CS138: Automata and Formal Languages
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UC Santa Barbara
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Homework 5

Posted: Monday, May 7, 2018 – 11:59pm
Due: Wednesday, May 16, 2018 – 2pm (Gradescope only)

Task 1 – Context-Free Grammars (8 points)

Give context-free grammars for the following languages with alphabet $\Sigma = \{a, b\}$:

a) $L_1 = \{a^n b^{n+3} : n \geq 1\}$

b) $L_2 = \{ww^R : w \in \{a, b\}^*\}$

c) $L_3 = \{w \in \{a, b\}^* : n_a(w) = n_b(w)\}$, where $n_a(w)$ and $n_b(w)$ indicate the number of occurrences of a and b, respectively, in w.

d) $L_4 = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$.

e) $L_5 = \{a^m b^n a^{n+2} b^{2m+2} : n \geq 1, m \geq 0\}$

Task 2 – Grammars and Derivation Trees (3 points)

Consider the grammar G with the following productions (and start variable S):

$$S \to AB|\lambda\ , \quad A \to aB\ , \quad B \to Sb\ .$$

a) Show that the string $w = aabbbb$ is in $L(G)$ by giving the corresponding derivation tree.

b) Give a left-most and a right-most derivation of $w = aabbbb$ using the above derivation tree.

Task 3 – Ambiguity (6 points)

a) Show that the following grammar G with terminals $T = \{a, b, c\}$ and variables $V = \{S, A, B, C, D\}$ is ambiguous:

$$\begin{aligned} S &\to AB \mid CD \\ A &\to aA \mid \lambda \\ B &\to cBb \mid \lambda \\ C &\to aCc \mid \lambda \\ D &\to Db \mid \lambda\ . \end{aligned}$$

That is, find a $w \in L(G)$ with (at least) *two* different derivation trees. Describe both w and two derivation trees *explicitly*.

b) Show that a regular language is not *inherently ambiguous*, i.e., show that every regular language L admits an *unambiguous* CFG G such that $L(G) = L$.

c) Show that a right-regular grammar can be ambiguous.

Hint: Give a concrete example of right-regular grammar for which you can show it is ambiguous.

Task 4 – Simplifying Grammars (9 points)

a) Eliminate all useless productions from the following grammar:

$$\begin{aligned} S &\to aaSb \mid AbB \mid \lambda \\ A &\to CbbS \\ B &\to Ba \mid \lambda \\ C &\to aCa\ . \end{aligned}$$

b) Eliminate all useless productions from the following grammar:

$$\begin{aligned} S &\to bbaA \mid aA \mid aa \\ A &\to bA \mid b \mid C \\ B &\to A \mid baD \\ C &\to aC \\ D &\to bS\ . \end{aligned}$$

c) Eliminate all λ-productions from the following grammar:

$$\begin{aligned} S &\to abC \mid baB \\ B &\to CD \\ C &\to aDa \mid a \mid \lambda \\ D &\to aB \mid b \mid \lambda\ . \end{aligned}$$

In all three cases, use the algorithms from class and clearly indicate all steps!

Task 5 – Closure Properties – Part I (4 points)

Let L_1 and L_2 be arbitrary context-free languages with some common alphabet Σ. Prove that the following two languages are both context free:

a) $L_1 \cup L_2$

b) $L_1 L_2$

Hint: Given two context-free grammars G_1 and G_2 for L_1 and L_2, respectively, explain how to find a context-free grammar for the resulting language. In particular, explain why every string in the language is generated by your grammar, and why every string generated by the grammar is in the language.