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Interconnecting Rectangular Areas by Corridors and Tours

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***Abstract*—Given a rectilinear polygonal boundary partitioned into rectangles, the Minimum-Length Corridor (MLC-R) problem consists of finding a corridor of least total length. A corridor is a set of connected line segments, each of which must lie along the line segments that form the polygonal boundary and/or the boundary of the rectangles, and must include at least one point from the boundary of every rectangle and from the polygonal boundary. The MLC-R problem is known to be NP-hard. We present an alternative polynomial time constant ratio approximation algorithm for the MLC-R problem. Our algorithm is based on the restriction and relaxation approximation techniques.**

***Index Terms*—Corridors, Approximation Algorithms, Restriction and Relaxation Techniques, Complexity, Computational Geometry**

I. Introduction

An instance I of the Minimum-Length Corridor (MLC-R) problem consists of a pair (F, R), where F is a rectilinear polygon partitioned into a set R of rectangles[1] (or rooms) $R_1, R_2, \ldots, R_r$. A *corridor* $T(I)$ for instance I is a set of connected line segments, each of which lies along the line segments that form F and/or the boundary of the rooms, and must include at least one point from every room and from rectangle F. The objective of the MLC-R problem is to construct a corridor of least total length. A generalization of the MLC-R problem where the rooms are rectilinear polygons is called the MLC problem. The MLC problem was initially defined by Naoki Katoh [1] and subsequently Eppstein [2] discussed the MLC-R problem. Experimental evaluations of several heuristics for the MLC problem are discussed in Ref. [3]. The question as to whether or not the decision version of each of these problems is NP-complete is raised in the above three references.

Recently Gonzalez-Gutierrez and Gonzalez [4], and independently Bodlaender, et al. [5], proved that the decision version of the MLC problem is strongly NP-complete. Gonzalez-Gutierrez and Gonzalez [4] also showed that the decision version of the MLC-R problem is strongly NP-complete as well as some of its variants. In virtue of these results, attention has shifted to the corresponding approximation problems. Gonzalez-Gutierrez and Gonzalez [6] presented a constant ratio approximation algorithm for the MLC-R problem. In this paper we present an alternate approximation algorithm for the MLC-R problem. The benefit of having two or more approximation algorithms for the same problem is that one may run all of them and then to select the best of the solutions generated. This hybrid approach will generate solutions closer to the optimal.

Bodlaender, et al. [5] consider several restricted versions of the MLC problem. One of these restricted versions of the MLC problem is called the *geographic clustering* problem. However, not all instances of the MLC-R problem are geographic clustering problem instances. It is not known whether the decision version of the geometric clustering problem is NP-complete. A polynomial time approximation scheme (PTAS) for the geographic clustering version of the MLC and related problems is presented in Ref. [5].

Another restricted version of the MLC problem considered in Ref. [5] is when each room has a size ρ_i defined as the side length of the smallest enclosing square of the room, and each room R_i has perimeter at most $4\rho_i$. A room with this property is said to be a room with *square perimeter*. A room R_i is called α-fat if for every square Q whose boundary intersects R_i and its center is inside R_i, the intersection area of Q and R_i is at least $\frac{\alpha}{4}$ times the area Q. In general $\alpha \in [0, 1]$. For square rooms α is equal to one, but for rectangular rooms α tends to zero. It is not even known whether the decision version of the MLC problem, when all the rooms have square perimeter and are α-fat, is an NP-complete problem; however, there is a polynomial time approximation algorithm with approximation ratio $\frac{16}{\alpha} - 1$ [5]. In the case when all the rooms are squares the approximation ratio is 15. Clearly, $\frac{16}{\alpha} - 1$ is not bounded above by a constant for the MLC-R.

The *Group Steiner Tree* (GST) problem may be viewed as a generalization of the MLC problem. There is a simple and straight forward reduction from the MLC problem to the GST problem, which can be used to show that any constant ratio approximation algorithm for the GST problem is a constant ratio approximation algorithm for the MLC problem.

Slavik [7], [8], [9] defined a more general version of the

[1] Throughout this paper we assume that all the rectangles (and rectilinear polygons) consist only of horizontal and vertical line edges.

