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Multicasting using WDM in Multifiber Optical Star Networks

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ABSTRACT
This paper examines the wavelength assignment problem with single, dual, and multimessage multicasting. We use a star network as our model where the messages are routed using Wavelength Division Multiplexing over single, dual, and multifiber optical networks. The specific problem we consider is given any star network, with a finite number of nodes, a predetermined number of fibers that connect these nodes, and a set of multicasts to be sent in one communication round, determine the number of wavelengths required per fiber to achieve conflict free transmission of all of the messages.

KEY WORDS
Optical Networks, Wavelength Division Multiplexing, Multicasting, Routing Algorithms, Star Networks, Wavelength Assignment Problem

1 Introduction

The ever increasing need for faster data transmission has led to extended research into the area of Optical Networks using Wavelength Division Multiplexing (WDM). Optical Networks in general allow much greater data transmission speeds than Electrical Networks, and optical switching has allowed us to retain these transmission speeds even when direct links between nodes are not available [1, 2]. Furthermore, WDM allows messages to be transmitted on different wavelengths (or channels) over the same fiber. A passive star coupler is utilized to join the nodes in these networks, making transmission between all nodes along the same wavelength completely optical [2]. Networks can be single, dual, or multifiber networks depending on the number of fibers that connect adjacent nodes in the system. In dual fiber and multifiber networks, messages can be switched from one fiber to another along the same wavelength [2, 3].

These high transmission speeds are needed for applications such as video conferencing, distributed data processing, scientific visualization, high speed supercomputing, and real-time medical imaging to name a few [4]. The need for multicasting is also growing and it is likely that future communication networks will include a large amount of multi-destination traffic [1, 4]. Furthermore, since one of the biggest costs to implement an optical network is the actual physical laying of the optical fibers, often many fibers may be installed at the same time resulting in multifiber networks [5]. Wide area testbeds are currently being developed, employing WDM technology to pass data over various wavelengths in real-time [6]. The problem we consider in this paper is given any star network, a predetermined number of fibers, and a set of multicasts to be sent between nodes in one communication round, find the least number of wavelengths per fiber required to send all of the messages.

We use *multicast* to indicate that a node sends a message to one or more nodes in the system (also called a *unicast* when sending to only one other node and a *broadcast* when sending to all other nodes). A star network is a system of nodes that communicate with each other by sending messages through an internal routing node (the passive star coupler in our case). The messages are then routed to all of the appropriate receiving nodes. The benefit of multifiber networks is that even though nodes must receive messages on the same wavelength from which they were sent, the receiving nodes are able to receive the message on *any* fiber. We call this optical rerouting of messages onto a *different fiber*, "switching" the message. This is the central process that allows for better utilization of individual fibers in dual and multifiber networks. In this paper we consider a single time phase, meaning messages are sent and received in the same phase.

We consider the wavelength assignment problem (WAP) which is to determine the wavelengths on which to send the required messages. A related problem is the scheduling and wavelength assignment (SWA) problem. The goal of SWA is to schedule the required messages on the available wavelengths in order to minimize the finish time. Even more closely related is the wavelength and routing assignment problem (WRAP), in which both the routes and the wavelengths that each message uses must be determined.

1.1 Problem Definition

Given n nodes, with node i sending s_i multicasts, the j^{th} multicast of each node, for $1 \leq j \leq s_i$, is sent to the set of nodes $d_{i,j}$. Note that since $d_{i,j}$ is a set, a multicast cannot be sent to the same node more than once. Also, $i \notin d_{i,j}$, $\forall$ i and j since it does not make sense for a node to send a message to itself. Consider Example 1 with $n = 4$, 3 fibers, and 2 λ/f. The routing requests s_i and $d_{i,j}$ are given in Table 1.

