Use of Task Graph Model for Parallel Program Design

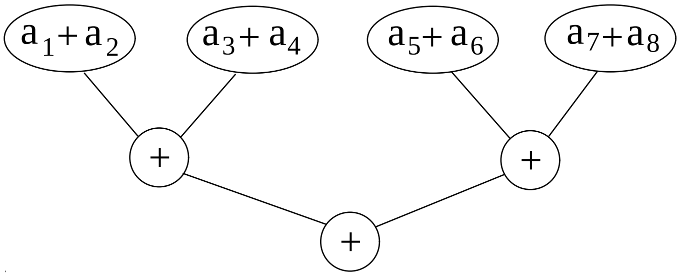
Detailed steps for parallel program design and implementation

1. Preparing Parallelism
   - Computational task partitioning. Aggregate tasks when needed.
   - Dependence analysis to derive a task graph
2. Mapping & Scheduling of parallelism
   - Map tasks $\Longrightarrow$ processors (cores)
   - Order execution
3. Parallel Programming
   - Coding
   - Debugging
4. Performance Evaluation

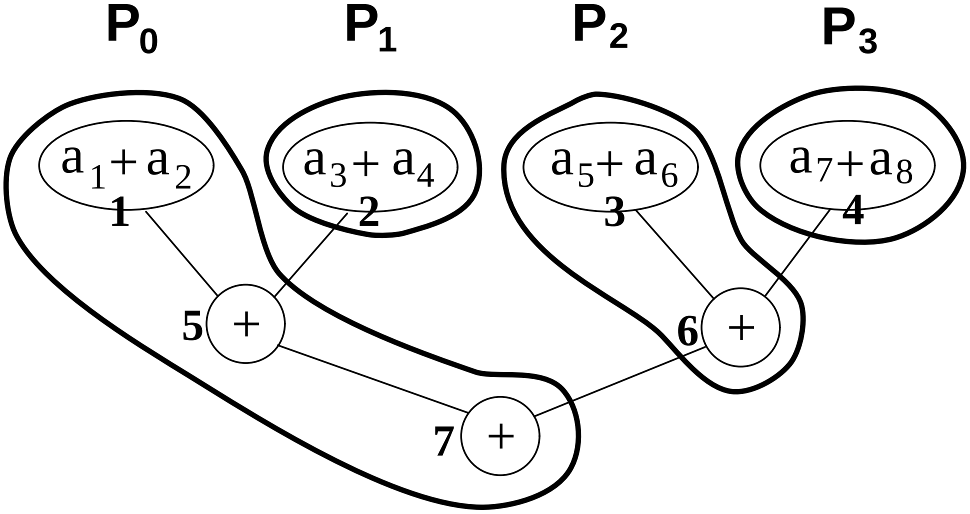
Example

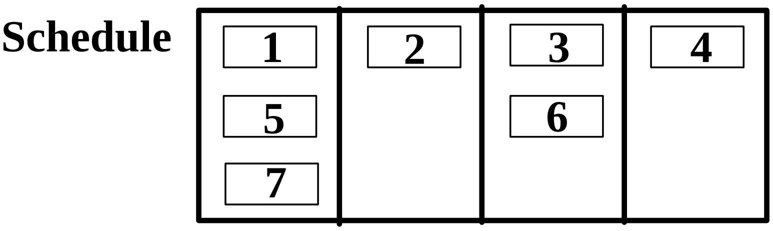
1. Parallelism

   $x = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8$



2. Processor mapping and scheduling

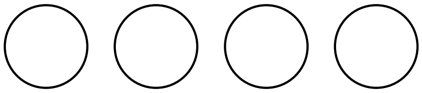




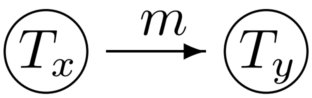
Task Graphs with Scheduling

A Simple Model for Parallel Computation

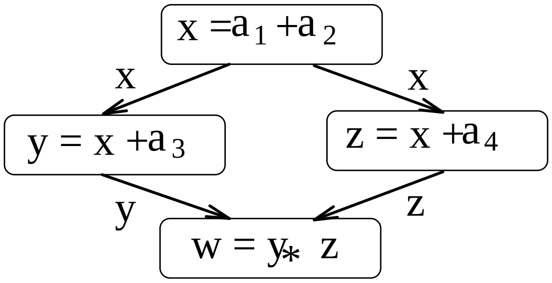
- A set of Tasks



- Data dependence among tasks



- Task Graph



$w = (a_1 + a_2 + a_3)(a_1 + a_2 + a_4)$

