Collective Communication in MPI and Advanced Features

Pacheco. Chapter 3

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Part of slides from the text book, CS267 K. Yelick from UC Berkeley and B. Gropp, ANL

Outline

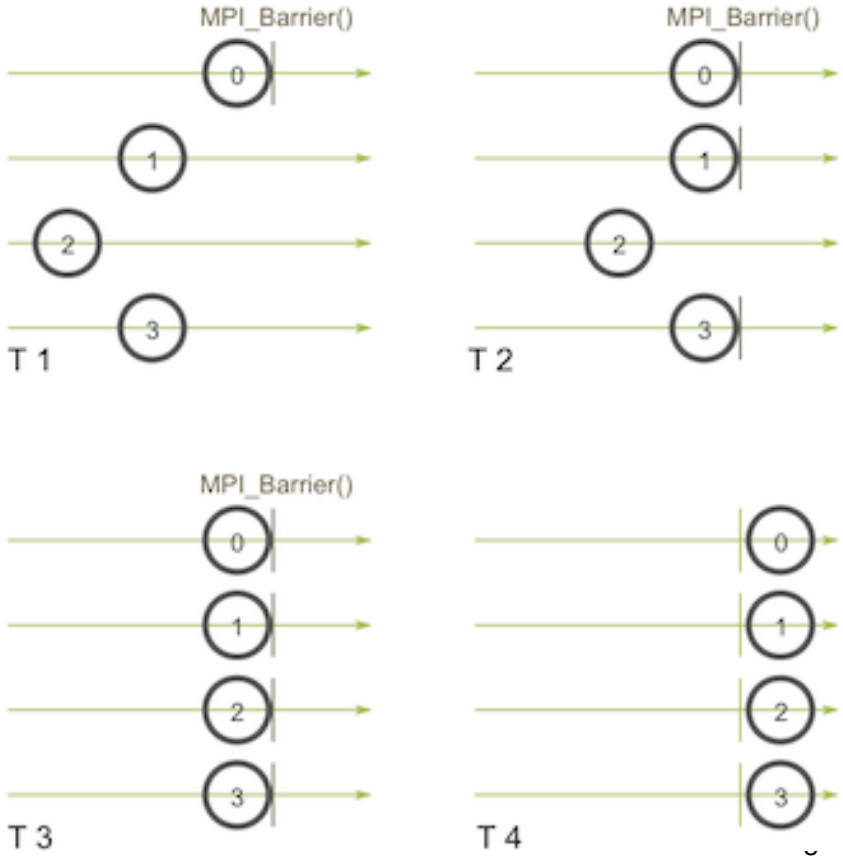
- **Collective group communication**
- **Application examples**
  - Pi computation
  - Summation of long vectors
- **More applications**
  - Matrix-vector multiplication
    - performance evaluation
  - Parallel sorting
- **Safety and other MPI issues.**

MPI Collective Communication

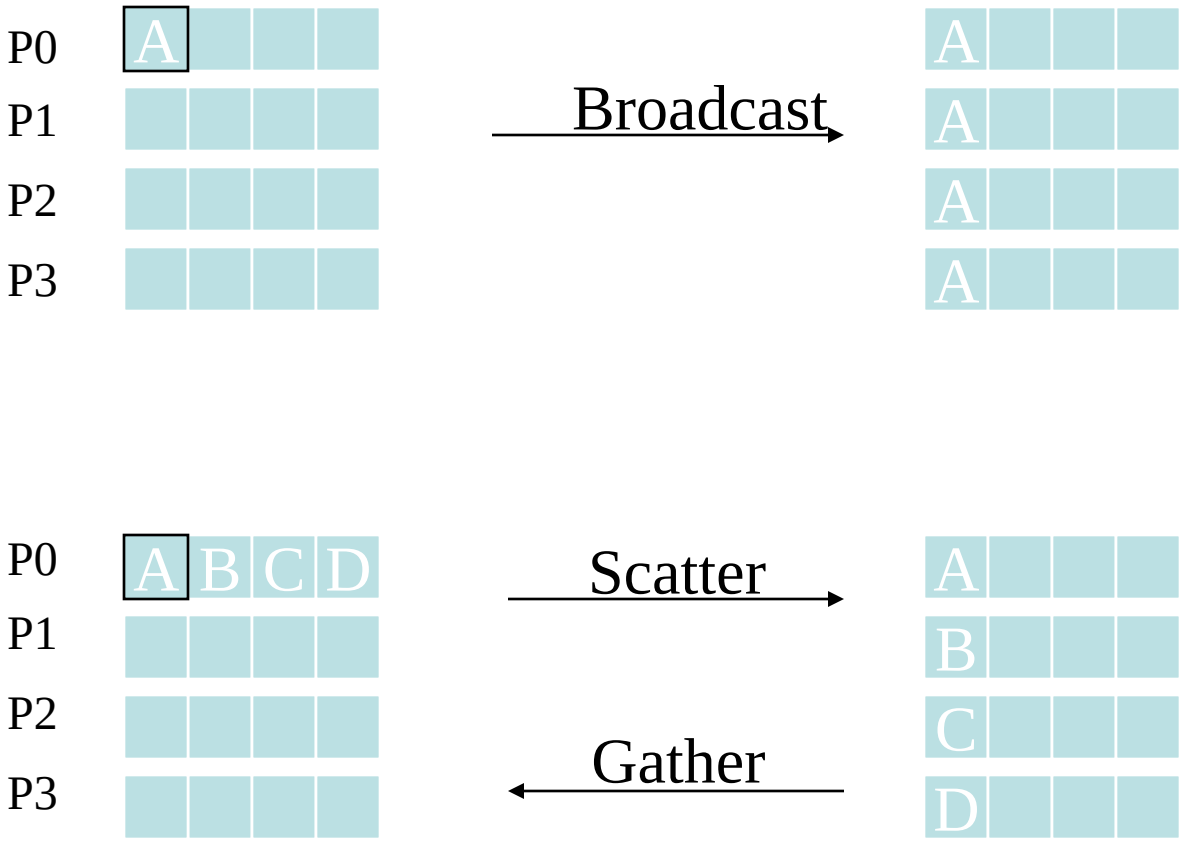
- ***Collective* routines provide a higher-level way to organize a parallel program**
  - Each process executes the same communication operations
  - Communication and computation is coordinated among a group of processes in a communicator
  - Tags are not used
  - No non-blocking collective operations.
- **Three classes of operations:** synchronization, data movement, collective computation.

