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Minimum-energy Broadcast in Simple Graphs with Limited Node Power

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Abstract

The minimum-energy broadcasting problem in wireless networks consists of finding a transmission radius vector for all stations in such a way that the total transmission power of the whole network is least possible. The minimum-energy broadcast problem may be modeled by an edge weighted complete graph in which each vertex in the graph represents a station and the weight of the edge is distance between the two nodes it joins. The weighted graph version of the minimum-energy broadcast problem has been shown to be NP-hard.

We show that the weighted graph minimum-energy broadcast problem is NP-hard in metric space when transmissions are restricted to a given set of power levels by means of an upper bound d on the allowed transmission radius. We also show that the problem remains NP-hard even when defined on an unweighted graph and d is an arbitrary integer. The power limitation is justified because it is unrealistic to expect transmissions with unbounded power which may be needed in an optimal solution for large diameter networks. The problem can be solved in polynomial time when there is an optimal solution with a fixed number of transmitter nodes.

The general 2D or geometric version of the minimum-energy broadcast problem has also been shown to be NP-hard. However, we show that the given reduction is incorrect as it is, and propose a way to correct the metric version of the problem. This still leaves open whether the corrected version itself is embeddable in 2D.

1 Introduction

We study a graph version of the problem of *broadcasting* from a source station to all the stations in a static ad-hoc wireless network using minimum total energy. We assume the n

stations are located in the plane and the *source station* is s. In a wireless network each station is represented by a *node* and there are no fixed links joining pairs of nodes, but when a node transmits with power r^α, all the nodes within distance r will receive the transmission. This power function and the specific value of α, which is normally between 2 and 4, are derived from physical considerations. We state our results for $\alpha = 2$, but can be easily generalized to cover all other values of α. The nodes other than s that transmit with power greater than zero are called *retransmitters* or *relay stations* and their purpose is to receive information and send it to other nodes. The broadcasting problem in static wireless networks consists of finding for each node v a transmission radius $r(v)$ so that s broadcasts to all the nodes either directly or indirectly through the relay nodes. In other words, for every node v there is a sequence of nodes $s = v_{i_1}, v_{i_2}, \ldots, v_{i_l} = v$, for some $l \geq 2$, such that the distance from v_{i_j} to $v_{i_{j+1}}$ is at most $r(v_{i_j})$, for $1 \leq j < l$. The total energy or power used is $\sum_{v \in G} r(v)^2$. Static and slow changing wireless networks have received considerable attention because of their applications to battlefield, emergency disaster relief, etc. as well as in situations when it is not economically practical or physically possible to provide Internet or Intranet connectivity for distributed sensor data.

Table 1 gives the transmission vector r for a set of 10 stations with source station $s = 10$. Figure 1(a) depicts the set of points in the plane, with circles representing the range of node transmissions. The source station ($s = 10$) transmits directly to stations 1, 2, 3, 6 and 7. Stations 3, 6 and 7 relay the transmission to 4, 5 and 8, respectively. Node 8 retransmits to station 9.

Table 1: Transmission radius vector r for Example 1.

	1	2	3	4	5	6	7	8	9	10
Vector r	0	0	2	0	0	2	3	1	0	5

Thus the minimum-energy broadcasting problem in wireless networks consists of finding a transmission radius vector for the vertices in such a way that the total transmission power is least possible. Each transmission radius vector defines a spanning broadcast tree for the network in which the root is the source node s and each path from the root to a vertex