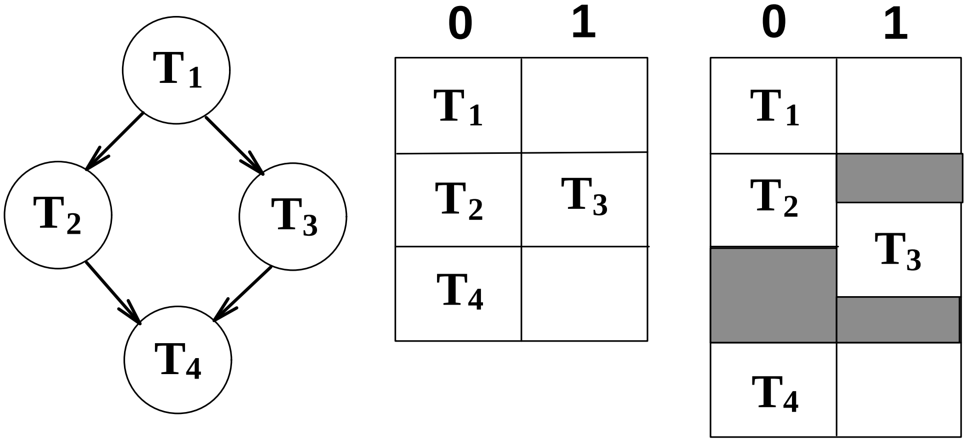
Scheduling of task graph

Use a *gantt chart* to represent a schedule.

I) Assign tasks to processors.

II) Order execution within each processor. Each task

1) Receives data from parents.
2) Executes computation.
3) Sends data to children.



$\tau = 1$
$c = 0$

$\tau = 1$
$c = 0.5$

The left schedule can be expressed as:

	T_1	T_2	T_3	T_4
Proc Assign.	0	0	1	0
Start time	0	1	1	2

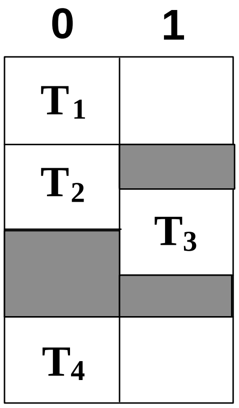
Performance Evaluation

- *Seq* — Sequential Time ($\sum$ task weights)
- PT_p — Parallel Time (Length of the schedule)

$$\text{Speedup} = \frac{Seq}{PT_p}$$

$$\text{Efficiency} = \frac{Speedup}{p}$$

Ex.



$$Seq = 4 \quad p = 2, \quad PT_p = 4$$

$$Speedup = 1$$

$$Efficiency = \frac{1}{2} = 50\%$$

Performance Limited by

- Parallelism availability
- Task granularity $(\frac{Computation\ Cost}{Communication\ Cost})$

Revisit Amdahl's Law: Given sequential time Seq, define α as fraction of computation that has to be done sequentially.