Synchronization

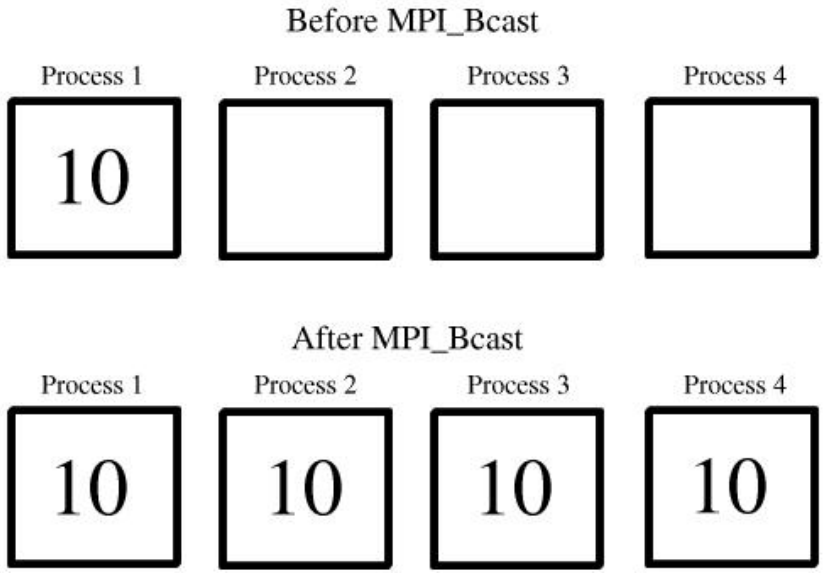
- **`MPI_Barrier( comm )`**
- Blocks until all processes in the group of the communicator `comm` call it.
- Not used often. Sometime used in measuring performance and load balancing



Collective Data Movement: Broadcast, Scatter, and Gather



Broadcast

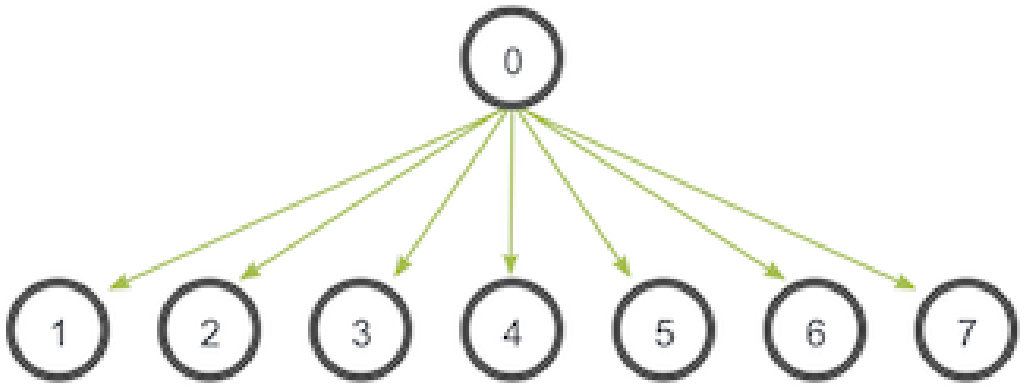


- **Data belonging to a single process is sent to all of the processes in the communicator.**

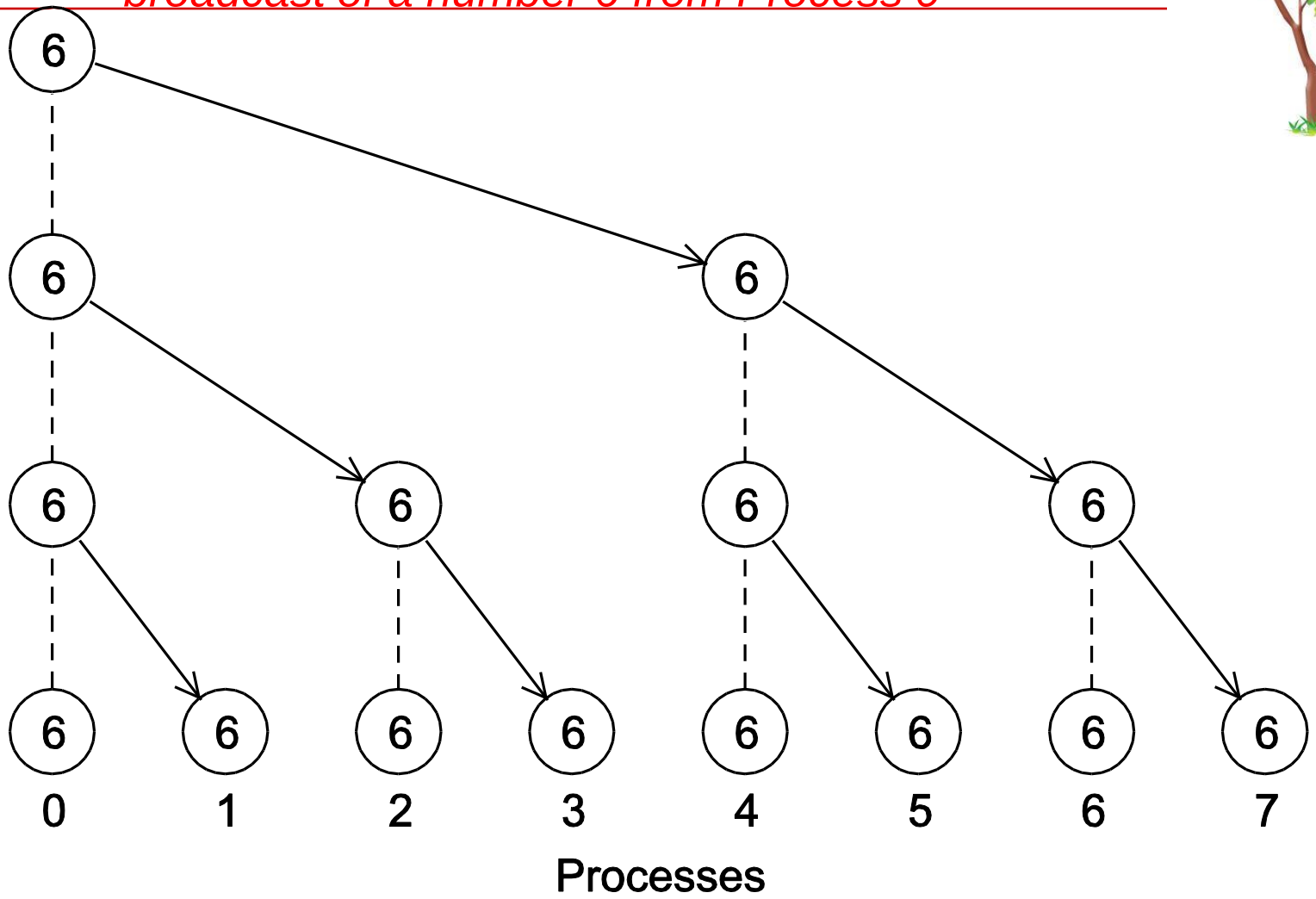
```
int MPI_Bcast(
        void*          data_p      /* in/out */,
        int            count       /* in     */,
        MPI_Datatype   datatype    /* in     */,
        int            source_proc /* in     */,
        MPI_Comm       comm        /* in     */);
```