Table 1. s_i and $d_{i,j}$ for Example 1

$s_1 = 3$	$d_{1,1} = \{2\}$ $d_{1,2} = \{2,3\}$ $d_{1,3} = \{2,3\}$
$s_2 = 0$	
$s_3 = 3$	$d_{3,1} = \{1,2,4\}$ $d_{3,2} = \{2\}$ $d_{3,3} = \{4\}$
$s_4 = 1$	$d_{4,1} = \{1,2,3\}$

We now specify the constraints for sending and receiving messages in our model. Nodes cannot both send and receive on the same fiber-wavelength pair at the same time. Furthermore, nodes can receive at most one message on each fiber-wavelength pair and send at most one multicast on each fiber-wavelength pair, but not both. There is no reason to send a message on multiple fibers using the same wavelength because switching the messages across fibers can achieve this result. However, it might be advantageous to send the same message on different wavelengths so that different nodes can receive the message on several wavelengths and avoid conflicts with other messages. As we stated above, we can switch a message from one fiber to another on the same wavelength, but cannot switch the wavelength on which a message is sent. Given a system with n nodes, g fibers, and the sets of multicasts $d_{i,j}$, an assignment using w wavelengths is to specify for each wavelength w in the system, the messages it will transmit, i.e. a partition of the nodes and fibers into sets for each w as specified in (1).

$$\begin{array}{l} \forall w \;\; partition \;\; \{1,2,...,n\} \; X \; \{1,2,...,g\} \\ into \;\; S_0^w, \; S_{1j}^w, \; S_{2j}^w, \; ..., \; S_{nj}^w \;\; for \;\; 1 \le j \le g \end{array} \quad (1)$$

The set S_{ij}^w, in (1), indicates that a multicast sent from node i on fiber j is transmitted to node k on fiber l for each $(k,l) \in S_{ij}^w$. The set S_0^w represents the unused receiving node-fiber pairs; and, if $S_{ij}^w \neq \emptyset$, then $(i,j) \in S_0^w$ (since a node-fiber pair used to send a message cannot be used to receive a message). Additionally, $k \neq i$ for all cases since a node cannot send a message to itself, and a node cannot send the same multicast to the same destination more than once. Consider the assignments in Figure 1 which contains 4 nodes and 3 fibers. Two wavelengths are shown (w_1 and w_2). On the left side of each wavelength representation, the nodes are shown with their respective sent messages. The messages are then routed by the passive star coupler and finally received by the nodes on the right. Each node has three lines on which messages can be transmitted with the top line representing fiber 1 (f_1), the next f_2, and the bottom f_3.

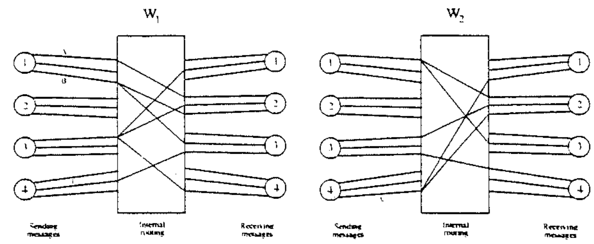

Figure 1. Routing Example

On w_1, the multicast labeled "A" is sent from node 1 on f_1 and received by node 2 on f_1. So, $S_{11}^1 = \{(2,1)\}$. No messages are sent from node 1 on f_2, so $S_{12}^1 = \emptyset$. The multicast labeled "B' is sent from node 1 on f_3 and received by both node 2 on f_3 and by node 3 on f_2, so $S_{13}^1 = \{(2,3),(3,2)\}$. Remember that since nodes cannot send and receive on the same fiber-wavelength pair, once node 1 sends on $f_1 w_1$ it cannot receive on $f_1 w_1$. Multicast "C" cannot be sent from node 4 to node 2 on w_1 since node 2 does not have any fibers available. Instead, node 4 sends multicast "C" to node 2 on w_2. The nonempty sets for w_1 and w_2 are defined in Table 2.

