EFFICIENT ALGORITHMS AND ROUTING PROTOCOLS FOR HANDLING TRANSIENT SINGLE NODE FAILURES

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Abstract

Single node failures represent more than 85% of all node failures[7] in the today's large communication networks such as the Internet. Also, these node failures are usually transient. Consequently, having the routing paths globally recomputed does not pay off since the failed nodes recover fairly quickly, and the recomputed routing paths need to be discarded. Instead, we develop algorithms and protocols for dealing with such transient single node failures by suppressing the failure (instead of advertising it across the network), and routing messages to the destination via alternate paths that do not use the failed node. We compare our solution to that of [11], which also discusses such a proactive recovery scheme for handling transient node failures. We show that our algorithms are faster by an order of magnitude while our paths are equally good. We show via simulation results that our paths are usually within 15% of the optimal for randomly generated graph with 100-1000 nodes.

KEY WORDS: Network Protocols, Node Failure Recovery, Transient Node Failures, Alternate Path Routing.

1 Introduction

Let G=(V,E) be an edge weighted graph that represents a computer network, where the weight (positive real number), denoted by cost(e), of the edges represents the cost (time) required to transmit a packet through the edge (link). The number of vertices (|V|) is n and the number of edges (|E|) is m. It is well known that a shortest paths tree of a node s, \mathcal{T}_s , specifies the fastest way of transmitting a message to node s originating at any given node in the graph under the assumption that messages can be transmitted at the specified costs. Under normal operation the routes are the fastest, but when the system carries heavy traffic on some links these routes might not be the best routes. These trees can be constructed (in polynomial time) by finding a shortest path between every pair of nodes. In this paper we consider the case when the nodes in the network are

susceptible to transient faults. These are sporadic faults of at most one node¹ at a time that last for a relatively short period of time. This type of situation has been studied in the past [11] because it represents most of the node failures occurring in networks. *Single* node failures represent more than 85% of all node failures [7]. Also, these node failures are usually *transient*, with 46% lasting less than a minute, and 86% lasting less than 10 minutes [7]. Because nodes fail for relative short periods of time, propagating information about the failure throughout the network is not recommended.

In this paper we consider the case where the network is biconnected (2-node-connected), meaning that the deletion of a single node does not disconnect the network. Based on our previous assumptions about failures, a message originating at node x with destination s will be sent along the path specified by \mathcal{T}_s until it reaches node s or a node (other than s) that failed. In the latter case, we need to use a recovery path to s from that point. Since we assume single node faults and the graph is biconnected, such a path always exists. We call this problem of finding the recovery paths the Single Node Failure Recovery (SNFR) problem. It is important to recognize that the recovery path depends heavily on the protocol being deployed in the system. Later on we discuss our (simple) routing protocol.

1.1 Preliminaries

Our communication network is modeled by an edge-weighted biconnected undirected graph G=(V,E), with n=|V| and m=|E|. Each edge $e\in E$ has an associated cost (weight), denoted by cost(e), which is a non-negative real number. $p_G(s,t)$ denotes a shortest path between s and t in graph G and $d_G(s,t)$ to denote its cost (weight).

A shortest path tree \mathcal{T}_s for a node s is a collection of n-1 edges $\{e_1,e_2,\ldots,e_{n-1}\}$ of G which form a spanning tree of G such that the path from node v to s in \mathcal{T}_s is a shortest path from v to s in G. We say that \mathcal{T}_s is rooted at node s. With respect to this root we define the set of nodes that are the *children* of each node x as follows. In \mathcal{T}_s we say that every node y that is adjacent to x such that x is on

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¹The nodes are *single*- or *multi*-processor computers

the path in \mathcal{T}_s from y to s, is a child of x. For each node x in the shortest paths tree, k_x denotes the number of children of x in the tree, and $\mathcal{C}_x = \{x_1, x_2, \dots x_{k_x}\}$ denotes this set of children of the node x. Also, x is said to be the parent of each $x_i \in \mathcal{C}_x$ in the tree \mathcal{T}_s . With respect to s, the parent node, p, of a node c is sometimes referred to as the primary neighbor or primary router of c, while c is referred to as an upstream neighbor or upstream router of p. The children of a particular node are said to be siblings of each other. $V_x(\mathcal{T})$ denotes the set of nodes in the subtree of x in the tree \mathcal{T} and $E_x \subseteq E$ denotes the set of all edges incident on the node x in the graph x. We use x nextHopx to denote the next node from x on the shortest path tree from x to y. Note that by definition, x nextHopx is the parent of x in x.