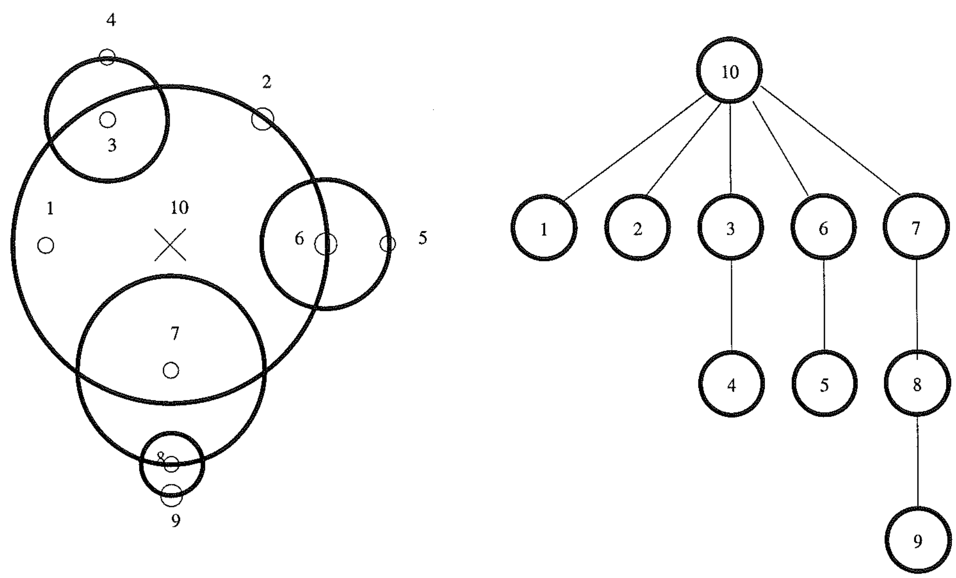

Figure 1: Example.

v corresponds to the sequence of transmissions by which s communicates its message to v. Note that there may be many possible transmissions from s to v, but the broadcast tree will just represent one. Internal nodes of the tree are the transmitter nodes (or relays for non-root internal nodes). Figure 1(b) represents the broadcast spanning tree for the transmission radius vector in Table 1. The cost of transmission from a transmitter node in a broadcast tree is the minimum power required by the internal node to transmit to all its children. A minimum-energy broadcast tree is one with the minimum sum of the cost of the transmissions from each internal node in the tree. We call this problem the *geometric or 2D minimum-energy broadcast* problem (*2D-MEB*). This problem was initially introduced by Wieselthier, Nguyen and Ephremides [15] where their main result was developing heuristics for its solution.

In wired networks that allow only unicasting communications (transmission from one node to just another one) the total cost of the broadcast is the sum of the weights on the links of the spanning tree. Therefore, it can be formulated and solved efficiently by the algorithms for the minimum-cost spanning tree (MST) problem. In wireless networks, there is a significant difference in the calculation of the energy required for the broadcast, since all of the nodes within the communication range of a transmission node may receive a transmission without additional transmitter power. Details of this model and the physical assumptions made can be found in [15]. Wired networks that allow multicasting fall somewhere between the above two communication models.

The minimum-energy broadcast problem may by modeled by an edge weighted complete graph in which each vertex in the graph represents a station, and the weight of the edge is the distance between the two nodes it joins. We call this problem with arbitrary weights the *weighted graph minimum-energy broadcast* problem (*MEB*). Clementi, Penna and Silvestri [6, 5] show that the minimum range assignment problem that guarantees the strong connectivity of the corresponding transmission graph is NP-hard. Wan, Calinescu, Li and Frieder [14] showed that for arbitrary weights, this problem is an NP-hard problem. They also show that approximating this problem is as hard as approximating the dominating set problem, which its known to be NP-hard to approximate [10]. Clementi, *etal.* [2, 3] adapted the reduction in [6, 5] and showed that the 2D-MEB problem is NP-hard. A current survey of results for this problem appears in [4]. Clementi etal. [3] also show that the 2D-MEB problem can be approximated by the so called MST-ALG, initially proposed by [15], by the ratio $5^{\alpha/2} \cdot 2^{\alpha}$. They also present approximation algorithms for the problem defined in multidimensional space. To establish that the 2D-MEB problem is an NP-complete problem, Clementi etal. [3] gave a clever and elegant argument; however, it is incorrect as it is. In the Appendix we summarize their reduction and show where it errs. Then we explain how to fix the main problem with their reduction, but it remains open as to whether the modified reduction is now embeddable in 2D space in a straightforward manner by fixing the embedding part of the proof. In other words, we fix the problem that arises in the properties of an intermediate structure in the proof, but not the overall proof.

If the set of weights satisfies the triangle inequality, then the problem is said to be in *metric space*. In this paper we show that the weighted graph version of the problem is NP-hard in metric space when transmissions are restricted to a given set of power levels. This means that the transmission radius of each vertex has to be no more than some bound d. We also show that the problem remains NP-hard even when defined on an unweighted graph and d is an arbitrary integer. The weighted graph problem is shown to be solvable in polynomial time when there is an optimal solution where only a fixed number of nodes transmit.

