Remarks on the Via Assignment Problem

by

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ABSTRACT: In [TKS] it was shown that the via assignment problem is NP-complete when each net is restricted to use at most one via column and it is conjectured that the problem remains NP-complete even when this restriction is relaxed. In this note we resolve this conjecture by showing that this problem is NP-complete.

Keywords: Via Assignment, NP-complete problems.

In this short note, we use the notation introduced in [TKS]. It will be shown that the unrestricted minimum forest (UMF) problem is NP-hard. This problem is defined as follows:

Unrestricted Minimum Forest (UMF)

Input: G_k associated with net list L and k via columns.

Property: A forest F satisfying Theorem II.3 exists in G_q such that $q \leq k$. []

In [TKS] it was conjectured that this problem is NP-complete. In this note we resolve this conjecture by showing that this problem is NP-complete. Our reduction is similar to the one used in [TKS] to prove that the minimum forest problem is NP-complete (the proof of this result is based on the proof of theorem II.2).

In order to prove our result we reduce a version of the 3-graph coloration problem to our problem. It is simple to show that this version of the graph coloration problem is NP-complete.

3-graph coloration problem:

Input: Given an arbitrary undirected graph (G) such that each node is of degree ≥ 2.

Property: Is G three colorable? []

<u>Theorem 1</u>: The UMF problem is NP-complete.

<u>Proof</u>: It is simple to prove that this decision problem is NP, so we only prove that the 3-graph coloration problem reduces to it. The reduction is as follows:

Given an arbitrary undirected graph $G=(V,E)$, we define the following instance of the UMF problem that we denote INS. INS is constructed as follows:

Let $V = \{v_1, v_2, \ldots, v_n\}$, where n is the number of nodes in G and let $E = \{e_1, e_2, \ldots, e_r\}$, where r is the number of edges in G. There are n+r nets. We shall denote these nets by the symbols $N_1, N_2, \ldots, N_n$ and $Y_1, Y_2, \ldots, Y_r$. There are 2*r rows and k, the maximum number of via columns that one can use for the connections, is 3. Net N_i has a generalized pin in rows (2*j)-1 and 2*j if edge e_j is incident upon node v_i. Net Y_j has a generalized pin in rows (2*j)-1 and 2*j. It is easy to see that the construction process can be carried out in polynomial time.

We claim that the instance INS we construct from G can be connected by using three via columns if and only if G is three colorable. It is simple to see that if G is three colorable then INS can be connected by using three columns. We now show that if INS can be connected by using three columns then G is three colorable.

Assume that INS can be connected by using at most three columns. It is simple to show that all the N nets will use at least 2*r vias for their connections and each of the Y nets will use at least 2 vias. Since there are only 3*r vias then all N nets use exactly 2*r vias and each Y net uses 2 vias. Hence, each net has to be assigned to precisely one via column. Assign color j to each node n_i if net N_i is connected by the vias in column number j. It is simple to show that this assignment gives a proper three coloration for G.

Hence, INS can be connected by using three colors if and only if G is three colorable. This completes the proof of the theorem. []

REFERENCES

[TKS] Ting, Kuh and Sangiovanni-Vincentelli,"Via Assignment problem in Multilayer Printed Circuit Board," IEEE Transactions on Circuits and Systems, Vol. CAS-26, No. 4, April 1979.