Finally, we use ρ_x to denote the escape edge in $G(E)\backslash \mathcal{T}_s$ that the node x uses to recover from the failure of its parent. As we discuss later, having the information of a single escape edge ρ_x for each node $x\in G(V)$ and $x\neq s$ is sufficient to construct the entire alternate path for any node to recover from the failure of its parent, even though the path may actually contain multiple non-tree edges.

1.2 Related Work

One popular approach of tackling the issues related to transient failures of network elements is that of using *proactive recovery schemes*. These schemes typically work by precomputing alternate paths at the network setup time for the failure scenarios, and then using these alternate paths to reroute the traffic when the failure actually occurs. Also, the information of the failure is suppressed in the hope that it is a transient failure. The local rerouting based solutions proposed in [1, 6, 9, 10, 11] fall into this category.

Refs. [8, 11] present protocols based on local rerouting for dealing with transient single link and single node failures respectively. They demonstrate via simulations that the recovery paths computed by their algorithm are usually within 15% of the theoretically optimal alternate paths.

Wang and Gao's Backup Route Aware Protocol [10] also uses some precomputed backup routes in order to handle transient single *link* failures. One problem central to their solution asks for the availability of *reverse paths* at each node. However, they do not discuss the computation of these reverse paths. Interestingly, the alternate paths that our algorithm computes qualify as the reverse paths required by the BRAP protocol of [10].

Slosiar and Latin [9] studied the single link failure recovery problem and presented an $O(n^3)$ time for computing the link-avoiding alternate paths. A faster algorithm, with a running time of $O(m+n\log n)$ for this problem was presented in [1]. Our central protocol presented in this paper can be generalized to handle single link failures as well. Unlike the protocol of [8], this single link failure recovery protocol would use *optimal* recovery paths.

1.3 Problem Definition

The Single Node Failure Recovery problem, is defined as follows: (SNFR) Given a biconnected undirected edge weighted graph G=(V,E), and the shortest paths tree $\mathcal{T}_s(G)$ of a node s in G where $\mathcal{C}_x=\{x_1,x_2,\ldots x_{k_x}\}$ denotes the set of *children* of the node x in \mathcal{T}_s , for each node $x\in V$ and $x\neq s$, find a path from $x_i\in\mathcal{C}_x$ to s in the graph $G=(V\setminus\{x\},E\setminus E_x)$, where E_x is the set of edges adjacent to vertex x.

In other words, for each node x in the graph, we are interested in finding alternate paths from each of its children to the source² node s when the node x fails. Note that we don't consider the problem to be well defined when the node s fails.

The above definition of alternate paths matches that in [10] for *reverse paths*: for each node $x \in G(V)$, find a path from x to the node s that does not use the primary neighbor (parent node) y of x in T_s .

1.4 Main Results

We discuss our efficient³ algorithm for the SNFR problem that has a running time of $O(m \log n)$ (by contrast, the alternate path algorithms of [6, 8, 11] have a time complexity of $\Omega(mn \log n)$ per destination). We further develop protocols based on this algorithm for recovering from single node *transient* failures in communication networks. In the *failure free* case, our protocol does not use any extra resources.

The recovery paths computed by our algorithm are not necessarily the shortest recovery paths. However, we demonstrate via simulation results that they are very close to the optimal paths.

We compare our results with those of [11] wherein the authors have also studied the same problem and presented protocols based on local rerouting for dealing with transient single node failures. One important difference between the algorithms of [6, 8, 11] and our's is that unlike our algorithm, these are based primarily on recomputations. Consequently, our algorithm is faster by an order of magnitude than those in [6, 8, 11], and as shown by our simulation results, our recovery paths are usually comparable, and sometimes better.

2 Algorithm for Single Node Failure Recovery

A naive algorithm for the SNFR problem is based on recomputation: for each node $v \in G(V)$ and $v \neq s$, compute the shortest paths tree of s in the graph $G(V \setminus v, E \setminus E_v)$. Of interest are the paths from s to each of the nodes $v_i \in \mathcal{C}_v$. This naive algorithm invokes a shortest paths algorithm

²We use *source* and *destination* in an interchangeable way

³The primary routing tables can be computed using the Fibonacci heaps [3] based implementation of Dijkstra's shortest paths algorithm [2] in $O(m+n\log n)$ time

n-1 times, and thus takes $O(mn+n^2\log n)$ time when it uses the Fibonacci heap [3] implementation of Dijkstra's shortest paths algorithm [2]. While these paths are *optimal* recovery paths for recovering from the node failure, their *structure* can be much different from each other, and from the original shortest paths (in absence of any failures) - to the extent that routing messages along these paths may involve recomputing large parts of the primary routing tables at the nodes through which these paths pass. The recovery paths computed by our algorithm have a well defined structure, and they overlap with the paths in the original shortest paths tree (\mathcal{T}_s) to an extent that storing the information of a single edge, ρ_x , at each node x provides sufficient information to infer the entire recovery path.