Comments on Broadcast

- **All collective operations must be called by *all* processes in the communicator**
- **MPI_Bcast is called by both the sender (called the root process) and the processes that are to receive the broadcast**
  - MPI_Bcast is not a “multi-send”
  - “root” argument is the rank of the sender; this tells MPI which process originates the broadcast and which receive



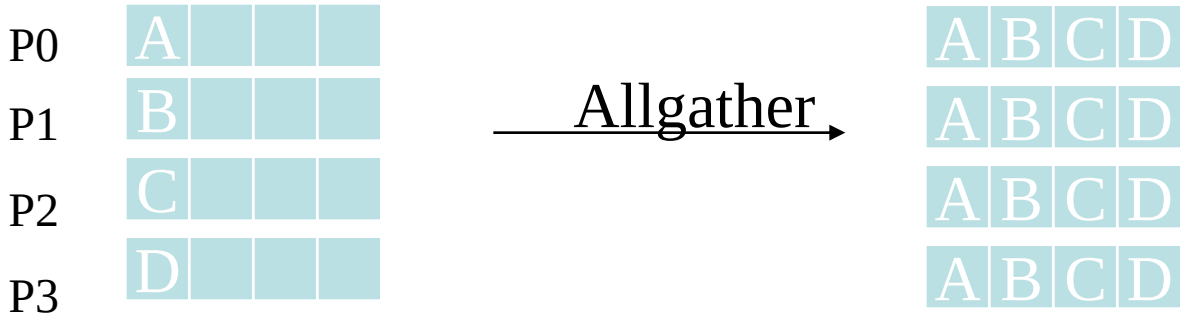
Implementation View: A tree-structured broadcast of a number 6 from Process 0



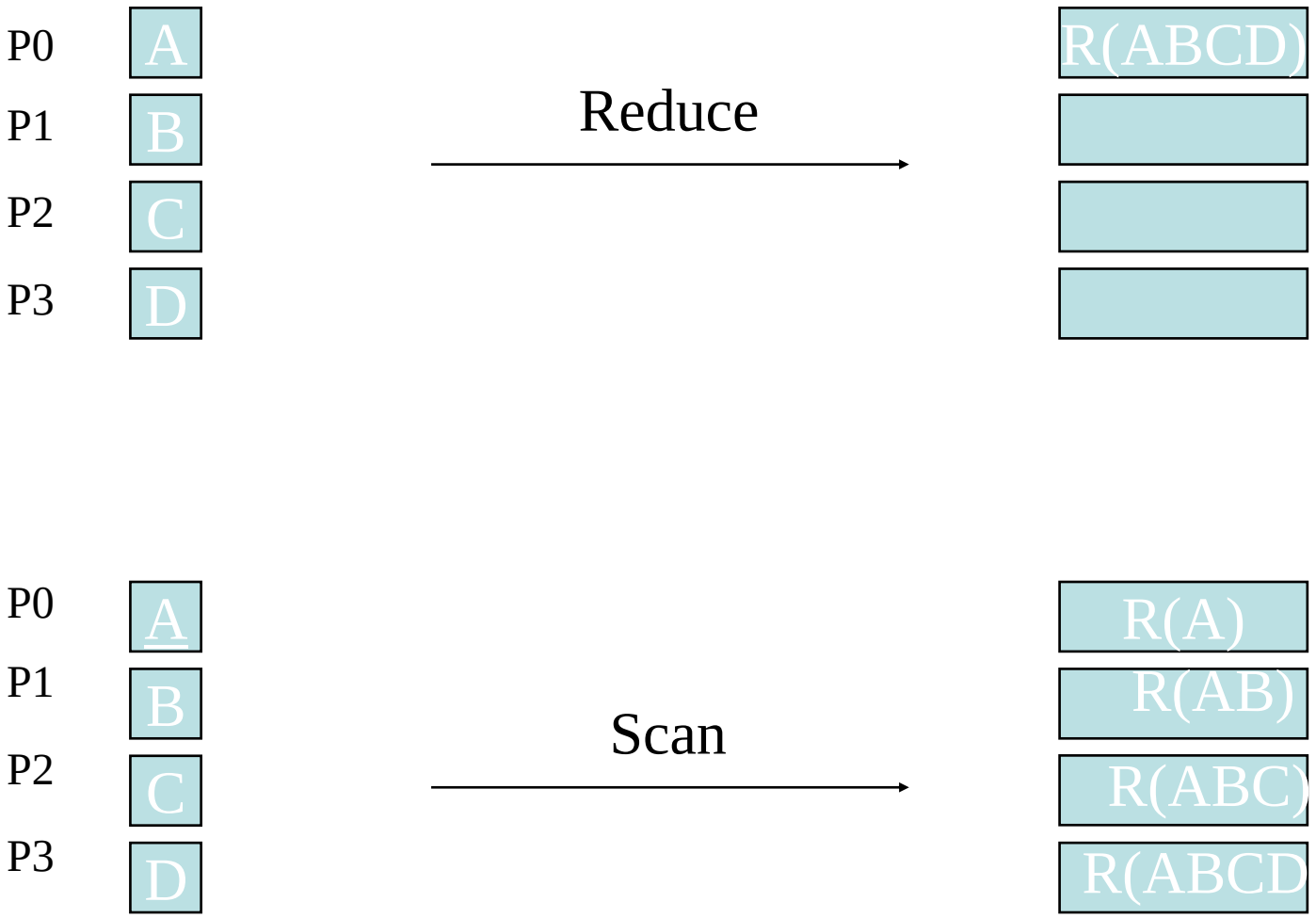
A version of Get_input that uses MPI_Bcast in the trapezoidal program

```
void Get_input(
      int       my_rank  /* in  */,
      int       comm_sz  /* in  */,
      double*   a_p      /* out */,
      double*   b_p      /* out */,
      int*      n_p      /* out */) {

   if (my_rank == 0) {
      printf("Enter a, b, and n\n");
      scanf("%lf %lf %d", a_p, b_p, n_p);
   }
   MPI_Bcast(a_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
   MPI_Bcast(b_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
   MPI_Bcast(n_p, 1, MPI_INT, 0, MPI_COMM_WORLD);
}  /* Get_input */
```