Table 2. Set definitions for Figure 1

$S_0^1 = \{(1,1),(1,3),(3,1),(4,1),(4,2)\}$	$S_0^2 = \{(1,1),(1,2),(3,1),(3,3),(4,2),(4,3)\}$
$S_{11}^1 = \{(2,1)\}$	$S_{11}^2 = \{(2,1),(3,2)\}$
$S_{13}^1 = \{(2,3),(3,2)\}$	$S_{31}^2 = \{(2,2)\}$
$S_{31}^1 = \{(1,2),(2,2),(4,3)\}$	$S_{33}^2 = \{(4,1)\}$
$S_{42}^1 = \{(3,3)\}$	$S_{43}^2 = \{(1,3),(2,3)\}$

An assignment is said to be feasible if the assignment can transit all of the multicasts, $d_{i,j}$, to their destinations. In other words, $d_{i,j}$ will be associated with a set of S_{kl}^w which we call $t_{i,j}$ such that $d_{i,j} = \{k \mid S_{k,l}^w \in t_{i,j}\}$ and each S_{kl}^w is assigned to at most one set $t_{i,j}$. Table 3 shows some of the $d_{i,j}$ sets for Example 1.

Table 3. $d_{i,j}$ and $t_{i,j}$ for Example 1

$d_{1,1}$: $\{S_{11}^1\} = t_{1,1}$
$d_{1,2}$: $\{S_{13}^1\} = t_{1,2}$
...
$d_{3,3}$: $\{S_{33}^2\} = t_{3,3}$
$d_{4,1}$: $\{S_{42}^1, \; S_{43}^2\} = t_{4,1}$

1.2 Related Work

The SWA problem was shown to be NP-Hard [7] for both preemptive (operations can be stopped, or preempted, and resumed at a later time) and non-preemptive (operations cannot be preempted) cases. Bampis and Rouskas [7] develop efficient approximation algorithms for both cases. Li and Simha [3] consider the offline WAP over multiple fibers in a unicast only environment. The main result in [3] is that in a multifiber network, switching messages between the fibers increases wavelength utilization. For star networks, WAP is known to be NP-Complete over a single fiber [5], but in [3], optimal polynomial time algorithms for the cases of dual and multifiber networks are developed. For ring networks, the dual and multifiber cases are shown to be NP-Complete and upper bounds are established for both cases. Several papers consider multicast environments over a single fiber network including [1, 4], and these architectures use all optical networks and WDM. Thaker and Rouskas [2] survey multicast scheduling algorithms (MSAs) in single fiber star networks.

We extend the above research by considering the WAP in multifiber multicast networks. We use an optical star network as our model and develop bounds for single, dual, and multifiber networks using WDM. We consider the offline version of the WAP.

1.3 Conventions and Outline

We introduce the notation defined ($\alpha \mid \beta \mid \gamma$). The first and second terms, α and β, specify that every node in the system can receive at most α messages and send at most β multicasts. Additionally, there is at least one node in the system that receives α messages and sends β multicasts. We consider the cases where the values of α and β are 1, 2, or n. The cases are called single, dual and multimessage multicasting, respectively. The number of fibers is γ. We assume that all fibers have the same number of wavelengths.

For every system, ($\alpha \mid \beta \mid \gamma$), we exhibit a *least possible upper bound* (which we denote as *LPU bound*) and an *upper bound* on the number of wavelengths required per fiber. By LPU bound, we mean that *there exists* a problem instance, in this specific system, that in order to achieve conflict free transmission of all messages, requires at least this number of wavelengths per fiber. By upper bound, we mean that *every* problem instance, in this specific system, can achieve conflict free transmission of all messages with at most this number of wavelengths per fiber.

Throughout the paper we use the terms *wavelength* and *color* interchangeably. Additionally, although the internal routing node is always present, in this paper we simplify our descriptions and figures by ignoring it and showing all messages going from one node in the system directly to another node in the system.