A particular case of the weighted graph minimum-energy broadcast problem is obtained by the application of the terms "distance", "diameter", etc., in their usual graph theoretical sense. Here we start with a simple unweighted graph $G = (V, E)$. The distance between $u, v \in V$ is then the length of the shortest path between u and v in G. If vertex u transmits to a distance r in G, this incurs a cost of r^α, and the transmission is received by all $v \in V$ within distance (in the graph theoretical sense) r of u in G. The quantity r is then the transmission radius of u. As before, the total energy (cost) of a broadcast tree is the sum of the energies r^α of the internal nodes of the tree.

One may view our problem as an instance of the metric space problem by applying the following transformation. ¿From a simple graph G we construct a weighted complete graph G' by assigning a weight to each edge in G' equal to the length of the shortest path joining the two vertices in G. The resulting graph satisfies the triangle inequality but not all graphs in metric space can be obtained in this fashion.

Given a simple graph $G = (V, E)$ and a source vertex $s \in V$, there are two problems of interest:

> **Unrestricted minimum-energy broadcast (MEB):** Find a minimum-energy broadcast tree in G rooted at source vertex s.
>
> **Restricted minimum-energy broadcast (RMEB):** Find a minimum-energy broadcast tree in G rooted at vertex s in which the permitted transmission radii are bounded by some $d \geq 1$.

The *metric restricted minimum-energy broadcast (M-RMEB)* problem is a generalization of the RMEB problem to metric graphs. The *2D or restricted restricted minimum-energy broadcast (2D-RMEB)* is the M-RMEB problem when the graph is embeddable in 2D space.

The RMEB problem for $d = 1$ was show to be NP-complete in [9]. As discussed in [9, 2], the reduction in [10] can be used to show that this problem as hard to approximate as the minimum set cover, which is not approximable within a sub-logarithmic factor unless $P = NP$. As pointed out in a personal communication with the authors of the paper [3] the NP-hardness reduction for the 2D-MEB given in [3] can be easily extended to the 2D-RMEB. However, as we remarked before, there is a problem with that reduction.

In this paper we consider the decision versions of these problems and show that the restricted problem is NP-complete. This restriction is justified because it is unrealistic to expect transmissions with unlimited power which may be needed in an optimal solution for large diameter networks.

We formally define the restricted minimum-energy broadcast decision problem (RMEB) as follows:

RESTRICTED MINIMUM-ENERGY BROADCAST PROBLEM (RMEB):
INSTANCE: A 4–tuple (G, s, d, K) where $G = (V, E)$ is a simple graph, $s \in V$ is the source node, $d < |V|$, $K < |V|^2$ are positive integers.
QUESTION: Is there a spanning broadcast tree rooted at s and with total energy K or less, in which each transmission radius used is at most d?

Thus in RMEB, we permit nontrivial transmission radii only from the set $\{1, 2, \ldots, d\}$. If the radius is unrestricted, then the decision problem is MEB, defined as

UNRESTRICTED MINIMUM-ENERGY BROADCAST PROBLEM (MEB):
INSTANCE: A 3–tuple (G, s, K) where $G = (V, E)$ is a simple graph, $s \in V$ is the source node, $K < |V|^2$ is a positive integer.
QUESTION: Is there a spanning broadcast tree rooted at s and with total energy K or less?

In Section 2, we present our reduction to establish the NP-completeness of RMEB. Issues relating to MEB, as well as the the corresponding approximation problems are discussed in Section 3.

2 NP-completeness of RMEB

We begin by defining the vertex cover problem (VC). Let $G = (V, E)$ be an undirected graph with vertex set V, and edge set E. A subset $V' \subseteq V$ of vertices is said to be a vertex cover for G iff every edge in E is incident to at least one vertex in V'.

VERTEX COVER (VC):
INSTANCE: A pair (G, K), where $G = (V, E)$ is an undirected graph, and $K < |V|$ is an integer.
QUESTION: Does G have a vertex cover with at most K vertices?