2.1 Basic Principles and Observations

We start by describing some basic observations about the characteristics of the recovery paths. We also categorize the graph edges according to their *role* in providing recovery paths for a node when its parent fails.

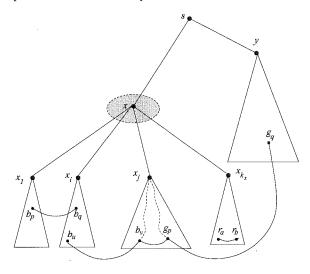


Figure 1. Recovery paths for recovering from the failure of x.

Figure 1 illustrates a scenario of a single node failure. In this case, the node x has failed, and we need to find recovery paths to s from each $x_i \in \mathcal{C}_x$. When a node fails, the shortest paths tree of s, \mathcal{T}_s , gets split into $k_x + 1$ components - one containing the source node s and each of the remaining ones contain one subtree of a child $x_i \in \mathcal{C}_x$.

Notice that the edge $\{g_p,g_q\}$ (Figure 1), which has one end point in the subtree of x_j , and the other outside the subtree of x provides a candidate recovery path for the node x_j . The complete path is of the form $p_G(x_j,g_p) \leadsto \{g_p,g_q\} \leadsto p_G(g_q,s)$. Since g_q is outside the subtree of x, the path $p_G(g_q,s)$ is not affected by the failure of x. Edges of this type (from a node in the subtree of $x_j \in \mathcal{C}_x$ to a node outside the subtree of x) can be used by $x_j \in \mathcal{C}_x$ to

escape the failure of node x. Such edges are called green edges. For example, edge $\{g_p, g_q\}$ is a green edge.

Next, consider the edge $\{b_u,b_v\}$ (Figure 1) between a node in the subtree of x_i and a node in the subtree of x_j . Although there is no green edge with an end point in the subtree of x_i , the edges $\{b_u,b_v\}$ and $\{g_p,g_q\}$ together offer a candidate recovery path that can be used by x_i to recover from the failure of x. Part of this path connects x_i to x_j $(p_G(x_i,b_u) \leadsto \{b_u,b_v\} \leadsto p_G(b_v,x_j))$, after which it uses the recovery path of x_j (via x_j 's green edge, $\{g_p,g_q\}$). Edges of this type (from a node in the subtree of x_i to a node in the subtree of a sibling x_j for some $i \neq j$) are called blue edges. Another example of a blue edge is edge $\{b_p,b_q\}$ which can be used the node x_1 to recover from the failure of x.

Note that edges like $\{r_a, r_b\}$ and $\{b_v, g_p\}$ (Figure 1) with both end points within the subtree of the same child of x do not help any of the nodes in \mathcal{C}_x to find a recovery path from the failure of node x. We do not consider such edges in the computation of recovery paths, even though they may provide a shorter recovery path for some nodes (e.g. $\{b_v, g_p\}$ may offer a shorter recovery path to x_i). The reason for this is that routing protocols would need to be quite complex in order to use this information. We carefully organize the *green* and *blue* edges in a way that allows us to retain only the useful edges and eliminate useless (red) ones efficiently.

We now describe the construction of a new graph \mathcal{R}_x , the recovery graph for x, which will be used to compute recovery paths for the elements of \mathcal{C}_x when the node x fails. A single source shortest paths computation on this graph suffices to compute the recovery paths for all $x_i \in \mathcal{C}_x$.

The graph \mathcal{R}_x has k_x+1 nodes, where $k_x=|\mathcal{C}_x|$. A special node, s_x , represents the source node s in the original graph G=(V,E). Apart from s_x , we have one node, denoted by y_i , for each $x_i\in\mathcal{C}_x$. We add all the *green* and blue edges defined earlier to the graph \mathcal{R}_x as follows. A green edge with an end point in the subtree of x_i (by definition, green edges have the other end point outside the subtree of x) translates to an edge between s_x and y_i . A blue edge with an end point in the subtree of x_i and the other in the subtree of x_j translates to an edge between nodes y_i and y_j . However, the weight of each edge added to \mathcal{R}_x is not the same as the weight of the green or blue edge in G=(V,E) used to define it. The weights are specified below.