GST problem where $Q = \{C_1, C_2, \ldots, C_k\}$ is not necessarily a partition of the subset C of vertices, but each errand i can be performed at any vertex in $C_i \subseteq C$ and $\cup_i C_i = C$, i.e., a vertex in C may be in more than one set C_i. This version of the GST problem is called by Slavik [8], [9] the *Tree (Errand) Cover* (TEC) problem. The TEC problem is formally defined as follows. Given a connected undirected edge-weighted graph $G = (V, E, w)$, where $w : E \rightarrow \mathbb{R}^+$ is an edge-weight function; a non-empty set C, $C \subseteq V$, of *terminals*; and a set $Q = \{C_1, C_2, \ldots, C_k\}$, where $C_i \subseteq C$ and $\cup_i C_i = C$, we want a tree $T(Q) = (V', E')$, where $E' \subseteq E$ and $V' \subseteq V$, such that at least one terminal from each set C_i is in the tree $T(Q)$ and the total edge-length $\sum_{e \in E'} w(e)$ is minimized.

Slavik [8], [9] developed an approximation algorithm for the TEC problem with approximation ratio 2ρ, when each errand can be performed in at most ρ locations.

As we have seen, our problems are restricted versions of more general problems reported in the literature. However, previous results for those problems do not establish NP-completeness results, inapproximability results, nor constant ratio approximation algorithms for our problems.

In this paper we present a polynomial time approximation algorithm for the MLC-R problem with approximation ratio 30. The approach is similar to the one in [6], but it differs mainly on restriction points and the correctness proof. The presentation in this paper is complete, but additional details and examples appear in [10].

An application for the MLC problem is when laying optical fiber in metropolitan areas and every block (or set of blocks) is connected through its own gateway which may be placed anywhere on the boundary of the set of blocks. The objective is to find a minimum-length corridor interconnecting all the gateways (one for each set of blocks) in the area. Our problems also have applications in VLSI and floorplanning when laying wires for clock signals or power, and when laying wires for an electrical network or optical fibers for data communications. There has been recent research activity for related problems arising in intelligent transportation as well as in modern spatial database systems for trip planning queries [11].

In Section II we discuss preliminary results and define the p-MLC-R problem, a restricted version of the MLC-R problem, used to approximate the solution of the MLC-R problem. Then in Section III we present our parameterized algorithm for the p-MLC-R problem. The parameterized algorithm takes in a parameter S that identifies a subset of boundary points from each rectangle and calls them *critical points*. The p-MLC-R$_S$ is exactly like the p-MLC-R problem except that every feasible corridor must include a critical point from each rectangle. Then Slavik's approximation algorithm for the TEC problem [8], [9] is used to generate a corridor for the p-MLC-R$_S$ problem instance. This is the corridor that our algorithm generates for the p-MLC-R problem instance. When the maximum number of critical points identified at each rectangle is k_S, our corridor has length at most $2k_S$ times the length of an optimal corridor for the p-MLC-R$_S$ problem instance. The approximation ratio for this algorithm depends on the ratio (r_S) between optimal solutions for p-MLC-R$_S$ and p-MLC-R problem instances. Therefore, the approximation ratio for the parameterized algorithm is $2k_S \cdot r_S$. There are many selector functions S for which k_S is a constant, e.g., the selector function that identifies from each rectangle its corners as critical points has $k_S = 4$. However for some functions the ratio r_S cannot be bounded above by a constant. In Section IV we present a method for selecting a set of points called special points. This method sheds some light for possible design principles that result in constant ratio approximation algorithms. A constant ratio approximation algorithm arising from the design principles is discussed in Section V. The corresponding selector function identifies from each rectangle its four corners and a special point. The main trust of the paper is Section V where we establish that r_S is bounded above by a constant for the above selector function. In Section VI we discuss results for related problems, conclusions, and open problems.