The *chromatic index* (or edge coloring problem) of a graph is the minimum number of colors required to color the graph's edges so that no two edges emanating from the same vertex have the same color. The problem is known to be NP-Complete [8] and there have been several approximation techniques developed [9]. Our problem reduces to the edge coloration problem only in the specific case where all messages are unicasts and the number of fibers is equal to one.

For every section in the paper the trivial LPU bound is simply $\left\lceil \frac{(\alpha+\beta)}{\gamma} \right\rceil$. About half of the LPU bounds we obtain are equal to this trivial bound and the remaining LPU bounds are obtained using examples that range from simple to much more complex problem instances. A summary of the results is shown in Figure 2. This paper contains

Fibers	send ≤ 1, rec. ≤ 1	send ≤ 1, rec. ≤ 2	send ≤ 1, rec. ≤ n
1	3	LP = 4 UB = 5	LP = 2n UB = $2n + \lceil \frac{n}{2} \rceil$
2	1	2	n
g	1	1	$\lceil \frac{n}{g-1} \rceil$
Fibers	**send ≤ 2, rec. ≤ 1**	**send ≤ 2, rec. ≤ 2**	**send ≤ 2, rec. ≤ n**
1	3	6	3n
2	2	3	LP = n UB = $\lceil \frac{3n}{2} \rceil$
g	1	1	$\lceil \frac{n}{g-2} \rceil$ *
Fibers	**send ≤ n, rec. ≤ 1**	**send ≤ n, rec. ≤ 2**	**send ≤ n, rec. ≤ n**
1	n+1	2n+2	n^2+n
2	$\lceil \frac{n+1}{2} \rceil$	LP = $\lceil \frac{n+2}{2} \rceil$ UB = n+1	LP = $\lceil \frac{\frac{n^2}{2}+n}{2} \rceil$ UB = $\lceil \frac{n^2+n}{2} \rceil$
g	$\lceil \frac{n+1}{g} \rceil$	LP = $\lceil \frac{n+2}{g} \rceil$ UB = $\lceil \frac{2n+2}{g} \rceil$	LP = $\lceil \frac{\frac{n^2}{g}+n}{g} \rceil$ UB = $\lceil \frac{n^2+n}{g} \rceil$

Figure 2. LPU and Upper Bounds on the Number of Wavelengths required per Fiber. LP = LPU Bound, UB = Upper Bound. Values are tight bounds (i.e. LP=UB) unless otherwise stated. $n > 2$ and $g > 2$ unless otherwise stated.
* LP valid for $g \geq 4$; and, UB valid for $g \geq 3$

proofs of some selected results; detailed proofs for all of the results from Figure 2 can be found in [10]. The paper is organized as follows. Instances of LPU bound and upper bound results for single message multicasting are described in Sections 2 through 5. Section 6 presents a result for dual message multicasting, and Section 7 gives a multimessage multicasting example. We conclude the paper in Section 8.

2 Bounds for (1 | 1 | 1)

In this case, the LPU bound and the upper bound are both equal to 3 wavelengths per fiber (3 λ/f). The LPU bound follows from the problem instance given in Figure 3(a) which requires three colors because each of the three edges must be colored differently than the other two.

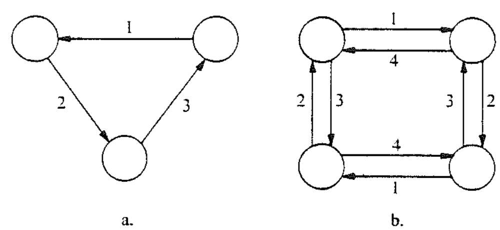

Figure 3. a. Requires 3 Colors, b. Requires 4 Colors

Next, we show that the upper bound is also 3 λ/f. Every problem instance can be viewed as a collection of disjoint subgraphs. Each subgraph is a tree, plus there could be a single additional edge from a node to the root. We color all of the edges in each tree with colors 1 and 2, and the edge that goes from a node to the root with color 3.