VC was among the first set of problems shown to be NP-complete [11], [8]. The class of NP-complete problems is the set of all decision problems $Q \in NP$ such that SAT $\propto$ Q, where SAT is the satisfiability problem defined in [8], and $\propto$ represents polynomial time reducibility [8]. The class of NP-complete problems is very rich. We refer the reader to [7], [11], and [8] for additional details about the theory of NP-completeness. We establish our intractability results by constructing a polynomial time reduction from VC to the RMEB problem.

2.1 The general reduction for RMEB

First we define a certain gadget that we build to transform an edge between two vertices u, v. Given positive integers c and d, the graph $g(c, d)$ is constructed as follows.
Case 1: d is even

1. Make c copies of a path with $d/2$ nodes.
2. Construct the multigraph obtained by making c edges between u and v.
3. Subdivide each of these c edges by inserting $d-1$ additional vertices (this turns each edge between u and v into a path with $d+1$ vertices).
4. Connect one extreme vertex of each of the c paths to the central vertex of the subdivisions in a one-to-one manner to obtain $g(c, d)$.

As an example, the gadget $g(3, 4)$ is shown in Figure 2.
Case 2: d is odd

1. Make c copies of a path with $(d+1)/2$ nodes.
2. Construct the multigraph obtained by making c edges between u and v.
3. Subdivide each of these c edges by inserting $d-1$ additional vertices (this turns each edge between u and v into a path with $d+1$ vertices).

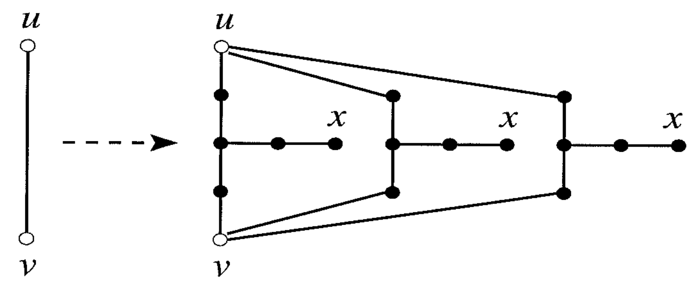

Figure 2: The gadget $g(3,4)$.

4. Connect one extreme vertex of each of the c paths to the two central vertices of the subdivisions in a one-to-one manner to obtain $g(c,d)$.

As an example, the gadget $g(3,5)$ is shown in Figure 3.

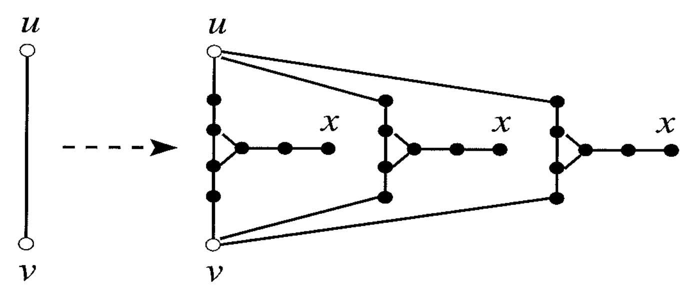

Figure 3: The gadget $g(3,5)$.

If we construct $g(c,d)$ starting with the edge $\{u,v\}$, we say that the gadget is *built* on the edge $\{u,v\}$. In our reduction, we will be using the gadgets $g(c,d)$ with $c \gg d$. We observe that for any gadget $g(c,d)$ built on the edge $\{u,v\}$, the minimum energy required to transmit from either u or v (or from a vertex outside $g(c,d)$) to all of the extreme vertices of $g(c,d)$ labeled as x in Figures 2 and 3, is at least $\min\{d^2, c\}$. The quantity c comes from at least c transmissions of radius 1 or more required if neither u nor v has transmission radius d.

We construct a direct polynomial time reduction from VC to RMEB.

Proposition 1 *VC polynomial time reduces to RMEB. Thus RMEB is NP-hard.*

Proof Given an instance of the vertex cover problem (G,K) with $G=(V,E)$ and $K<|V|$, we construct an instance of an RMEB problem with parameters $(G_2, s, d, (1+K)d^2)$ as follows. Suppose $n = |V|, e = |E|$. Construct a graph G_2 from G by using the gadgets $g(c, d)$ and following the steps below:

1. First add an extra vertex s and add the edges $\{s, v\}$ for all $v \in V$ to E. Call the resulting graph $G_1 = (V_1, E_1)$. Then $|V_1| = 1 + n$ and $|E_1| = n + e$.