II. Preliminaries

The *p-access point* version of the problems, or simply the p-MLC and p-MLC-R problems, are restricted versions of the MLC and MLC-R problems where one identifies an access point p located on the edges of the polygon F, and the solution must include this access point p. NP-complete in Ref. [4]. An optimal corridor for the p-MLC-R problem instance given in Figure 1 is represented by the thick line segments. The solution to any instance of the MLC (resp. MLC-R) problem can be obtained by finding a corridor for the p-MLC (resp. p-MLC-R) problem at each intersection point p located along the edges of polygon F, and then selecting the best of these corridors. Based on this observation we state the following theorem.

Theorem 2.1: Any polynomial time constant ratio approximation algorithm for the p-MLC-R problem is also a polynomial time constant ratio approximation algorithm for the MLC-R problem. The approximation ratio is identical for both algorithms.

As a result of Theorem 2.1, we have reduced the MLC-R approximation problem to the p-MLC-R approximation problem. Hereafter we concentrate our efforts on the p-MLC-R approximation problem.

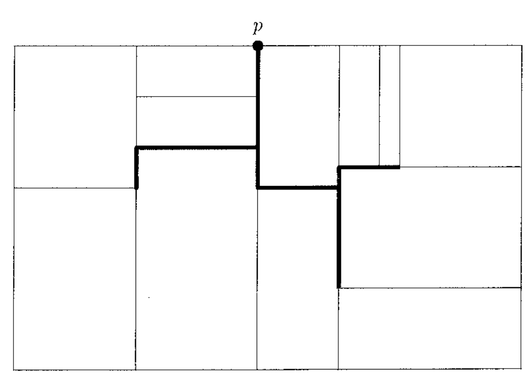


Fig. 1. Optimal corridor for a p-MLC-R problem instance.

It is convenient to transform the geometric representation of the p-MLC-R problem into the following graph representation. There is a vertex for every distinct point located at the intersection of two orthogonal line segments representing the edges of rectangles and the polygonal boundary F. A vertical (resp. horizontal) line segment in the instance I of the p-MLC-R problem is called an *edge* if it includes exactly two points represented by vertices, and the two points are the segment's endpoints. We assume without loss of generality that p is located at a vertex representing a point located on F. Every instance I of the p-MLC-R problem is represented by the graph $G(I) = (V, E, w)$, where the set V of vertices and the set of E of edges is defined above, and the weight of an edge ($w(e)$) corresponds to the length of the line segment represented by the edge. In this paper we use the geometric and graph representation of the p-MLC-R problem interchangeably, and mix the two notations. We use $V(R_i)$ to denote all the vertices located along the boundary of rectangle R_i. We use $C(R_i)$ to denote the set of vertices that corresponds to the corners of R_i. Every vertex is a non-corner point of at most one rectangle.

The instance of the TEC problem corresponding to the instance (F, R, p) of the p-MLC-R problem is defined for the (metric) graph $G(V, E, w)$ constructed from (F, R, p) with an errand E_i for each rectangle R_i located at all the vertices $V(R_i)$, plus the errand E_0 located at vertex p. Clearly every feasible solution to the p-MLC-R problem instance is also a feasible solution for the corresponding TEC problem instance and vice versa. Furthermore, the objective function value of every feasible solution to both problem instances is identical.

Let I represent any instance of the p-MLC-R problem. Let $T(I)$ be any corridor for instance I and $t(I)$ be its edge-length. Let $OPT(I)$ be an optimal corridor for instance I and let $opt(I)$ be its edge-length. An approach to generate suboptimal solutions for the p-MLC-R problem instance (F, R, p) is to construct an instance of the TEC problem and then invoke an approximation algorithm for the TEC problem instance. The solution generated by the algorithm for the TEC problem instance is the solution to the p-MLC-R problem instance. Currently one uses Slavik's [8], [9] approximation algorithm for the TEC problem, which is based on *relaxation* techniques. A direct application of this approach to the p-MLC-R problem generates a corridor whose total edge-length is at most $2\rho \cdot opt(I)$, where $\rho = max_i\{|V(R_i)|\}$. Unfortunately, this simple approach does not result in a constant ratio approximation for the p-MLC-R problem since, as we pointed out before, $|V(R_i)|$ is not bounded above by any constant. Our parameterized approximation algorithm is a refined version of this approach.

