Homework 4

Posted: Monday, April 23, 2018 – 11:59pm
Due: Wednesday, May 2nd, 2018 – 5:00pm (Gradescope)

Task 1 – The Pumping Lemma (9 points)

Use the Pumping Lemma to show that the following languages with alphabet $\Sigma = \{0, 1\}$ are *not* regular.

a) $L_1 = \{ww \,:\, w \in \Sigma^*\}$.

b) $L_2 = \left\{0^n 1^\ell \,:\, n \geq \ell\right\}$.

c) $L_3 = \left\{0^{2^n} \,:\, n \geq 0\right\}$.

Hint: You need to show that for every possible m, there exists $w \in L$ with length $|w| \geq m$ such that for every splitting $w = xyz$ where $|xy| \leq m$ and $|y| \geq 1$, there exists $i \neq 1$ with $w_i = xy^iz \notin L$. In particular, you need to clearly describe how you construct w given m, and why $w_i \notin L$ for some i no matter how x, y, and z are chosen.

Task 2 – Myhill-Nerode and Regularity (4 points)

Let $L \subseteq \Sigma^*$ be a language with alphabet Σ. We say that two strings $x, y \in \Sigma^*$ are *indistinguishable* (wrt L) – denoted $x \sim_L y$ – if for all $w \in \Sigma^*$, $xw \in L$ holds if and only if $yw \in L$ holds. Note that $\sim_L$ partitions Σ^* into *equivalence classes*. The Myhill-Nerode Theorem says that L is regular if and only if the number of these equivalence classes is finite.

Describe the equivalence classes of $\sim_L$ for the following languages with alphabet $\Sigma = \{0, 1\}$.

a) $L_1 = \{00w \,:\, w \in \Sigma^*\}$.

b) $L_2 = \{w \in \Sigma^* \,:\, w \text{ contains exactly three 1's}\}$.

Task 3 – Right-Regular Grammars (7 points)

Consider the following right-regular grammar $G = (\{A, B, C\}, \{a, b\}, A, P)$, where P consists of the following productions:

$$\begin{aligned} A &\to aaB \mid bbB \\ B &\to abB \mid baB \mid C \\ C &\to aC \mid aaB \mid \lambda\ . \end{aligned}$$

a) Which of the following derivations are possible? Justify your answer by giving every single step of the derivation (if it is possible), or a justification why the derivation is not possible.

- $A \overset{*}{\Rightarrow} aa$
- $A \overset{*}{\Rightarrow} aabab$
- $A \overset{*}{\Rightarrow} bbabbaa$
- $A \overset{*}{\Rightarrow} bbabbbb$

b) Give an NFA accepting the language $L(G)$.

c) Prove that for every right-regular grammar G, there exists a right-regular grammar G' such that $L(G) = L(G')$, and all productions have form $A \to xB$ or $A \to \lambda$ for $A, B \in V$, $x \in T$.

d) Prove that unless $L(G) = \emptyset$, every right-regular grammar $G = (V, T, S, P)$ contains at least a production of the form $A \to x$ where $A \in V$ and $x \in T^*$.

Task 4 – Right- and Left-Regular Grammars (6 points)

For each of the following languages L with alphabet $\Sigma = \{a, b\}$, give right-regular and left-regular grammars G and G' such that $L(G) = L(G') = L$.

a) $L_1 = L(ab^*(aba + \lambda)b^*)$

b) $L_2 = L((aa^*b)^*)$

c) $L_3 = \{w \in \Sigma^* : w \text{ contains an odd number of } a\text{'s}\}$.

Task 5 – More on Closure Properties (4 points)

Show in general that a language L is regular iff there exists a left-regular grammar G such that $L(G) = L$.

Hint. Many solutions are possible. One possible one goes over the following observation: Assume that there exists a *right*-regular grammar for L, how can you obtain a left-regular grammar for L^R? How can you use this fact to prove the above statement? You can use facts about right-regular grammars from class.