Note that the candidate recovery path of x_j that uses the green edge $g=\{g_p,g_q\}$ has total cost equal to:

$$greenWeight(g) = d_G(x_j, g_p) + cost(g_p, g_q) + d_G(g_q, s)$$

$$(1)$$

As discussed earlier, a blue edge provides a path connecting two siblings of x, say x_i and x_j . Once the path reaches x_j , the remaining part of the recovery path of x_i coincides with that of x_j . If $\{b_u, b_v\}$ is the blue edge connecting the subtrees of x_i and x_j (the cheapest one corre-

sponding to the edge $\{y_i, y_j\}$), the length of the subpath from x_i to x_j is:

$$blueWeight(b) = d_G(x_i, b_u) + cost(b_u, b_v) + d_G(b_v, x_j)$$
(2)

We assign this weight to the edge corresponding to the blue edge $\{b_u, b_v\}$ that is added in \mathcal{R}_x between y_i and y_i .

The construction of our graph \mathcal{R}_x is now complete. Computing the shortest paths tree of s_x in \mathcal{R}_x provides enough information to compute the recovery paths for all nodes $x_i \in \mathcal{C}_x$ when x fails.

2.2 Description of the Algorithm and its Analysis

We now incorporate the basic observations described earlier into a formal algorithm for the SNFR problem. Then we analyze the complexity of our algorithm and show that it has a nearly optimal running time of $O(m \log n)$.

Our algorithm is a *depth-first* recursive algorithm over \mathcal{T}_s . We maintain the following information at each node x:

- Green Edges: The set of green edges in G = (V, E) that offer a recovery path for x to escape the failure of its parent.
- Blue Edges: A set of edges {p, q} in G = (V, E) such that x is the nearest-common-ancestor of p and q with respect to the tree T_s.

The set of green edges for node x is maintained in a \min heap (priority queue) data structure, which is denoted by \mathcal{H}_x . The heap elements are tuples of the form $< e, greenWeight(e) + d_G(s,x) >$ where e is a green edge, and $greenWeight(\cdot) + d_G(s,x)$ defines its priority as an element of the heap. Note that the extra element $d_G(s,x)$ is added in order to maintain invariance that the priority of an edge in any heap \mathcal{H} remains constant as the path to s is traversed. Initially \mathcal{H}_x contains an entry for each edge of x which serves as a green edge for it (i.e. an edge of x whose other end point does not lie in the subtree of the parent of x). A linked list, \mathcal{B}_x , stores the tuples < e, blueWeight(e) >, where e is a blue edge, and blueWeight(e) is the weight of e as defined by the equation (2).

The heap \mathcal{H}_{x_i} is built by merging together the \mathcal{H} heaps of the nodes in \mathcal{C}_{x_i} , the set of children on x_i . Consequently, all the elements in \mathcal{H}_{x_i} may not be green edges for x_i . Using a dfs labeling scheme similar to the one in [1], we can quickly determine whether the edge retrieved by $findMin(\mathcal{H}_{x_i})$ is a valid green edge for x_i or not. If not, we remove the entry corresponding to the edge from \mathcal{H}_{x_i} via a $deleteMin(\mathcal{H}_{x_i})$ operation. Note that since the deleted edge cannot serve as a green edge for x_i , it cannot serve as one for any of the ancestors of x_i , and it doesn't need to be added back to the \mathcal{H}_x heap for any x. We continue deleting the minimum weight edges from \mathcal{H}_{x_i} till either \mathcal{H}_{x_i} becomes empty or we find a green edge valid for x_i to escape x's failure, in which case we add it to \mathcal{R}_x .

After adding the green edges to \mathcal{R}_x , we add the blue edges from \mathcal{B}_x to \mathcal{R}_x .

Finally, we compute the shortest paths tree of the node s_x in the graph \mathcal{R}_x using a standard shortest paths algorithm (e.g. Dijkstra's algorithm [2]). The escape edge for the node x_i is stored as the parent edge of x_i in \mathcal{T}_{s_x} , the shortest paths tree of s_x in \mathcal{R}_x . Since the communication graph is assumed to be bi-connected, there exists a path from each node $x_i \in \mathcal{C}_x$ to s_x , provided that the failing node is not s.