III. Parameterized Algorithm

To establish our algorithm we need to refine our previous strategy. The idea is to *restrict* the solution space by limiting in each rectangle R_i the possible vertices, from which at least one must be part of the corridor. Consider the p-MLC-R_S problem where the selector function S identifies from each rectangle R_i a set of at most k_S of its boundary points from which at least one must be included by every corridor. The points selected for each rectangle R_i are called the *critical points of* R_i. Usually, the k_S critical points for each rectangle R_i defined by S include a subset of its corner points as well as some points with a special connectivity property. This connectivity property will be defined later on. The objective function of the p-MLC-R_S problem is to find a minimum edge-length corridor that includes for each rectangle R_i at least one of its critical points.

Given S and an instance I of the p-MCL-R problem, we use I_S to denote the instance of the corresponding p-MLC-R_S problem. The instance of the TEC problem, denoted by J_S, is constructed from the instance I_S of the p-MLC-R_S problem using the same approach as the one used for the p-MLC-R problem, but limiting the errands from each rectangle to the critical points of the rectangle. Clearly every feasible solution to the p-MLC-R_S problem instance I_S is also a feasible solution to the instance J_S of the TEC problem, and vice versa. Furthermore, the objective function value of every feasible solution to both problems is identical. Slavik's algorithm applied to the instance J_S of the TEC problem generates a solution $T(J_S)$ from which we construct a corridor $T(I)$ with edge-length $t(I)$ for the p-MLC-R problem. We call our approach the parameterized algorithm $Alg(S)$, where S is the parameter. Since Slavik's approximation algorithm is based on relaxation techniques and we apply it to a restricted version of the p-MLC-R problem, we say that our approximation algorithm is based on *restriction* and *relaxation* approximation techniques. Let $OPT(I_S)$ be an optimal corridor for I_S and let $opt(I_S)$ be its edge-length. Theorem 3.1 establishes the approximation ratio for our parameterized algorithm $Alg(S)$. It is simple to see that the total edge-length of an optimal solution of the instance I_S, $opt(I_S)$, corresponding to the p-MLC-R_S problem, is at least as large as the total edge-length of an optimal solution of the instance I, $opt(I)$, of the p-MLC-R problem. We define the ratio between $opt(I_S)$ and $opt(I)$ as r_S (with $r_S \geq 1$). In other words, one needs to prove that $opt(I_S) \leq r_S \cdot opt(I)$ for every instance I of the p-MLC-R problem, in order to use the following theorem.

Theorem 3.1: Parameterized algorithm $Alg(S)$ generates for every instance I of the p-MLC-R problem a corridor $T(I)$ of length $t(I)$ at most $2k_S \cdot r_S$ times $opt(I)$, provided that $opt(I_S) \leq r_S \cdot opt(I)$.

Proof: Applying Slavik's approximation algorithm [8], [9] we generate a corridor $T(I_S)$ of length $t(I_S) \leq 2 \cdot k_S \cdot opt(I_S)$. Clearly $T(I_S)$ is also a corridor for I, so the solution generated, $T(I)$, is simply $T(I_S)$. By the condition of the theorem, $opt(I_S) \leq r_S \cdot opt(I)$. It then follows that the length of the corridor generated by our parameterized algorithm $Alg(S)$ is $t(I) \leq 2k_S \cdot r_S \cdot opt(I)$.

□

For the above approach to yield a constant ratio approximation algorithm we need both k_S and r_S to be bounded above by constants. For example, when S selects from each rectangle R_i its four corner points, k_S is four. However, r_S cannot be bounded above by any constant [10].

For most selector functions S, proving that $opt(I_S) \leq r_S \cdot opt(I)$, for every instance I of the p-MLC-R problem, is difficult because we do not have the optimal solutions at hand. Instead we establish a bound for all corridors. That is, we prove that for every corridor $T(I)$ with edge-length $t(I)$

there is a corridor for I_S denoted by $T(I_S)$ with edge-length $t(I_S) \leq r_S \cdot t(I)$. Applying this to $T(I) = OPT(I)$ we know that there is a corridor $T(I_S)$ such that $t(I_S) \leq r_S \cdot opt(I)$. Since $opt(I_S) \leq t(I_S)$, we know that $opt(I_S) \leq r_S \cdot opt(I)$.