2. Construct $G_2 = (V_2, E_2)$ from G_1 by replacing each edge $\{u, v\} \in E_1$ by the gadget $g(c, d)$.

We show later on that we can set c equal to nd^2 in this construction.

We view V_2 as consisting of $1 + n$ *old* vertices which are s and the original vertices in V, and a number of *new* vertices which are the vertices introduced by the gadgets $g(c, d)$ (these are the vertices distinct from u, v in Figures 2, and 3). We claim that in the resulting instance G_2, it is possible to broadcast from s with total energy $(1 + K)d^2$ to all vertices in V_2 with transmission radius bounded by d iff G has a vertex cover of size at most K.

First assume that G has a vertex cover V' of size K. With radius d, s can transmit to all of the new vertices on the gadgets formed from edges $\{s, v\}$ in G_1, and to all of the vertices in the vertex cover V' (in fact to all vertices in V_1). Now each $v \in V'$ can broadcast with radius d to all other vertices. Thus in G_2, it is possible to broadcast from s by using total energy $(1 + K)d^2$.

Suppose now that in G_2, s can broadcast with total energy $(1 + K)d^2$ or less for some $K < n$. Note that just as VC always has a solution for $K = n$, s can always broadcast in G_2 using total energy energy $(1 + n)d^2$.

Let W be the set of vertices that are transmitters in such a broadcast tree of total energy $(1 + K)d^2$ with root s. We'll show that in addition to s, W contains K old vertices (i.e. vertices in V_1) that have transmission radius d, and that these K vertices form a VC for G. We'll also show that all transmitters in the broadcast tree have radius d.

First we show that s has transmission radius $r = d$. Since the allowed radii are at most d in RMEB, it is enough to show that $r \geq d$. By way of contradiction, assume that s transmits with radius $r < d$. Then by our previous observation on the power required to reach extreme vertices labeled x in a gadget, we need at least an additional nd^2 energy to send to all gadgets built on edges $\{s, v\}, v \in V$, or one of them requires power c. Choose

$c = nd^2$. Then the total power required to broadcast from s in G_2 is at least $r^2 + nd^2$. Since $K < n$, $r^2 + nd^2 \geq 1 + nd^2 > (1 + K)d^2$, which is a contradiction. Thus s has transmission radius d.

Now given that $c = nd^2$ and s has transmission radius d, it follows that if W contains some new vertex y in some $g(c, d)$ built on an edge $\{u, v\} \in E_1$, then it contains either u or v (or both), since the only way to reach y from the source s is by relaying the transmission through u or v, for otherwise the power required will at least be c, which is too large. Therefore for every edge $\{u, v\} \in E_1$, either u or v is a transmitter. It follows that $V' = W \setminus \{s\}$ is a vertex cover for G of cardinality at most K.

The number of vertices of G_2 is found to be $|V_2| = O(d^3ne)$. Note also that the reduction itself can be carried out in polynomial time with respect to n, i.e. the reduction is polynomial time. •

3 Polynomial Time Algorithm for Weighted Graph Version

In this section we present a simple $O(n^{k+2})$ algorithm that finds an optimal solution to the MEB problem when there is an optimal solution in which no more than k nodes are transmitters.

The algorithm is simple. First it will try all subsets of at most k nodes. For each node there are at most n different power levels at which it may transmit, since transmission at intermediate levels will not reach other stations. We then try all these $O(n^k)$ possible power level choices for all the nodes selected and check to see if it is a solution and if so keep track of the one with smallest objective function value. Clearly this can be done in $O(n^2)$, so the total time complexity bound becomes $O(n^{k+2})$. Since there are $\binom{n}{k}$ subsets with k stations, the total time complexity bound is $O(n^{2k+2})$.

The RMEB problem is as hard to approximate as the set cover problem, which we know it is NP-hard to approximate. This reduction is not in this paper but it is similar to the ones in [10].