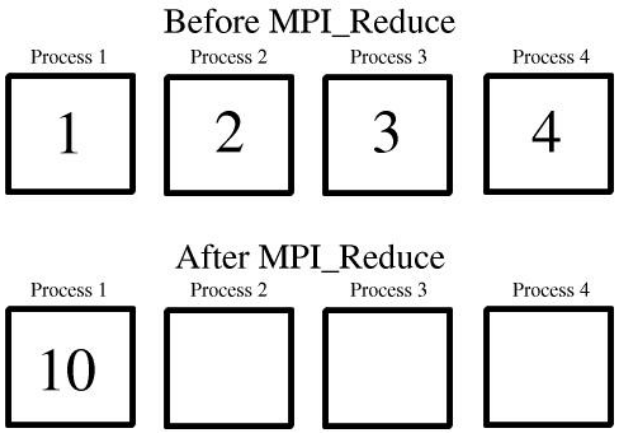
Collective Data Movement: Allgather and AlltoAll



Collective Computation: Reduce vs. Scan



MPI_Reduce




```
int MPI_Reduce(
      void*         input_data_p    /* in  */,
      void*         output_data_p   /* out */,
      int           count           /* in  */,
      MPI_Datatype  datatype        /* in  */,
      MPI_Op        operator        /* in  */,
      int           dest_process    /* in  */,
      MPI_Comm      comm            /* in  */);
```

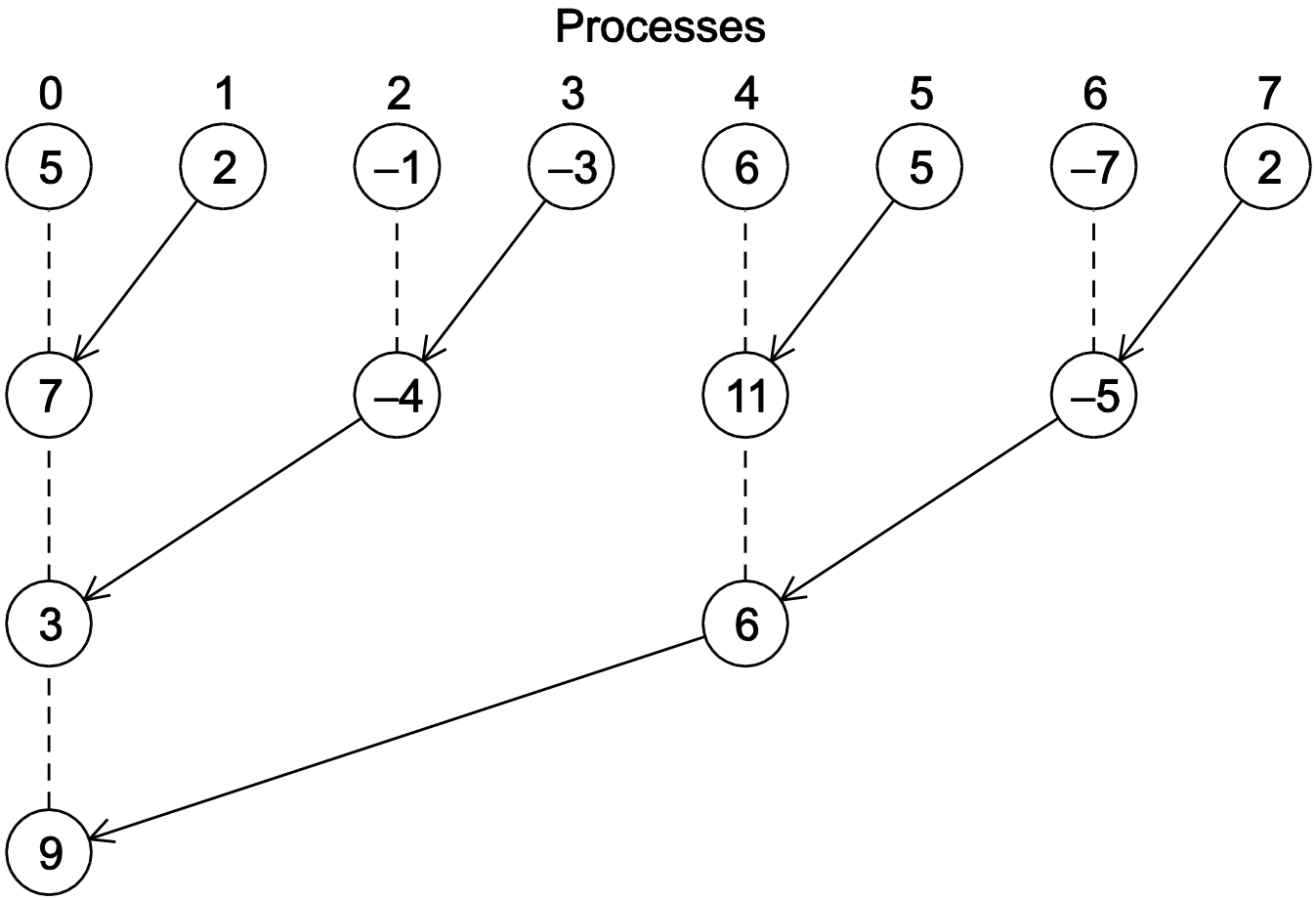
```
MPI_Reduce(&local_int, &total_int, 1, MPI_DOUBLE, MPI_SUM, 0,
      MPI_COMM_WORLD);
```

```
double local_x[N], sum[N];
. . .
MPI_Reduce(local_x, sum, N, MPI_DOUBLE, MPI_SUM, 0,
      MPI_COMM_WORLD);
```