In the next section, we define formally a special connectivity property. We use this property to define the *special points* of rectangle R_i. In Section V, we present our constant ratio approximation algorithm, which results from incorporating to our parameterized algorithm $Alg(S)$ a selector function S. This function S identifies the four corners as well as an special point from each rectangle R_i.

IV. Special Connectivity Property

We can show that several different basic selector functions S do not result in constant ratio approximations when they are incorporated into our parameterized algorithm $Alg(S)$. These facts point us in the direction of a method for selecting a set of points called *special points*. Special points will turn out to be very important when combined with other critical points to generate constant ratio approximation algorithms (Section V).

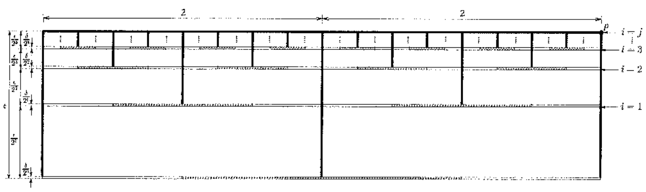


Fig. 2. Optimal solution $OPT(I(j))$ for the family of instances $I(j)$ of the p-MLC-R problem.

Considering the family of instances given in Figure 2, the only "good" selector functions S for our parametrized algorithm are those that include for every gray rectangle its middle point as a critical point. We call these points *special points*, and we formally define them below. For the definition of special point, assume that "rectangle" R_0, which is just point p, is included in R, and p is said to be a corner of R_0. Depending on the selector function S, a subset of the corner points $C(R_i)$ are called the *fixed points* $F(R_i)$ of rectangle R_i. Now, the set of critical points consists of the union of the disjoint sets of fixed points and special points.

The middle point of each gray rectangle in Figure 2 has the *minimum connectivity distance property*. By this we mean, in very general terms, that if given all partial corridors that do not include a point from rectangle R_i, but include points from all other rectangles, then a *special point* of R_i is a vertex in $V(R_i)$ that is not in $F(R_i)$, and the maximum edge-length needed to connect it to each one of the partial corridors is least possible. Finding special points in this way is in general time consuming. Also, this definition is not valid for all problem instances as the set of partial corridors, where every corridor includes vertices from all the rectangles except from R_i, may be empty. In what follows we define special points precisely for all problem instances in a way that is computationally easy to identify a set of special points for each rectangle. Special points are identified using an upper bound on the connectivity distance.

Given that we have selected a set $F(R_i)$ of fixed points for each room (rectangle) R_i, we define a special point as follows. Let $u \in V(R_i)$ and let T_u be a tree of shortest paths rooted at u to all other vertices $(\bigcup_{j \neq i} V(R_j))$ along the edges of polygon F and the edges of the rooms. Let $SP(u, v)$ be the length of the (shortest) path from vertex u to vertex v along T_u. Let $FP(u, R_j)$ be the length of the (shortest) path from point $u \in V(R_i)$ to the "farthest" vertex of rectangle R_j along T_u, for $i \neq j$, i.e.,

$$FP(u, R_j) = max_{v \in V(R_j)} \{SP(u, v) | u \in V(R_i), j \neq i\}.$$

In other words, the edge-length needed to connect vertex u of room R_i to any corridor through the connection of room R_j is at most $FP(u, R_j)$.

We define the *connectivity distance* $CD(u, R)$ of vertex u in room R_i as

$$min_{j \neq i} \{FP(u, R_j) | R_j \in R\}.$$

If $F(R_i) \subset V(R_i)$ we define the connectivity distance $CD(R_i, R)$ of room R_i as

$$min_{u \in V(R_i) \setminus F(R_i)} \{CD(u, R)\}.$$

In other words, $CD(R_i, R)$ is the edge-length needed to connect some specific vertex in $V(R_i) \setminus F(R_i)$ to any corridor through the connection of another room. The special point of R_i is a vertex $u \in V(R_i) \setminus F(R_i)$ such that $CD(u, R) = CD(R_i, R)$. Notice that there may be more than one point satisfying this condition, in which case we select any of these points as the special point. When $F(R_i) = V(R_i)$ then there is no special point. It is important to remember that for the definition of special point, R_0 which is simply p is included in R.