4 Discussion

We have shown that the weighted graph minimum-energy broadcast problem is NP-hard in metric space when transmissions are restricted to a given set of power levels by means of an upper bound d on the allowed transmission radius. This restriction is justified because it is unrealistic to expect transmissions with unlimited power which may be needed in an optimal solution for large diameter networks. We have also shown that our problem can be solved in polynomial time when there is an optimal solution with a fixed number of transmitter nodes.

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5 Appendix

To establish that the 2D-MEB problem in 2D space is an NP-complete problem, Clementi etal. [3] gave a clever and elegant argument; however, it is incorrect as it is. In what follows we summarize their reduction and show where it errs. Then we explain how to fix the main problem with their reduction, but it remains open as to whether the modified reduction is now embeddable in 2D space in a straightforward manner by fixing the embedding part of their proof. In other words, we fix the problem that arises in the properties of an intermediate structure in the proof, but not the overall proof.

Clementi etal. [3] proposed a multi-step reduction from a restricted version of the vertex cover (VC) problem, which is known to be NP-complete, to the 2D-MEB problem. First they reduce the problem to finding a vertex cover in a straight line drawing graph, and then replace every edge by a gadget and embed it in 2D space in such a way that certain properties needed by the reduction are preserved.

Specifically, given an instance of the vertex cover problem consisting of an undirected planar graph G which is at most cubic and an integer k, the first step in the reduction constructs a planar orthogonal drawing in which every edge is represented by a polyline with two bends. Then to each bend two vertices are added to obtain a straight line drawing $D(G)$. Clearly, if graph G has a vertex cover with cardinality k, and $2h$ nodes are added when transforming G to $D(G)$, then $D(G)$ has a vertex cover with cardinality $k + h$. Figure 4(a) depicts a graph G, and Figures 4(b) and 4(c) represent the resulting graphs after applying the above two steps. Note that the length of the line segments is not uniform.

In this example the graph G has a vertex cover of cardinality $k = 2$, and $D(G)$ has a vertex cover with cardinality $k + h = 8$. It is important to note that every vertex cover for $D(G)$ given in Figure 4(c) contains has two vertices that are adjacent in $D(G)$.

In the next step we replace each straight line with the (flexible) gadget given in Figure

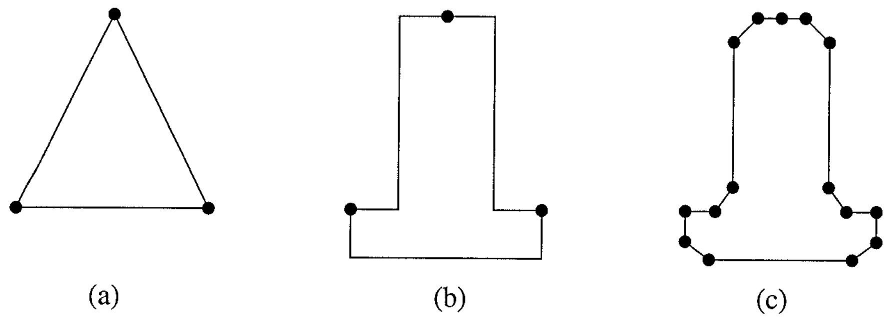

Figure 4: (a) Given graph G; (b) Planar orthogonal drawing of G in which every edge is represented by a polygonal line with two bends; (c) Straight line drawing $D(G)$ obtained by adding two vertices to each bend.

2 and obtain a weighted graph which we call $F(G)$[1]. The only edges in this graph are the ones from the gadgets and their weights and are shown in Figure 2. This graph can be easily transformed into a complete metric graph. This weighted graph $F(G)$ is embeddable in 2D by preserving the inter-node distances for each gadget (Figure 5) and modifying the number of x and/or z nodes. Note that one may need to add the bumps for the x (z) nodes (e.g., nodes z_5 and x_3 in Figure 2) when the total length segment is not a multiple of δ (δ'). The final embedding in 2D, called $S(G)$, must also satisfy some additional properties.