3 Bounds for (1 | 1 | 2)

In this situation, the LPU bound and the upper bound are both equal to 1 λ/f. The algorithm to "color" the messages is a simple one. All messages are sent out on f_1w_1 (fiber one and wavelength one). Next, we will utilize the dual fiber network and switch the messages from one fiber to another. So, every message is switched to f_2w_1. Since every message is sent out on f_1w_1, received on f_2w_1, and every node can receive at most one message, there will not be any conflicts. Therefore, every graph can be colored with 2 fibers and 1 color per fiber. Note that having additional fibers available would not decrease the upper bound.

4 Bounds for (2 | 1 | 1)

For this case, we show an LPU bound of 4 λ/f and an upper bound of 5 λ/f. We establish the LPU bound with the problem instance given in Figure 3(b). The proof that this problem instance requires 4 colors can be found in [10].

To show the upper bound of 5 λ/f we first consider the subproblem where every node has exactly two outgoing edges and two incoming edges. We present a constructive proof that shows that every such problem instance is colorable with 5 colors. Then we show how to use this result to color all problem instances with 5 colors. The resulting algorithm takes $O(n)$ time.

Theorem 4.1 *Every problem instance where every node has exactly 2 outgoing edges and exactly 2 incoming edges can be colored with 5 colors (which correspond to f_1w_1 through f_1w_5).*

Proof: Our proof is constructive. We consider each node one at a time. When considering a node x we color its incoming edges and in some special cases, we must recolor some, previously colored, edges. We will refer to the nodes where the two incoming messages to node x originate as the "parents" of node x and label them $P1$ and $P2$. There are three cases to consider when coloring node x depending on the number of different colors of the incoming messages to $P1$ and $P2$.

The cases where the number of colors that overlap is 0 or 1 are very similar and straightforward. We now discuss the 3^{rd} case, when the incoming edges to node $P1$ are colored identically to the incoming edges to node $P2$. Without loss of generality, we can assign the incoming messages of $P1$ and the incoming messages of $P2$ each to colors 1 and 2. If an edge leaving node x is an incoming edge to $P1$ or $P2$, then clearly the incoming edges to node x can be colored with the remaining colors. In all other cases, we proceed as follows. Without considering the edges emanating from node x, the edge from $P1$ to x and the edge from $P2$ to x could each be colored 3, 4, or 5. If the outgoing edges from node x are colored, then remove such coloring. Those edges, leaving node x, may each be colored with two colors in such a way that they will not conflict with the coloring of the other edges at the nodes where they terminate. Let S_1 be the set of colors of which the first edge leaving node x can be colored and let S_2 be the set of colors of which the second edge leaving node x can be colored.

If S_1 and S_2 have a color in common, then these edges (the edges leaving node x) can be assigned such color and there will be at least two possible colors that can be assigned to node $x's$ incoming edges. Therefore, a valid coloration is possible in this case.

On the other hand, we have the case where S_1 and S_2 do not have a color in common. Since S_1 and S_2 have two colors each, there is at least one color in S_1 or S_2 that is not color 3, 4, or 5. Assume, without loss of generality, that such color is $s \in S_1$. Now, assign color s to the first edge leaving node x, and assign one of the colors from S_2, let us call it t, to the second edge leaving node x. The incoming edges to node x may then be assigned to colors $\{3, 4, 5\} - t$. Therefore, a valid coloration is also possible in this case. []

Let us now return to the more general problem where any node can send up to one multicast (to any number of destinations) and receive up to two messages and show that this problem can be colored using 5 colors.

Theorem 4.2 *Every problem instance where every node is sending at most one multicast and receiving at most two messages can be colored with 5 colors.*

Proof: Let G_1 be the message directed graph. Every node in G_1 has an in-degree of at most two and an out-degree of at most n. Consider any node x of out-degree zero or one. Given any 5-coloration of $G_1 - \{incoming\ edges\ to\ x\}$ one can color the incoming edges to node x without any conflicts as follows. The incoming edges to node x cannot be the same color as the two edges being received by the node where the edge originates or as the edge leaving node x. This leaves at least 2 colors available to color 2 edges.