V. Constant Ratio Approximation Algorithms

In this section we present the selector function $S(4C+)$ which defines the four corners as well as a special point of each rectangle R_i. When we incorporate $S(4C+)$ into the parameterized algorithm, it results in a constant ratio approximation algorithm. This approximation is an alternative polynomial time constant ratio approximation algorithm for the p-MLC-R problem and, by Theorem 2.1, for the MLC-R problem. We already proved [6], [10] that for the selector function $S(2OC+)$, which defines two opposite corners plus one special point, the parameterized algorithm $Alg(S(2OC+))$ results also in a constant ratio approximation algorithm.

In Subsection V-A we briefly sketch the analysis for the approach applied to the selector function $S(4C+)$.

A. Selecting the four corners and one special point

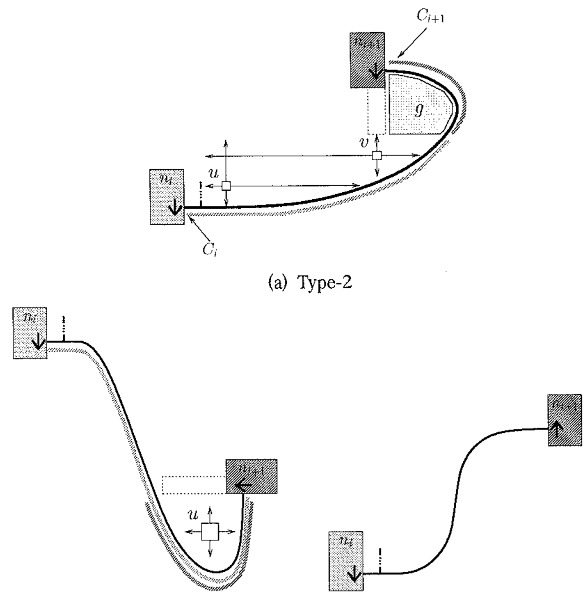
The critical points in $S(4C+)$ for each rectangle $R_i \in R$ are its four corners and a special point. For this case $k_{S(4C+)}$ = 5 and we can show that $r_{S(4C+)}$ = 3. Therefore, the approximation ratio of the parametrized algorithm is 30 (as in the case of $S(2OC+)$).

We now briefly discuss our proof strategy to show that given any corridor $T(I)$ there is a corridor $T(I_{S(4C+)})$ such that $t(I_{S(4C+)}) \leq 3 \cdot t(I)$. Given any corridor $T(I)$ we identify all the ncpe rectangles and establish an ordering $(n_1, n_2, \ldots, n_q)$ between them. Assume there exists at least one ncpe rectangle $(q \geq 1)$, otherwise $t(I_{S(4C+)}) = t(I)$ and the result follows. For each ncpe we find a shortest path from one of its critical points to the corridor $T(I)$. We then select this path to connect a critical point to the corridor $T(I)$. Clearly after deleting some edges to remove any cycle that may have been created, corridor $T(I)$ plus a subset of these connections give a corridor $T(I_{S(4C+)})$. Next we need to show that the sum of the length of the segments introduced is at most $2 \cdot t(I)$. This part is more complex than the one for the selector function $S(2OC+)$ [6].