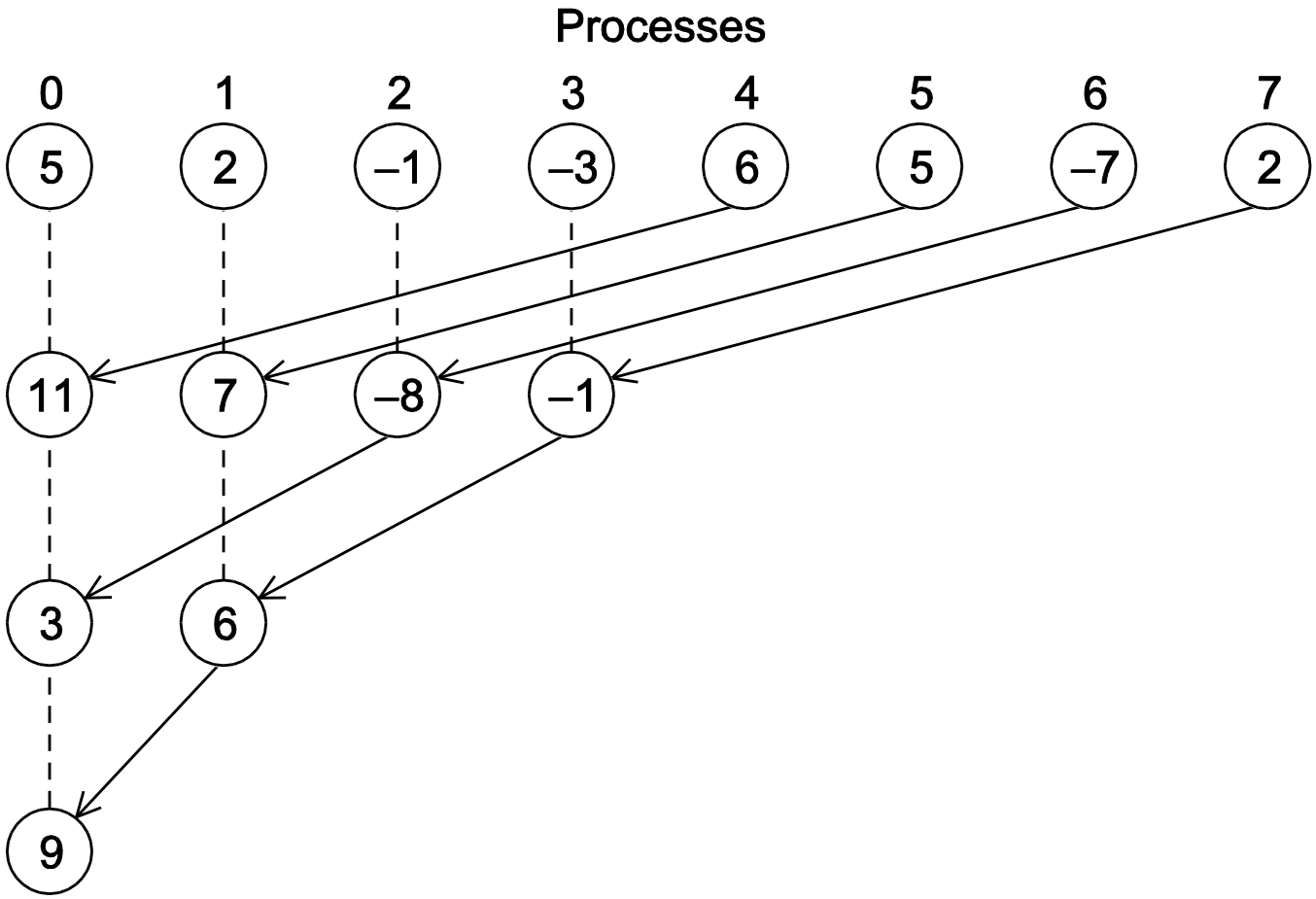
Predefined reduction operators in MPI

Operation Value	Meaning
MPI_MAX	Maximum
MPI_MIN	Minimum
MPI_SUM	Sum
MPI_PROD	Product
MPI_LAND	Logical and
MPI_BAND	Bitwise and
MPI_LOR	Logical or
MPI_BOR	Bitwise or
MPI_LXOR	Logical exclusive or
MPI_BXOR	Bitwise exclusive or
MPI_MAXLOC	Maximum and location of maximum
MPI_MINLOC	Minimum and location of minimum