The resulting problem is on a message directed graph G_2. Every node in G_2 is such that either the out-degree is one and the in-degree is zero *or* the out-degree is at least two and the in-degree is at most two. One can show that it must be that every node is of in-degree 2 and out-degree 2. By *Theorem 4.1*, such a problem is 5-colorable. Therefore, every problem instance where all nodes can send at most one multicast and receive at most two messages can be colored with 5 colors. []

5 LPU Bound for $(n \mid 1 \mid g)$

In this case, the LPU bound is equal to $\lceil \frac{n}{g-1} \rceil \ \lambda/f$ (where g is the number of fibers). To show the LPU bound we give a problem instance that cannot be colored using $(\lceil \frac{n}{g-1} \rceil - 1) \ \lambda/f$. The construction will be such that n nodes will have the same colors available for their multicasts; and additionally, they will have less than $\frac{n}{g}$ of such colors. Furthermore, all of these nodes will send a message to some node x. Therefore, we will have a conflict because there will not be enough colors available to color the messages that node x receives.

We now describe the structure of the problem instance. There are k levels of nodes with ϕ_i nodes in level i. Every node in the first level, receives n messages (from n nodes that are not in these levels of nodes, and that do not receive any messages), and sends one multicast. For every subset of n nodes in level 1, there are n nodes in level 2 receiving a message from each of these n nodes. The nodes in every subsequent level i, for $i > 1$, will receive n messages from the previous level. Also, for every subset of n nodes in level i, there are n nodes in level $i+1$ receiving a message from each of these n nodes. To guarantee that there are n nodes in level 1, all receiving identically colored messages, we let $\phi_1 = \binom{(\lceil \frac{n}{g-1} \rceil - 1)g}{n}(n-1) + 1$. Additionally, to guarantee that there are n nodes in a level i, all receiving identically colored messages, we let $\phi_i = \binom{\phi_{i-1}}{n} n$ for $i > 1$. Note that this implies a graph with a large number of nodes; however, since the number of nodes is finite, it is still a valid problem instance.

Let β colors be available in g fibers for the input messages to a node (at any level). Clearly, $\beta \leq \lceil \frac{n}{g-1} \rceil - 1$. It must be that $\beta g \geq n$, as otherwise there is no feasible coloring of the incoming messages at the node. We claim the following theorem.

Theorem 5.1 *If a node receives all messages in β different colors, then the maximum number of colors available for its multicasting send operation is less than β.*

The proof of *Theorem 5.1* can be found in [10], for brevity, we have omitted it from this paper. From *Theorem 5.1*, we have established that, in our problem instance, for any number of fibers greater than or equal to two ($g \geq 2$), the number of colors available as input to the next level of nodes is strictly less than the number of colors available as input to the current level of nodes. Since the number of colors available is always decreasing, by having at most $(\lceil \frac{n}{g-1} \rceil - \lceil \frac{n}{g} \rceil)$ levels of nodes, the number of colors available will fall below $\frac{n}{g}$. At this point, there will not be enough fiber-wavelength pairs to color the next level and we will have a conflict.

6 LPU Bound for $(2 \mid 2 \mid 2)$

Here, the LPU bound is equal to 3 λ/f. To establish the LPU bound, we give the problem instance in Figure 4. We refer the reader to [10] for a detailed proof.