Parallel time is modeled as

$$PT_p = \alpha\ Seq + \frac{(1-\alpha)Seq}{p}$$

$$Speedup = \frac{Seq}{PT_p} = \frac{1}{\alpha + (1-\alpha)/p}$$

Example:

$$\alpha = 0, \quad Speedup = p$$

$$\alpha = 0.5, \quad Speedup = \frac{2}{1+p^{-1}} < 2$$

Performance bounds for task graph executio

Define

- *Critical path* is the longest path (including computation weights). The length of critical path is also called *Span*.
- *Degree of parallelism* be the maximum size of independent task sets in the graph.
- Seq = Sequential time (or called work load)

Span Law

$$PT \geq \texttt{Length of the critical path.}$$

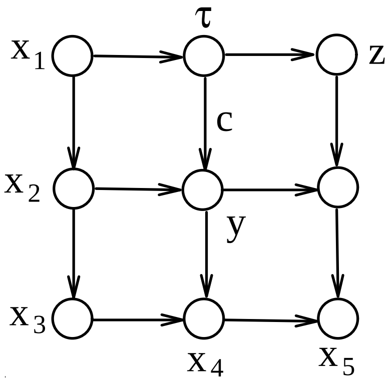
Work Law

$$PT \geq \frac{Seq}{p}$$

Additionally

$$Speedup \leq \texttt{Degree of parallelism}$$

Example.



No of processors $p = 2$. Task weight $\tau = 1$. Sequential time $Seq = 9$. Communication cost $c = 0$.

Maximum independent set $= \{x_3, y, z\}$.

Degree of parallelism =3.

CP = critical path = $\{\ x, x_2, x_3, x_4, x_5\}$.

Length(CP) = 5.

$PT \geq max(Length(CP), \frac{\texttt{seq}}{p}) = max(5, \frac{9}{2}) = 5.$

`Speedup` $\leq \frac{\texttt{Seq}}{5} = \frac{9}{5} = 1.8$

`Speedup` ≤ 3 `Degree of parallelism`

Pseudo Parallel Code

- **SPMD** - Single Program / Multiple Data
  - Data and program are distributed among processors, code is executed based on a predetermined schedule.
  - Each processor executes the same program but operates on different data based on processor identification.
- **Master/slaves:** One control process is called the master (or host). There are a number of slaves working for this master. These slaves can be coded using an SPMD style.

Pseudo Library Functions

- **mynode()**.

  Return the processor ID. p processors are numbered as $0, 1, 2, \cdots, p-1$.

- **numnodes()**. Return the number of processors allocated.

- **send(data,dest)**.

  Send data to a destination processor.

- **recv(data_buffer, source_id)** or **recv(data_buffer)**.

  Executing $recv()$ will get a message from a processor (or any processor) and store it in the space specified by $data\_buffer$.

- **broadcast(data)**.

  Broadcast a message to all processors.

Two examples of SPMD Code

- SPMD code:

```
Print
"hello";
```

Execute in 4 processors. The screen is:

```
hello
hello
hello
hello
```

- SPMD code:

```
x=mynode();
If x > 0, then Print "hello from " x.
```

Screen:

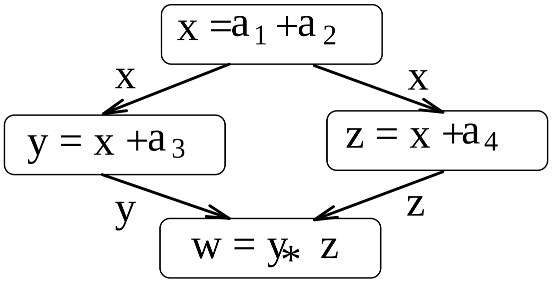
```
hello from 1
hello from 2
hello from 3
```

Example 3: Parallel Programming Steps

Sequential program:

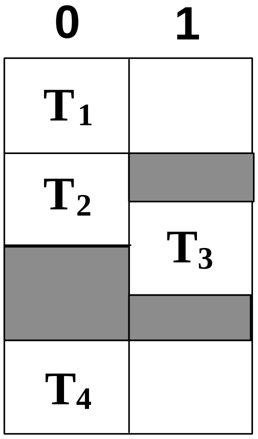
$$x = a_1 + a_2;$$
$$y = x + a_3;$$
$$z = x + a_4;$$
$$w = y * z;$$

