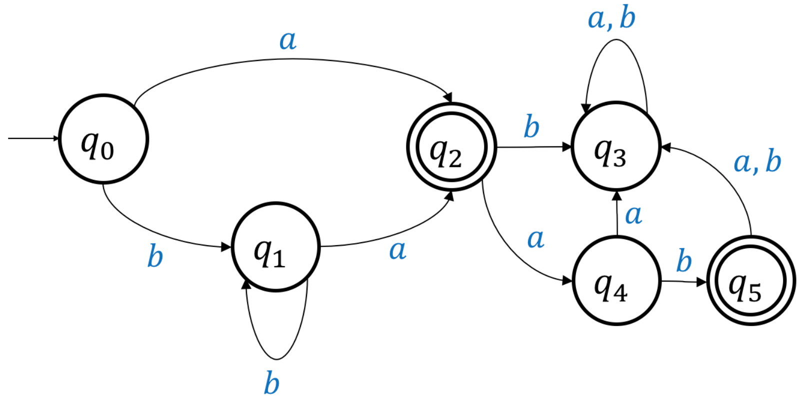
Homework 2

Posted: Monday, April 9, 2018 – 11:59pm
Due: Wednesday, April 18, 2018 – 2pm (Gradescope – instruction will follow)

Task 1 – DFAs (6 points)

Consider the DFA M with alphabet $\Sigma = \{a, b\}$ described by the following transition graph:



a) Represent the DFA as a tuple $M = (Q, \Sigma, \delta, q_0, F)$. In particular, describe Q, δ, and F.

b) What is $\delta^*(q_0, ab)$? What is $\delta^*(q_0, b^{20}aab)$? What is $\delta^*(q_0, aba^{34}b^{123})$?

c) Describe the language $L(M)$ accepted by M.

Hint: Any description is accepted, but It may be easier to represent $L(M)$ as the union of more than one language.

Task 2 – Regular Languages (8 points)

Show that each of the following languages with alphabet $\Sigma = \{0, 1\}$ is regular. Concretely, give corresponding DFAs accepting them, and justify (in words, but as precisely as possible) why your DFAs accept the corresponding languages.

a) $L_1 = \emptyset$

b) $L_2 = \Sigma^*$

c) $L_3 = \{\lambda\}$

d) $L_4 = \{01^30^i \,:\, i \geq 1\}$

e) $L_5 = \{u011v \,:\, u, v \in \{0,1\}^*\}$

f) $L_6 = \{00w \,:\, w \in \Sigma^*, w$ does not contain two consecutive 1's$\}$

g) $L_7 = \{w \in \Sigma^* \,:\, |w|$ is a multiple of $3\}$.

h) $L_8 = \{w \in \Sigma^* \,:\, w$ contains an even number of 0's$\}$

Task 3 – Finite Languages (6 points)

The goal of this task is to convince yourself that all finite languages (i.e., languages that only contain a finite number of sentences) are regular.

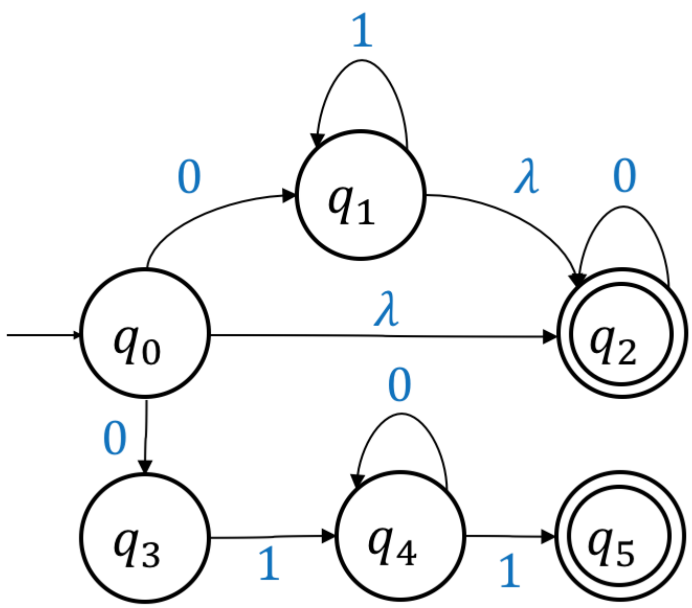
a) Let $L = \{\lambda, a, b, ba, bbb\}$ a language with alphabet $\Sigma = \{a, b\}$. Show that L is regular by giving a corresponding DFA.

b) Show that every *finite* language L is regular.

Hint: Given *any* Σ and *any* finite language $L \subseteq \Sigma^*$, explain a method to construct the corresponding DFA M such that $L(M) = L$. Take inspiration from what you did in **a)**.

Task 4 – NFAs (6 points)

Throughout this task, consider the NFA $M = (Q, \Sigma, \delta, q_0, F)$ with alphabet $\Sigma = \{0, 1\}$ defined by the following transition graph.



a) Complete the formal description of M, i.e., give Q, δ and F matching the above transition graph. Recall that here $\delta : Q \times (\Sigma \cup \{\lambda\}) \to 2^Q$, i.e., δ returns a *subset* of Q, which could be empty.

b) What is $\delta^*(q_0, 0)$? $\delta^*(q_0, \lambda)$? $\delta^*(q_0, 0110)$?

c) Describe the language $L = L(M)$ accepted by the NFA M.

Task 5 – Simplified NFAs (4 points)

Show that for every NFA M, there exists an NFA M' such that $L(M) = L(M')$, and moreover, M' has a *single* final state which is different from its initial state.

Hint: How you would reduce the number of final states in the NFA from **Task 4** without affecting the language accepted? How can you transform this idea to work with any NFA with multiple final states?