We characterize the region between every pair of adjacent ncpe rectangles. The region between two adjacent ncpe rectangles is said to be of type 0, 1 or 2 (see Figure 3). The region between ncpe rectangles n_i and n_{i+1} is of type-2 (see Figure 3 (a)), if the distance along the corridor between n_i and n_{i+1}, which we call l_i, is larger than the edge-length needed to connect both a critical point from n_i and one from n_{i+1} to the corridor. This is the most desirable case. If this were to be the case for every pair of adjacent ncpe rectangles the proof of the approximation ratio would be simple to establish (in fact we would even be able to establish a better ratio). However, this is not always the case. The region between ncpe n_i and n_{i+1} is of type-1 (see Figure 3 (b)), if l_i is larger than the edge-length needed to connect either a critical point from n_i to the corridor, or one from n_{i+1} to the corridor, but not both. If this were the case for every pair of adjacent ncpe rectangles the proof would also be simple. The main problem is when the region between ncpe n_i and n_{i+1} is of type-0 (see Figure 3 (c)). In this case, the edge-length needed to connect a critical point of either n_i or n_{i+1} cannot be bounded above by l_i. This is where the proof is complex because we need to consider a sequence of ncpe rectangles, not just three as in the proof for the case when we use the selector function $S(2OC+)$ [10].

Now suppose that there is a sequence of type-0 adjacent ncpe rectangles as shown in Figure 4. The connection of the first ncpe rectangle n_1 to the corridor has already been accounted for in l_0. Now we need to charge the connection of a critical point of each ncpe rectangles $n_2, \ldots, n_9$ to the corridor. The connection for the ncpe n_2 is charged to the horizontal distance from ncpe n_1 to ncpe n_2, and the vertical distance from ncpe n_2 to ncpe n_3 because one can show that the area includes at least one rectangle already connected to the corridor. Similarly, the cost of connecting ncpe n_3 can be charged to the horizontal distance from ncpe n_2 to ncpe n_3 and the vertical distance from ncpe n_3 to ncpe n_4. And so forth until ncpe n_9, where its connection is charged to the corridor after it because there is a rectangle inside the box in the center of Figure 4.

Other complex cases are given in Figure 5(a-b) which indicate how to deal with the sequence of adjacent rectangles of types 001000 and sequence 00111. By the sequence $Y_1 Y_2 \ldots Y_k$ we mean that the first pair of ncpe rectangles is of type Y_1 and the second pair is of type Y_2, and so on. There are more critical cases that need to be considered. A complete characterization of all critical cases is possible, but the proof that they can be resolved to match the approximation bound is quite complex.



(a) Type-2

(b) Type-1

(c) Type-0

Fig. 3. Region Types.

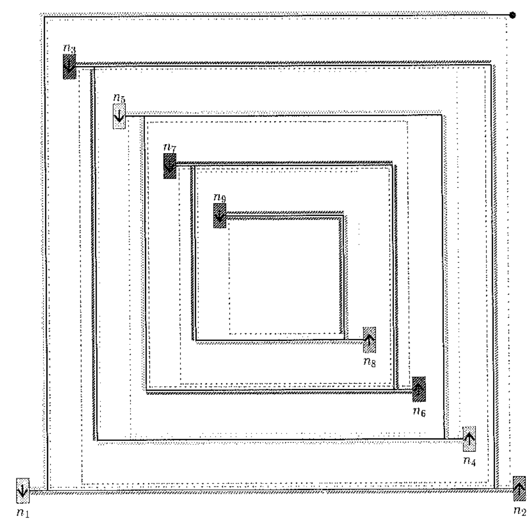


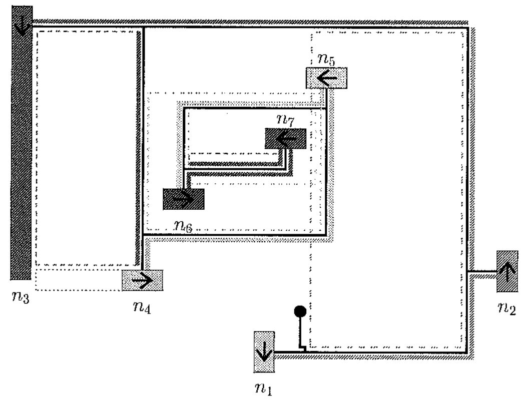
Fig. 4. Sequence of type-0 adjacent ncpe rectangles.

We claim without stating any further details that the approximation ratio of the parameterized algorithm Alg($S(4C+)$) is 30.