Implementation View of Global Reduction using a tree-structured sum

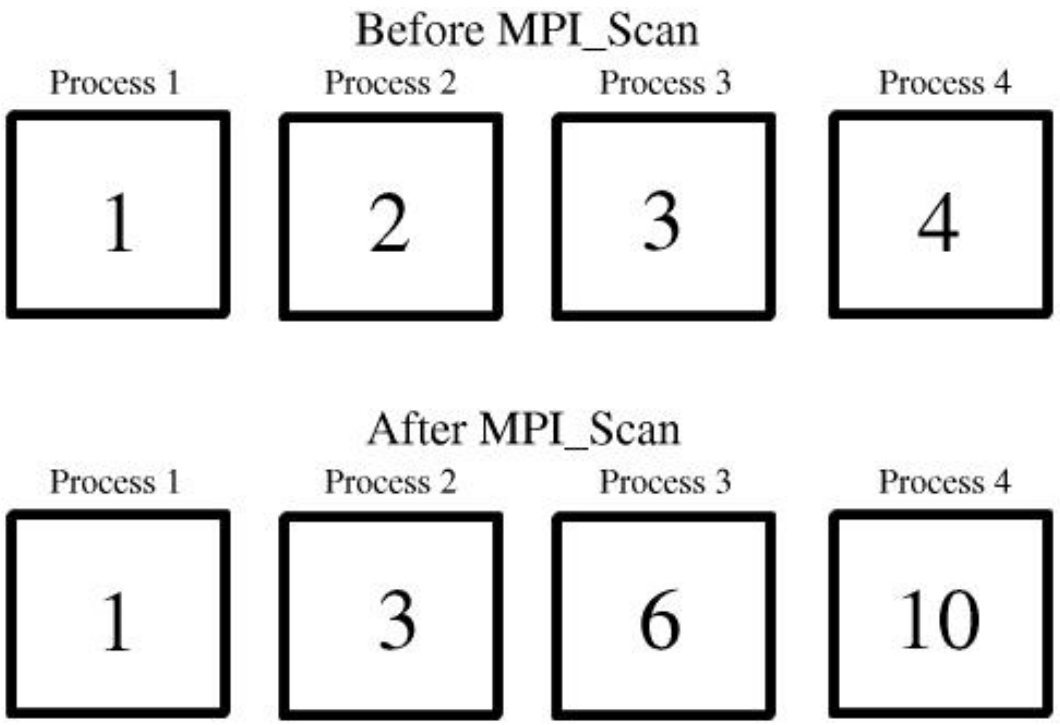


An alternative tree-structured global sum

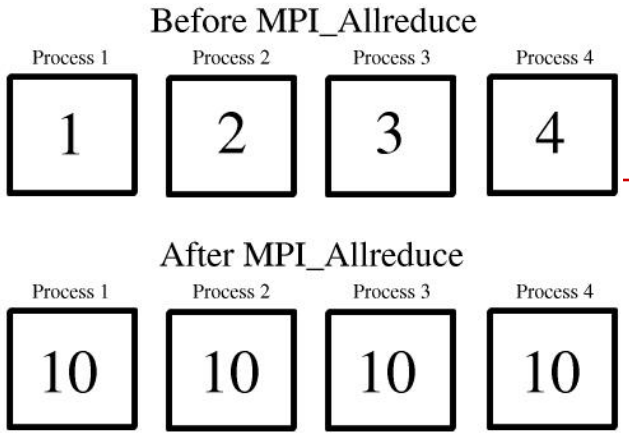


MPI Scan

MPI_Scan(void *sendbuf, void *recvbuf, int count,
MPI_Datatype datatype, MPI_Op op, MPI_Comm comm);

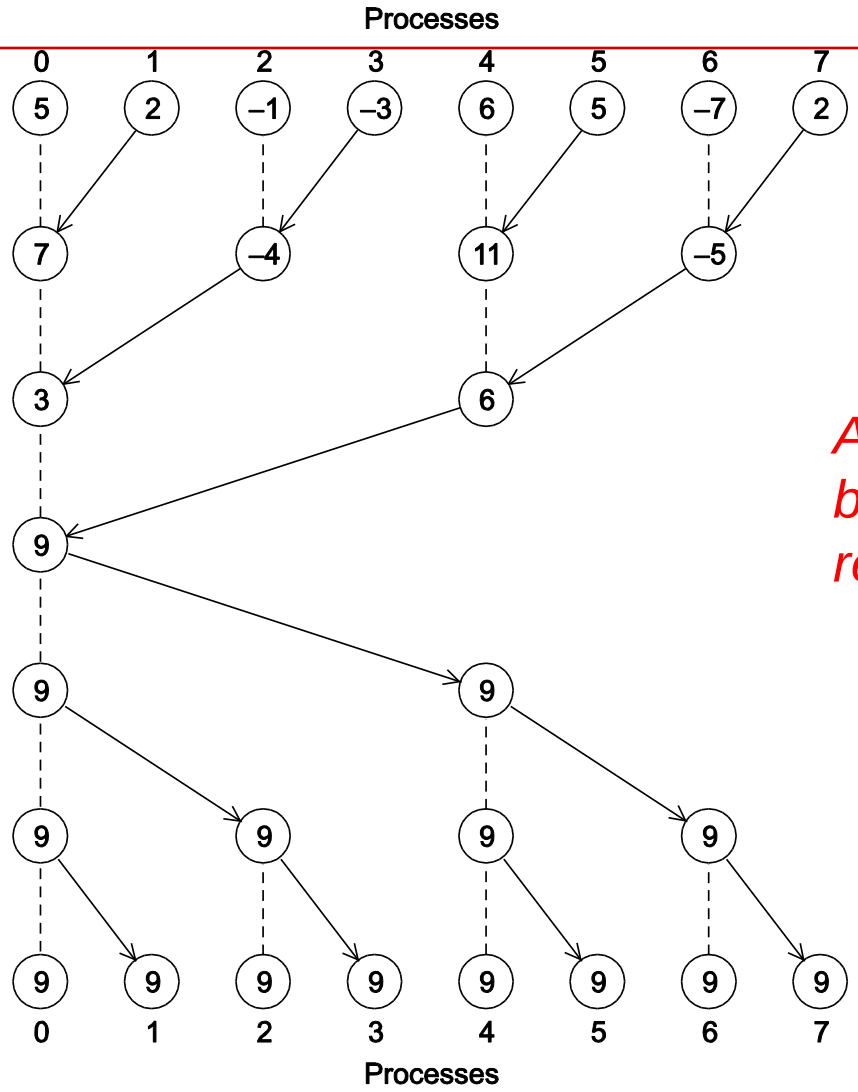


MPI_Allreduce



- **Useful in a situation in which all of the processes need the result of a global sum in order to complete some larger computation.**

```
int MPI_Allreduce(
        void*        input_data_p   /* in  */,
        void*        output_data_p  /* out */,
        int          count          /* in  */,
        MPI_Datatype datatype       /* in  */,
        MPI_Op       operator       /* in  */,
        MPI_Comm     comm           /* in  */);
```