For brevity, we omit the detailed analysis of the algorithm. The $O(m \log n)$ time complexity of the algorithm follows from the fact that (1) An edge can be a blue edge in the recovery graph of exactly one node: that of the nearest-common-ancestor of its two end points, and (2) An edge can be deleted at most once from any $\mathcal H$ heap. We state the result as the following theorem.

Theorem 2.1 Given an undirected weighted graph G = (V, E) and a specified node s, the recovery path from each node x_i to s to escape from the failure of the parent of x is computed by our procedure in $O(m \log n)$ time.

3 Single Node Failure Recovery Protocol

When routing a message to a node s, if a node x needs to forward the message to another node y, the node y is the parent of x in the shortest paths tree \mathcal{T}_s of s. The SNFR algorithm computes the recovery path from x to s which does not use the node y. In case a node has failed, the protocol re-routes the messages along these alternate paths that have been computed by the SNFR algorithm.

3.1 Embedding the Escape Edge

In our protocol, the node x that discovers the failure of y embeds information about the escape edge to use in the message. The escape edge is same as the ρ_x edge identified for the node x to use when its parent (y, i) in this example) has failed. We describe two alteratives for embedding the escape edge information in the message, depending on the particular routing protocol being used.

Protocol Headers

In several routing protocols, including TCP, the message headers are not of fixed size, and other header fields (e.g. Data Offset in TCP) indicate where the actual message data begins. For our purpose, we need an additional header space for two node identifiers (e.g. IP addresses, and the port numbers) which define the two end points of the escape edge. It is important to note that this extra space is required only when the messages are being re-routed as part of a failure recovery. In absence of failures, we do not need to modify the message headers.

Recovery Message

In some cases, it may not be feasible or desirable to add the information about the escape edge to the protocol headers. In such situations, the node x that discovers the failure of its parent node y during the delivery of a message \mathcal{M}_o , constructs a new message, \mathcal{M}_r , that contains information for recovering from the failure. In particular, the recovery message, \mathcal{M}_r contains (a) \mathcal{M}_o : the original message, and (b) $\rho_x = (p_x, q_x)$: the escape edge to be used by x to recover from the failure of its parent.

With either of the above two approaches, a light weight application is used to determine if a message is being routed in a *failure free* case or as part of a *failure recovery*, and take appropriate actions. Depending on whether the escape edge information is present in the messagae, the application decides which node to forward the message to. This process consumes almost negligible additional resources. As a further optimization, this application can use a special reserved port on the routers, and messages would be sent to it only during the failure recovery mode. This would ensure that no additional resources are consumed in the failure free case.

3.2 Protocol Illustration

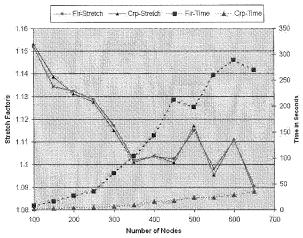
For brevity we do not formally specify our protocol, but only illustrate how it works. Consider the network in Figure 1. If x_i notices that x has failed, it adds information in the message (using one of the two options discussed above) about $\{b_u, b_v\}$ as the escape edge to use, and reroutes the message to b_u . b_u clears the escape edge information, and sends the message to b_v , after which it follows the *regular* path to s. If x has not recovered when the message reaches x_j , x_j reroutes with message to g_p with $\{g_p, g_q\}$ as the escape edge to use. This continues till the message reaches a node outside the subtree of x, or till x recovers.

Note that since the alternate paths are used only during failure recovery, and the escape edges dictate the alternate paths, the protocol ensures *loop free* routing, even though the alternate paths may form loops with the original routing (shortest) paths.

4 Simulation Results and Comparisons

We present the simulation results for our algorithm, and compare the lengths of the recovery paths generated by our algorithm to the theoretically optimal paths as well as with the ones computed by the algorithm in [11]. In the implementation of our algorithm, we have used *standard* data structures (e.g. binary heaps instead of Fibonacci heaps [3]: binary heaps suffer from a linear-time merge/meld operation as opposed to constant time for the latter). Consequently, our algorithms have the potential to produce much better running times than what we report.

