Homework 6

Posted: Monday, May 14, 2018 – 11:59pm
Due: Wednesday, May 23, 2018 – 2pm (Gradescope)

Task 1 – Simplifying Context-Free Grammars (6 points)

a) Remove all unit productions from the following CFG with start variable S:

$$\begin{aligned} S &\to AB \mid BC \mid a \\ A &\to B \mid aCb \mid bb \\ B &\to C \mid Aa \mid aa \\ C &\to S \mid B \mid BA \mid a\ . \end{aligned}$$

b) Remove all λ-, unit- and useless productions (in this order) from the following CFG with start variable S:

$$\begin{aligned} S &\to bAB \mid bb \mid C \\ A &\to BC \mid aCB \mid \lambda \mid a \\ B &\to bB \mid C \mid \lambda \\ C &\to aaC \mid bbC \mid D \\ D &\to a \mid b\ . \end{aligned}$$

Task 2 – Simplifying Context-Free Grammars – Theory (6 points)

a) Explain why removing unit productions from a CFG without λ-productions does not add new λ-productions.

b) Explain why removing useless productions from a CFG without λ- and unit-productions does not add any λ- and unit-productions.

c) Can removing λ-productions from a CFG G with $\lambda \notin L(G)$ create *new* unit productions? Explain!

d) Can removing unit-productions from a CFG without λ-productions create new useless productions? Explain!

Task 3 – Chomsky Normal Form (6 points)

Transform each of the two following CFGs into an equivalent CFG in Chomsky Normal Form.

a)

$$S \to aSSa \mid bSTb$$
$$S \to aa \mid bb$$
$$T \to a \mid b\ .$$

b)

$$S \to aAB \mid b \mid bCa$$
$$A \to aB \mid bC$$
$$B \to aB \mid bAbB \mid a$$
$$C \to aC \mid bCAa \mid \lambda\ .$$

Task 4 – The CYK Algorithm (8 points)

Consider the following CFG G with start variable S:

$$S \to AB \mid AD$$
$$C \to BA$$
$$D \to SC$$
$$A \to a$$
$$B \to b$$

Note that G is already in Chomsky Normal Form.

a) Run the CYK algorithm with input $w = aabba$. Is w in $L(G)$?

b) Run the CYK algorithm with input $w' = aaabba$. Is w' in $L(G)$?

Hint: For both **a)** and **b)**, complete the triangular table describing the complete execution of the CYK algorithm and use it to conclude whether the string w is in $L(G)$ or not. In particular, give all sets V_{ij} for $i \leq j$.

As explained in class, one can modify the CYK algorithm to build a derivation tree for its input w, whenever $w \in L(G)$: When a variable A is added to V_{ij} because of a production $A \to BC$ with $B \in V_{i,k}$ and $C \in V_{k+1,j}$, then one stores this production together with k and A in the table entry.

c) Use the modified CYK algorithm to give a derivation tree for $w = aaabbaba$.

Task 5 – Closure Properties – Part II (4 points)

Let L be an arbitrary context-free language with alphabet Σ. Prove that the following two languages are both context free:

a) L^*

b) L^R

Hint: Given a context-free grammar G for L, explain how to find a context-free grammar for the resulting language. In particular, explain why every string in the language is generated by your grammar, and why every string generated by the grammar is in the language.