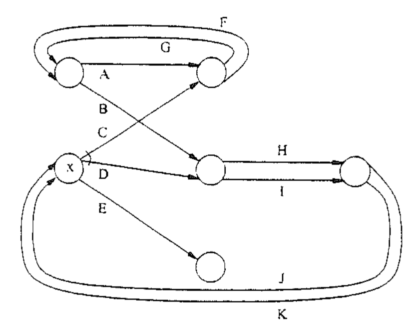

Figure 4. Requires 3 λ/f

7 Upper Bound for $(n \mid n \mid g)$

In this section, we exhibit an upper bound of $m \ \lambda/f$ (where $m \geq 1$) for the case when the number of fibers is equal to $(i + \frac{n^2}{im})$, where i represents any natural number. We minimize the product, fibers * wavelengths, by letting the value of $i = \frac{n}{\sqrt{m}}$. Therefore, $g = \frac{2n}{\sqrt{m}}$ and $m = \frac{4n^2}{g^2}$. This upper bound is essentially four times the LPU bound result and is therefore a better result than the general upper bound result when we are not limited to a fixed number of fibers.

The following descriptions assume that the values of i, m, and n when used to assign wavelengths always result in integer values for the expressions below. When this is not the case we refer the reader to [10]. The messages to be *received* by all nodes are assigned as follows.

The idea is to use i fibers (f_1, f_2, ... ,f_i) to receive the messages at every node. We will use each fiber for $\frac{n}{i}$ of the messages. Since each fiber has m wavelengths, we will allow the use of $\frac{m}{\frac{n}{i}}$ wavelengths for each message. So the first incoming message at every node can be assigned to fiber one (f_1) and any of the wavelengths from set S_1 in Figure 5. The second incoming message at every node can be assigned to f_1 and any of the wavelengths from set S_2; and so on up to the $(\frac{n}{i})^{th}$ message which can be assigned to f_1 and any of the wavelengths from set $S_{\frac{n}{i}}$. The next $\frac{n}{i}$ messages are assigned similarly, but using f_2. This coloring process continues until the last set of $\frac{n}{i}$ messages which use f_i and the same sets of wavelengths. At this point, using this technique, all of the incoming messages at every node can be assigned a unique fiber-wavelength pair. The appropriate wavelength to use for every incoming message will be determined based on the multicast's wavelength assignments, which are described below. Note that we have used a total of i fibers and m λ/f to assign the messages *received* at every node.

1	2	...	$\frac{im}{n}$	S_1
$\frac{im}{n}+1$	$\frac{im}{n}+2$	...	$\frac{2im}{n}$	S_2
⋮	⋮		⋮	⋮
$(\frac{n}{i}-1)\frac{im}{n}+1$	$(\frac{n}{i}-1)\frac{im}{n}+2$	...	m	$S_{\frac{n}{i}}$
T_1	T_2	...	$T_{\frac{im}{n}}$	

Figure 5. Sets of Wavelengths

Next we discuss the multicasts sent from every node. In order to avoid any conflicts every message in every multicast must be able to be sent using a wavelength in each of the sets $S_1, S_2, ..., S_{\frac{n}{i}}$ on some fiber; and no other multicast emanating from this node can use these fiber-wavelengths pairs. To accomplish this we define a set T_j as the set of all of the j^{th} elements in each of the sets $S_1, S_2, ..., S_{\frac{n}{i}}$ (when viewing these sets as order sets). Figure 5 gives a possible definition of the sets S and T. Clearly, there are $\frac{im}{n}$ different T sets. So, each fiber can be used on $\frac{im}{n}$ different multicasts. Therefore, the total number of fibers needed to send the n multicasts at every node is $\frac{n^2}{im}$ (fibers f_{i+1}, f_{i+2}, ... ,$f_{i+\frac{n^2}{im}}$).

8 Conclusion

We determined LPU and upper bounds on the number of wavelengths required per fiber for star networks where the messages are routed using Wavelength Division Multiplexing. Single, dual, and multimessage multicasting were considered along with single, dual, and multifiber optical networks. If, as networks develop, the available fibers increases beyond the amount of traffic in the network, ($i.e.$ $g \gg n$), many of our results reduce to 1 λ/f; however, this seems unlikely to happen given current trends. Future work could include continued efforts to obtain tight bounds for all of the remaining systems within star networks along with finding bounds for ring networks and more general network topologies.

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