A global sum followed by distribution of the result.

MPI Collective Routines: Summary

- Many Routines: `Allgather`, `Allgatherv`, `Allreduce`, `Alltoall`, `Alltoallv`, `Bcast`, `Gather`, `Gatherv`, `Reduce`, `Reduce_scatter`, `Scan`, `Scatter`, `Scatterv`
- `All` versions deliver results to all participating processes.
- V versions allow the hunks to have variable sizes.
- `Allreduce`, `Reduce`, `Reduce_scatter`, and `Scan` take both built-in and user-defined combiner functions.
- MPI-2 adds `Alltoallw`, `Exscan`, intercommunicator versions of most routines

Example of MPI PI program using 6 Functions

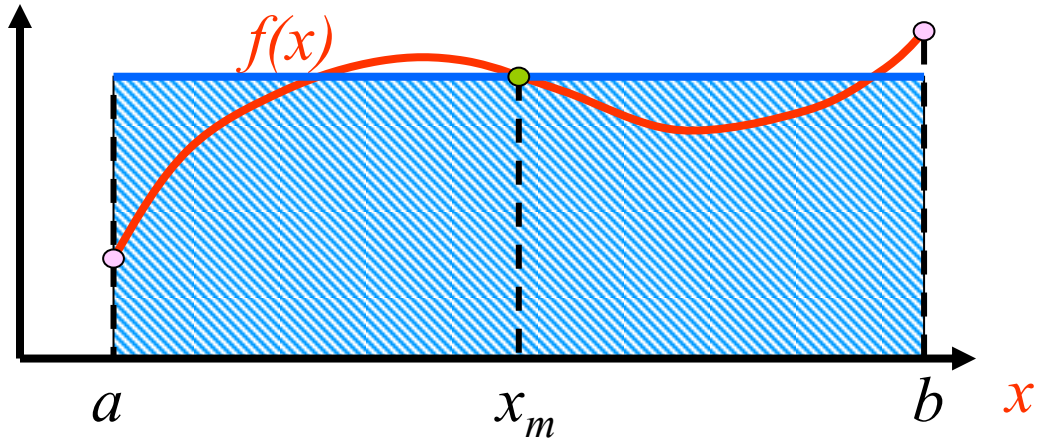
$$\pi = 4 \int_0^1 \frac{1}{1+x^2} \, dx$$

- Using basic MPI functions:
  - `MPI_INIT`
  - `MPI_FINALIZE`
  - `MPI_COMM_SIZE`
  - `MPI_COMM_RANK`
- Using MPI collectives:
  - `MPI_BCAST`
  - `MPI_REDUCE`

Slide source: Bill Gropp, ANL

Midpoint Rule for $\pi = 4\int_0^1 \frac{1}{1+x^2}\,dx$

$$\int_a^b f(x)dx \approx (b-a)\,f(x_m)$$



$$\int_{x=0}^{1} \frac{1}{1+x^2} \approx \sum_{i=1}^{n} \frac{1}{1+(\frac{i-0.5}{n})^2}$$

Example: PI in C - 1

$$\pi = 4 \int_0^1 \frac{1}{1+x^2} dx$$

```
#include "mpi.h"
#include <math.h>
   #include <stdio.h>
int main(int argc, char *argv[])
{
   int done = 0, n, myid, numprocs, i, rc;
   double PI25DT = 3.141592653589793238462643;
   double mypi, pi, h, sum, x, a;
   MPI_Init(&argc,&argv);
   MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
   MPI_Comm_rank(MPI_COMM_WORLD,&myid);
```

```
while (!done)  {
  if (myid == 0) {
    printf("Enter the number of intervals: (0 quits) ");
    scanf("%d",&n);
  }
MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);
if (n == 0) break;
```

Input and broadcast parameters

Slide source: Bill Gropp, ANL

Example: PI in C - 2

$$\int_{x=0}^{1} \frac{1}{1+x^2} \approx \sum_{i=1}^{n} \frac{1}{1+(\frac{i-0.5}{n})^2}$$

```
h   = 1.0 / (double) n;
sum = 0.0;
for (i = myid + 1; i <= n; i += numprocs) {
  x = h * ((double)i - 0.5);
  sum += 4.0 / (1.0 + x*x);
}
mypi = h * sum;
```

Compute local pi values

```
MPI_Reduce(&mypi, &pi, 1, MPI_DOUBLE, MPI_SUM, 0,
           MPI_COMM_WORLD);
if (myid == 0)
  printf("pi is approximately %.16f, Error is .16f\n",
         pi, fabs(pi - PI25DT));
}
MPI_Finalize();
return 0;
}
```

Compute summation

Slide source: Bill Gropp, ANL

Collective vs. Point-to-Point Communications

- **All the processes in the communicator must call the same collective function.**
  - For example, a program that attempts to match a call to MPI_Reduce on one process with a call to MPI_Recv on another process is erroneous, and, in all likelihood, the program will hang or crash.

```
if(my_rank==0)  MPI_Reduce(&a,&b,1, MPI_INT,
   MPI_SUM, 0, MPI_COMM_WORLD);
else MPI_Recv(&a, MPI_INT, MPI_SUM,0,0,
   MPI_COMM_WORLD);
```