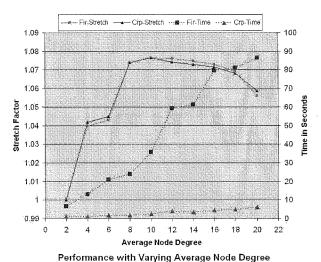
We ran our simulations on randomly generated graphs, with varying the following parameters: (a) Number of nodes, and (b) Average degree of a node. The edge weights are randomly generated numbers between 100 and 1000. In order to guarantee that the graph is 2-



Performance with Varying Number of Nodes

Figure 2.

node-connected (biconnected), we ensure that the generated graph contains a *Hamiltonian cycle*. Finally, for each set of these parameters, we simulate our algorithm on multiple random graphs to compute the *average* value of the of a *metric* for the parameter set. The algorithms have been implemented in the Java programming language (1.5.0.12 patch), and were run on an Intel machine (Pentium IV 3.06GHz with 2GB RAM).



-..-...g- ..-..g-

Figure 3.

The *stretch factor* is defined as the ratio of the lengths of recovery paths generated by our algorithm to the lengths of the theoretically optimal paths. The optimal recovery path lengths are computed by recomputing the shortest paths tree of s in the graph $G(V \setminus x, E \setminus E_x)$. In the figures

[2,3], the Fir labels relate to the performance of the alternate paths algorithm used by the Failure Insensitive Routing protocol of [11], while the Crp labels relate to the performance of our algorithm for the SNFR problem.

Though [11] doesn't present a detailed analysis of their algorithm, from our analysis, their algorithm needs at least $\Omega(mn \log n)$ time per sink node in the system. Figures [2,3] compare the performance of our algorithm (CRP) to that of [11] (FIR). The plots for the running times of our algorithm and that of [11] fall in line with the theoretical analysis that our algorithms are faster by an order of magnitude than those of [11]. Interestingly, the stretch factors of the two algorithms are very close for most of the cases, and stay within 15%. The running time of the algorithms fall in line with our theoretical analysis. Our CRP algorithm runs within 50 seconds for graphs upto 600-700 nodes, while the FIR algorithm's runtime shoots up to as high as 5 minutes as the number of nodes increase. The metrics are plotted against the variation in (1) the number of nodes (Figure [2]), and (2) the average degree of the nodes (Figure [3]). The average degree of a node is fixed at 15 for the cases where we vary the number of nodes (Figure [2]), and the number of nodes is fixed at 300 for the cases where we plot the impact of varying average node degree (Figure [3]). As expected, the stretch factors improve as the number of nodes increase. Our algorithm falls behind in finding the optimal paths in cases when the recovery path passes through the subtrees of multiple siblings. Instead of finding the best exit point out of the subtree, in order to keep the protocol simple and the paths well structured, our paths go to the root of the subtree and then follow its alternate path beyond that. These paths are formed using the blue edges. Paths discovered using a node's green edges are optimal such paths. In other words, if most of the edges of a node are green, our algorithm is more likely to find paths close to the optimal ones. Since the average degree of the nodes is kept fixed in these simulations, increasing the number of nodes increases the probability of the edges being green. A similar logic explains the plots in Figure [3]. When the number of nodes is fixed, increasing the average degree of a node results in an increase in the number of green edges for the nodes,⁴ as well as the stretch factors.

5 Concluding Remarks

In this paper we have presented an efficient algorithm for the SNFR problem, and developed protocols for dealing with transient single node failures in communication networks. Via simulation results, we show that our algorithms are much faster than those of [11], while the stretch factor of our paths are usually better or comparable.

Previous algorithms [6, 8, 11] for computing alternate paths are much slower, and thus impose a much longer network setup time as compared to our approach. The setup

time becomes critical in more dynamic networks, where the configuration changes due to events other than transient node or link failures. Note that in several kinds of configuration changes (e.g. permanent node failure, node additions, etc), recomputing the routing paths (or other information) cannot be avoided, and it is desirable to have shorter network setup times.

For the case where we need to solve the SNFR problem for all nodes in the graph, our algorithm would need $O(mn\log n)$ time, which is still very close to the time required $(O(mn+n^2\log n))$ to build the routing tables for the all-pairs setting. The space requirement still stays linear in m and n.

The directed version of the SNFR problem, where one needs to find the optimal (shortest) recovery paths can be shown to have a lower bound of $\Omega(\min(m\sqrt{n},n^2))$ using a construction similar to those used for proving the same lower bound on the directed version of SLFR[1] and replacement paths[4] problems. The bound holds under the path comparison model of [5] for shortest paths algorithms.

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⁴When the average degree is very small, there are only a few alternate paths available, and the algorithms usually find the better ones among them, resulting in smaller stretch factors.