ that starts at the top-right corner of $F$. We try to transform $Q$ into another corridor



(a) Corridor (FFFFF)    (b) Partial corridor (FFTFF)    (c) Partial corridor (TTTTT)

Fig. 21. Instance $f_c(I)$ constructed from Example 1.

with edge-length which is smaller than the original one. Then we show the resulting corridor must have edge-length at least $B$. But since the original corridor edge-length is at most $B$, then its edge-length must be exactly $B$. This is possible only if none of the following corridor transformations can be applied. Then we show that an assignment that satisfies all the clauses in instance $I$ can be constructed from the resulting corridor. Lets begin with the corridor transformations.

The first step is to eliminate from $Q$ any superfluous line segments, i.e. any line segments that can be deleted without affecting the connectivity of all the rectangles.

Define the window $WI$ as the area covering all the setting components, the setting-terminator components, and the top-frame-component. The window $WO$ is the area delimited by $F$ excluding $WI$.

Delete from $Q$ the set of all the vertical line segments along the center line of the vertical fixing regions, and all the horizontal line segments along the middle rectangles in the horizontal fixing regions. The resulting corridor line segments partitions the set of rectangles $R$ into $k$ groups such that there is a path along the resulting corridor between every pair of rectangles in a group, but there is no path along the resulting corridor between rectangles in different groups. Furthermore the length of the line segments deleted is at least $60\gamma k$. Now find the shortest path in $WI$ plus the top component that joins two groups of rectangles. By case analysis one can show that the length of these line segments is at most $25.5\gamma$. After repeating this corridor transformation at most $k-1$ times, we obtain another corridor with smaller length than before when $k > 0$, otherwise the corridor was not transformed.

We say that a vertical line segment $z \in Q$ is *exchangeable* if $L(z) \geqslant 30\gamma - 1$, $z$ is on the boundary of only two rectangles, and $z \in WO$.

While $Q$ has an exchangeable line segment $z$ do the following corridor transformation. Delete $z$ and all the superfluous line segments from $Q$. This creates at most two components. Find a shortest path that joins these two components and add it to $Q$. By case analysis one can show that the length of these line segments is at most $25.5\gamma$. Thus the resulting corridor will have smaller length than before.

At this point one can show that the corridor segments in the top frame component can be transformed to the ones given in Fig. 20 without increasing the edge-length of $Q$. Again, delete all the superfluous line segments from $Q$.

Proofs similar to the ones for Lemmas 6.2 and 6.5 can be used to show that the corridor $Q$ inside the top frame component and $WI$, has edge-length at least $B - \sum_{c\in N_2}(4I(c)+3) - \sum_{c\in N_3}(12I(c)+6)$. By using lemmas similar to Lemmas 6.3 and 6.4 we can show that the line segments required to expose all the rectangles in the clause-checking components have length at least $\sum_{c\in N_2}(4I(c)+3) + \sum_{c\in N_3}(12I(c)+6)$. Since the original corridor had edge-length at most $B$, it must be that the above segments have length exactly equal to $B - \sum_{c\in N_2}(4I(c)+3) - \sum_{c\in N_3}(12I(c)+6)$, and $\sum_{c\in N_2}(4I(c)+3) + \sum_{c\in N_3}(12I(c)+6)$, respectively. Note that this happens only if the above corridor transformations that decreased the corridor's edge-length were not possible. The resulting corridor $Q$ can be modified without changing its edge-length to be of the exact form given in Fig. 16 for each setting component. The Boolean variable can be assigned values as in Lemma 6.2. From Lemmas 6.3 and 6.4 we can show that the values assigned to the Boolean variable satisfy all the clauses. This implies that $I$ is satisfiable. This completes the proof of the theorem. □

## 7. Discussion

We have established that the MLC problem is NP-complete as well as its restricted versions of MLC-R, TRA-MLC, and TRA-MLC-R. We have also shown that the MLC-R problem remains NP-complete even when one allows a forest rather than a tree as the corridor (MLC$_f$-R and MA-MLC$_f$-R). The TRA-MLC-R problem remains NP-complete even when the partition of the rectangles is a guillotine partition, i.e., generated by recursive guillotine cuts. Our reductions can be easily modified to show that the TRA-MLC-R problem remains NP-complete even when we restrict the corridor to connect to one corner of two (fixed) opposite corners or a special point (defined in [5]) of each rectangle.

The most important open problem in this area is to either develop a constant ratio approximation algorithm or establish inapproximability results for the TRA-MLC problem. We have developed a constant ratio approximation algorithm for the TRA-MLC-R problem [5]. As we have seen, known results for more general problems do not resolve these issues for our corridor problems.

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