Task Graph:



$w = (a_1 + a_2 + a_3)(a_1 + a_2 + a_4)$

Schedule:



SPMD Code:

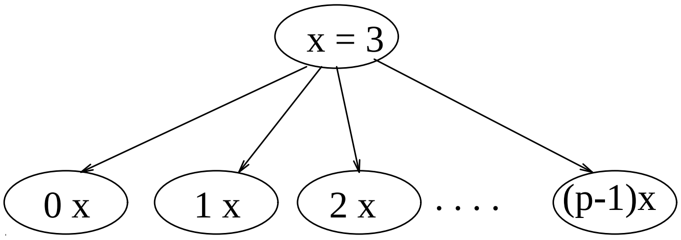
```
int i, x, y, z, w,  a[5];
i = mynode();
if (i==0) then {
      x=a[1]+a[2];
      send(x, 1);
      y=x+a[3];
      receive(z);
      w=y*z;
}
else{
      receive(x);
      z=x+a[4];
      send(z,0);
}
```

Example 4: Parallel Programming Steps

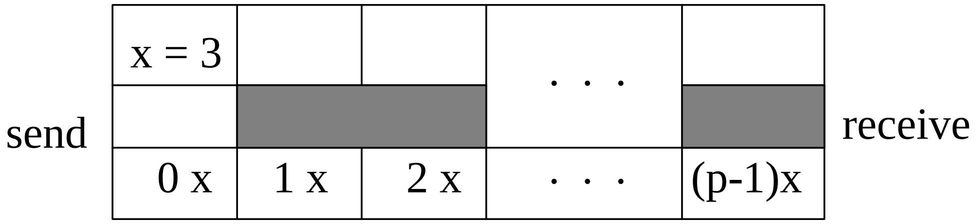
Sequential program:

```
x=3
For i = 0 to  p-1.
    y(i)= i*x;
Endfor
```

Task Graph:



Schedule:



SPMD Code:

```
int x,y,i;
i = mynode();
if (i==0) then { x=3;
                 broadcast(x); }
else receive(x);
y = i*x;
```

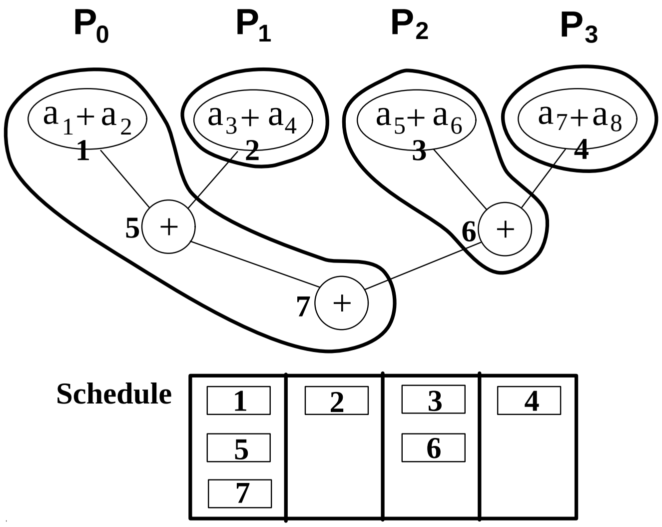
Evaluation:

Assume that each task takes one unit W and broadcasting takes C.

$$Seq = (p+1)W, \quad PT = W + C + W.$$

$$Speedup = \frac{(p+1)W}{2W+C}.$$

Partial SPMD Code for Tree Summation



```
me=mynode(); p= 4;
sum = sum of local numbers at this processor;
if(?for some leaf node?) Send sum to node ?f(me)?;
for i= 1 to tree depth do{
   if(?I am still used in this  depth?){
       x=receive partial sum from node ?f(me)?;
       sum = sum +x
       if (?I will not be used in next depth?)
             Send sum to node ?f(me)?;
   }}
```