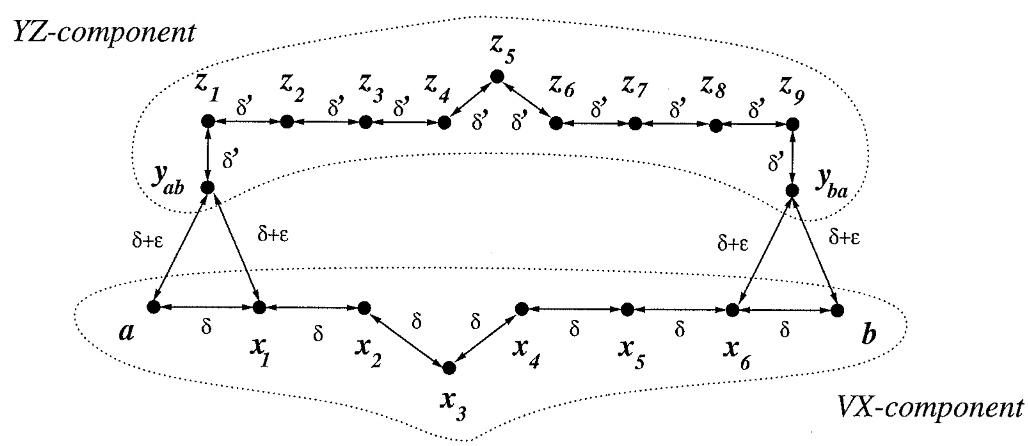

Figure 5: The gadget construction.

Clementi etal. [3] define a set of properties for the gadgets in $S(G)$. A modified version

[1] $F(G)$ does not appear in Clementi etal.'s [3] construction, but it is important to define it.

of these properties are the ones that are actually needed. These are given below.

Gadget Properties: Let δ, δ', $\epsilon \geq 0$ such that $\delta + \epsilon > \delta'$ and let $\beta > 1$ be a suitable parameter. For any edge (a, b) the corresponding gadget g_{ab} contains the set of points $X_{ab} = \{x_1, \ldots, x_{l_1}\}$, $Y_{ab} = \{x_{ab}, x_{ba}\}$, $Z_{ab} = \{z_1, \ldots, z_{l_2}\}$, and $V_{ab} = \{a, b\}$, where l_1 and l_2 depend on the length of the drawing of (a, b). These sets of points will be drawn in $\mathcal{R}^2$ so that in addition to the distances given in Figure 5, the following properties hold:

1. For every $i \geq 1$ and $i + 1 < j \leq l_1$, $d(x_i, x_j) > \delta$.

2. For every $i \geq 1$ and $i + 1 < j \leq l_2$, $d(z_i, z_j) > \delta$.

3. For every $x_i \in X_{ab}$ and $z_j \in Z_{ab}$, $d(x_i, z_j) > \delta + \epsilon$. Furthermore, for $2 < i \leq l_1$, $d(x_i, y_{ab}) > \delta + \epsilon$, and for $1 \leq i < l_1 - 1$, $d(x_i, y_{ba}) > \delta + \epsilon$.

4. Given any two different gadgets g_{ab} and g_{cd}, for any $u \in g_{ab} \backslash g_{cd}$ and $v \in g_{cd} \backslash g_{ab}$, we have that $d(u, v) > \delta$ and if $u \notin V_{ab} \cup X_{ab}$ or $v \notin V_{cd} \cup X_{cd}$ then $d(u, v) \geq \beta\delta$.

One may select as the source node s any node x in any of the gadgets. This node is the origin of the broadcast operation.

Let T be any spanning tree of $D(G)$. Clementi etal. [3] define a range assignment for $S(G)$ (and also $F(G)$). This assignment is defined for every gadget g_{ab} as follows: If (a, b) is in T, then all stations in VX_{ab} have range δ. If (a, b) is not in T, then we assign range δ to every station in VX_{ab} except one in X_{ab} that have a null range and it is always distinct from the source s. Every station in the YZ component has range δ'. This range assignment is not feasible because the broadcasting from source s can only reach the nodes in the union $\mathcal{U}$ of every VX component. In order to obtain a feasible range assignment, one needs to choose some "bridge-points" from $\mathcal{U}$ to every YZ-component. These bridge-points will have range assignment $\delta + \epsilon$.