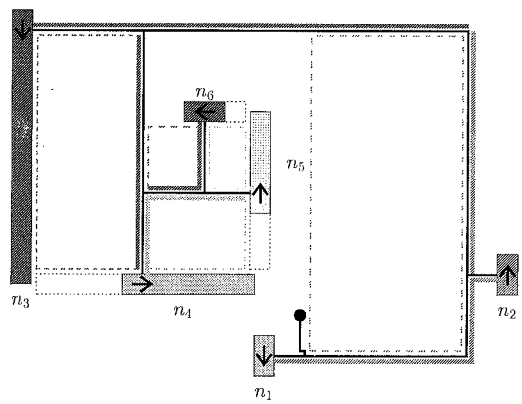
VI. Additional Results and Discussion

Our approximation algorithm is based on restriction and relaxation techniques. The analysis of our approximation algorithm applies (with the same time complexity and approximation ratio) when the boundary of the MLC-R problem is a rectilinear polygon rather than the rectangle F, or when the problem is to find a tree that is not necessary joined to the boundary of F.

When we restrict the MLC-R problem to $S(2OC+)$ or $S(4C+)$, we use Slavik's algorithm for the TEC problem



(a) Sequence 001000



(b) Sequence 00111

Fig. 5. Assignment of regions between adjacent ncpe rectangles.

to generate a suboptimal solution for the MLC-R problem instance. This is the most time consuming part of our procedures. The first open question is about the development of a faster approximation algorithm for the MLC-R problem restricted by $S(2OC+)$ or $S(4C+)$. The second open question has to do with the development of an algorithm for those problems with a smaller approximation ratio or even one that generates an optimal solution. But since the MLC-R problem is NP-hard when restricted by $S(2OC+)$ or $S(4C+)$, it is unlikely one can find an efficient algorithm for its solution. The NP-hardness proof follows the same lines as the one in Ref. [4], but we need to do some modifications to show that the the MLC-R restricted to $S(2OC+)$ ($S(4C+)$) (i.e. every rectangle must intersect the corridor at a critical point defined by $S(2OC+)$ (resp. $S(4C+)$)) is NP-hard.

A related problem studied by Slavik [8], [9] is the *Errand Scheduling* (ES) problem. In this case the problem is to find a *shortest partial tour* visiting a subset of vertices of the given metric graph G such that at least one vertex in $C_i \subset C$ is in the partial tour, where C_i is associated with the *errand* i. When each vertex represents a unique errand, the ES problem is an instance of the well-known *Traveling Salesperson Problem (TSP)*. Therefore the ES problem is NP-hard. The ES problem has also been referred to as the *group TSP (g-TSP)*. Slavik [8], [9] shows that the ES problem restricted to metric graphs problem can be approximated to within $\frac{3\rho}{2}$ when each errand can be performed in at most ρ nodes. Another interesting problem is the group-TSP problem when restricted to rectangles as in the case of the of the MLC-R problem, which we call the *rectangular group-TSP*. In this version of the TSP one may visit the same edge or vertex more than once.

We claim that the same approach that we use for the MLC-R problem can also be adapted to the rectangular group-TSP. It is simple to show that the selector functions that do not generate constant ratio approximations for the MLC-R problem do not generate constant ratio approximations for the rectangular group-TSP. However the parameterized algorithms using the selector function $S(4OC+)$ also generates a constant ratio approximation to the group-TSP. In fact we can just use the tour of the corridor (traversing each edge twice) as the solution to the rectangular TSP problem. The approximation ratio in this case will be 60 times the length of an optimal tour. However there is a better algorithm for this case. Instead of using Slavik's approximation algorithm for the TEC problem, we use the one for the ES problem [8], [9] in the parameterized algorithm. This results in an algorithm with approximation ratio $\frac{3}{2} \cdot k_{S(2OC+)} \cdot r_{S(2OC+)}$, where $k_{S(2OC+)} = 3$ and $r_{S(2OC+)} = 5$, which is 22.5 for the rectangular group-TSP.

For brevity we do not discuss additional results. Approximation algorithms for other versions of the group-TSP problem are discussed in Ref. [5].

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