# Homework 4.5

## Task 1 – Mealy and Moore Machines

We want to devise a machine for the following task:

- The machine takes as input a sequence of bits $b_1 b_2 \ldots$, and outputs a sequence of bits $c_1 c_2 \ldots$.
- The $i$-th output bit $c_i$ is the *parity* of all bits $b_1 \ldots b_i$ input so far, i.e., whether the number of $1$'s among $b_1 \ldots b_i$ is even (in which case the parity is $0$) or odd (here the parity is $1$).

For example, on input $01101110\ldots$, the machine would output $01001011\ldots$.

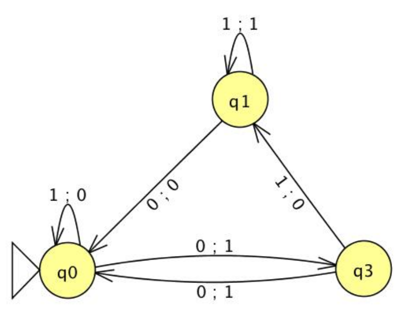
**a)** Give a Mealy machine implementing the above specification.

**b)** Give a Moore machine implementing the above specification.

In both cases, give a transition graph, but also describe both machines in terms of the corresponding tuples $(Q, \Sigma, \Gamma, \delta, \theta, q_0)$.

## Task 2 – Conversions between Mealy and Moore Machines

Describe a Moore machine $N$ equivalent to the following Mealy machine $M$. (Here, the notation $b : c$ indicates that the transition is triggered by the input symbol $b$ and produces output $c$.)



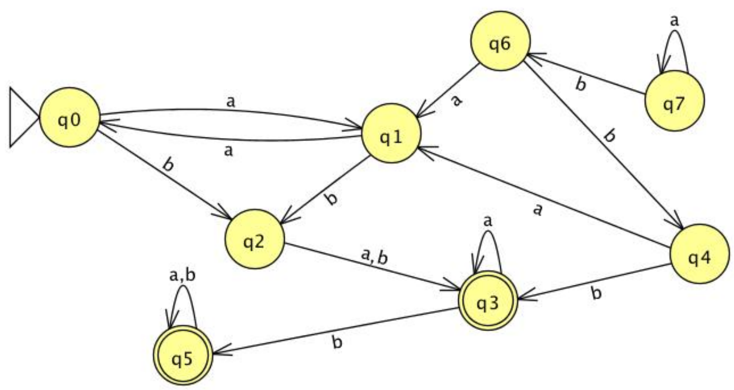
**Hint:** Recall that equivalent means that both machines produce the same output sequence when given the same input string.

## Task 3 – Limits of Mealy and Moore Machines

Prove that there exist functions $F_M : \Sigma^* \to \Gamma$ which cannot be computed by a Mealy (or Moore) machine.

## Task 4 – DFA Minimization (8 points)

Consider the DFA $M$ defined by the following transition graph.




**a)** Which states are *unreachable* in $M$, i.e., there is no way an execution of $M$ will ever reach these states?

**b)** Let $M'$ be the DFA obtained from $M$ by removing the unreachable states found in **a)**. Which pairs of states are *indistinguishable* in $M'$, and which ones are not?

**Hint:** You can either list all pairs, or give equivalence classes consisting of mutually indistinguishable states. You do not need to use any specific algorithm from the textbook, the task can be solved directly by inspecting the DFA.

**c)** Use the findings from **b)** to minimize the number of states in $M'$. Explicitly give the resulting DFA $M''$.

**d)** Alice claims she has found a four-state DFA for the language $L = \{a^i b^j \ : \ i \geq 2, j \geq 3\}$. Is she right? Justify your answer in detail!