Clementi etal. [3] define a *canonical solution* for $S(G)$ as a range assignment r for $S(G)$ if for every gadget g_{ab} of $S(G)$ (and also $F(G)$), the following properties hold:

1. For every $v \in \{a, b\}$, either $r(v) = \delta$ or $r(v) = \delta + \epsilon$. Furthermore, there exists $v \in \{a, b\}$ such that $r(v) = \delta + \epsilon$ (so, v is a bridge-point from the VX-component to the YZ-component).

2. Either $r(y_{ab}) = \delta'$ and $r(y_{ba}) = 0$ (so y_{ab} allows the broadcasting coming from the bridge-point along the VX-component) or vice versa.

3. For every $x \in X_{ab}$, either $r(x) = \delta$ or $r(x) = 0$. At most one station in X_{ab} has null range and such station is other than the source s.

4. The set $\bar{E} = \{(a,b) | r(x) = \delta, \forall x \in X_{ab}\}$ form a spanning tree of $D(G)$.

5. For every $z \in Z_{ab}$, $r(z) = \delta'$.

It is simple to establish that G has a vertex cover of cardinality k if, and only if, $D(G)$ has a vertex cover with cardinality $K = k + h$, where $2h$ is the number of vertices in $D(G)$ that are not in G. In order to establish the NP-completeness result one now needs to establish that $S(G)$ has a canonical solution with objective function value equal to $cost(r_K) = k(\delta+\epsilon)^\alpha + (N_{S(G)} - K)\delta^\alpha + M_{S(G)}$, where $M_{S(G)} = \sum_{(a,b)\in E'}[(YZ_{ab} - 1)(\delta')^\alpha]$ if, and only if, $D(G)$ has a vertex cover with cardinality $K = k + h$.

This last step is where the problems arise. The reason is that $M_{S(G)}$ is defined in such a way that item 5 in the definition of canonical solutions, does not always hold. The problem arises when adjacent points in $D(G)$ (say in line ab) are in the vertex cover. Since both nodes (a and b) have range assignment $\delta + \epsilon$, there will be two bridges to the YZ-component ab. Therefore, exactly $YZ_{ab} - 2$ nodes will transmit at a distance δ'. The savings from this line, as well as other ones will in some cases be enough to have another vertex transmit with power $\delta + \epsilon$ and thus a graph that does not have a vertex cover with cardinality k will have a transmission cost equal to $cost(r_k)$. In Figure 4, the graph $D(G)$ has the property that in every vertex cover with cardinality $K = k + h$ there are two adjacent vertices in it. In general one cannot predict how many of the edges in $D(G)$ will have this same condition (both vertices are in the vertex cover).

5.1 The modification

However we can modify the construction so that at least the reduction to $F(G)$ is correct. Now the question is whether $F(G)$ is embeddable in 2D. We suspect that this part of the proof should follow along the same lines of the embedding approach of Clementi etal. [3].

Our modification to the gadget is given Figure 6. The new points are called z'_i and are at a distance δ' from the corresponding z_i point. The distance from z'_i to z'_{i+1} is at least δ'. Note that by adding the spikes at the z vertices, we guarantee that item 5 in the canonical

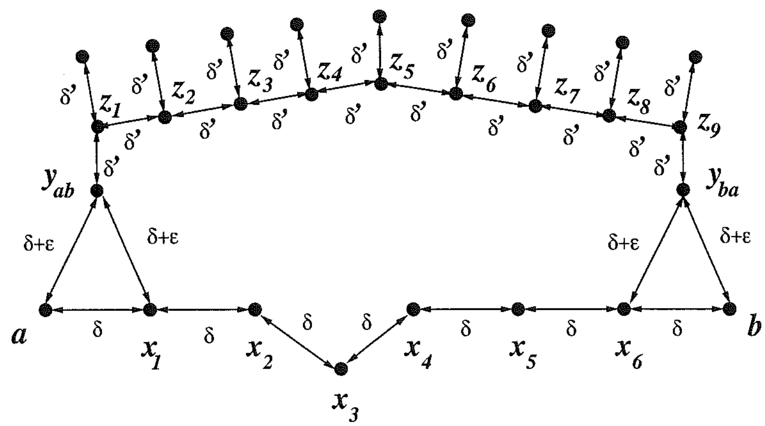

Figure 6: The modified gadget with spikes.

solution, as defined before, holds even when both the vertices a and b have range assignment $\delta + \epsilon$.