Collective vs. Point-to-Point Communications

- **The arguments passed by each process to an MPI collective communication must be "compatible."**
  - For example, if one process passes in 0 as the dest_process and another passes in 1, then the outcome of a call to MPI_Reduce is erroneous, and, once again, the program is likely to hang or crash.

```
if(my_rank==0)  MPI_Reduce(&a,&b,1, MPI_INT,
   MPI_SUM, 0, MPI_COMM_WORLD);
else MPI_Reduce(&a,&b,1, MPI_INT, MPI_SUM, 1,
   MPI_COMM_WORLD);
```

Example of MPI_Reduce execution

Time	Process 0	Process 1	Process 2
0	`a = 1; c = 2`	`a = 1; c = 2`	`a = 1; c = 2`
1	`MPI_Reduce(&a, &b, ...)`	`MPI_Reduce(&c, &d, ...)`	`MPI_Reduce(&a, &b, ...)`
2	`MPI_Reduce(&c, &d, ...)`	`MPI_Reduce(&a, &b, ...)`	`MPI_Reduce(&c, &d, ...)`

Multiple calls to MPI_Reduce with MPI_SUM and Proc 0 as destination (root)

Is b=3 on Proc 0 after two MPI_Reduce() calls?

Is d=6 on Proc 0?

Example: Output results

- However, the names of the memory locations are irrelevant to the matching of the calls to MPI_Reduce.

- The order of the calls will determine the matching so the value stored in b will be 1+2+1 = 4, and the value stored in d will be 2+1+2 = 5.

Summation of Two Long Vectors

Collective Communication Application

Textbook p.109-111

Application Example: Distributed Summation of Two Long Vectors

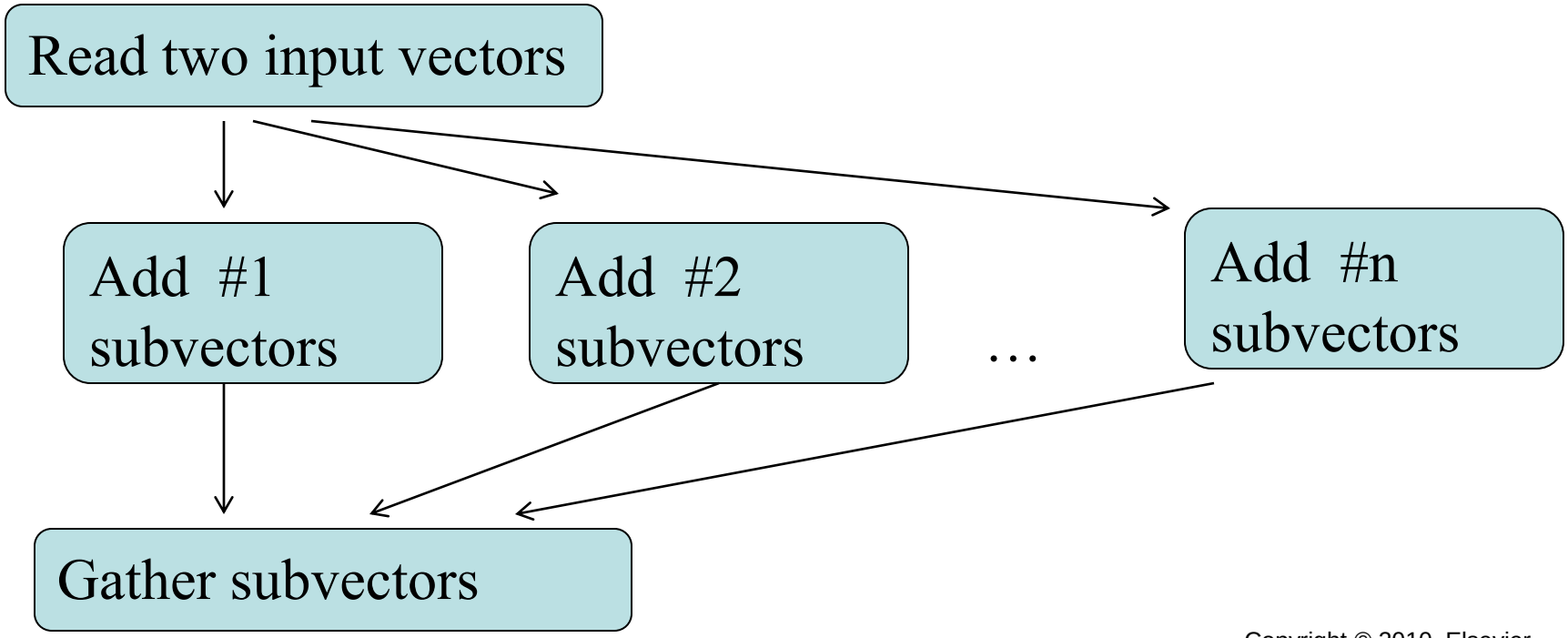
$$
\begin{aligned}
\mathbf{x}+\mathbf{y} &= (x_0, x_1, \ldots, x_{n-1}) + (y_0, y_1, \ldots, y_{n-1}) \\
&= (x_0 + y_0, x_1 + y_1, \ldots, x_{n-1} + y_{n-1}) \\
&= (z_0, z_1, \ldots, z_{n-1}) \\
&= \mathbf{z}
\end{aligned}
$$

Sequential code for computing a vector sum.

```
void Vector_sum(double x[], double y[], double z[], int n) {
   int i;

   for (i = 0; i < n; i++)
      z[i] = x[i] + y[i];
}  /* Vector_sum */
```

Parallel implementation of vector addition

1. Divide each vector into n subvectors and distribute data
2. Add two subvectors at each process in parallel.
3. Gather the sum subvector from each process.



Partitioning options for distributing vectors

- **Block partitioning**
  - Assign blocks of consecutive components to each process.
- **Cyclic partitioning**
  - Assign components in a round robin fashion.
- **Block-cyclic partitioning**
  - Use a cyclic distribution of blocks of components.

Partitioning examples for data distribution

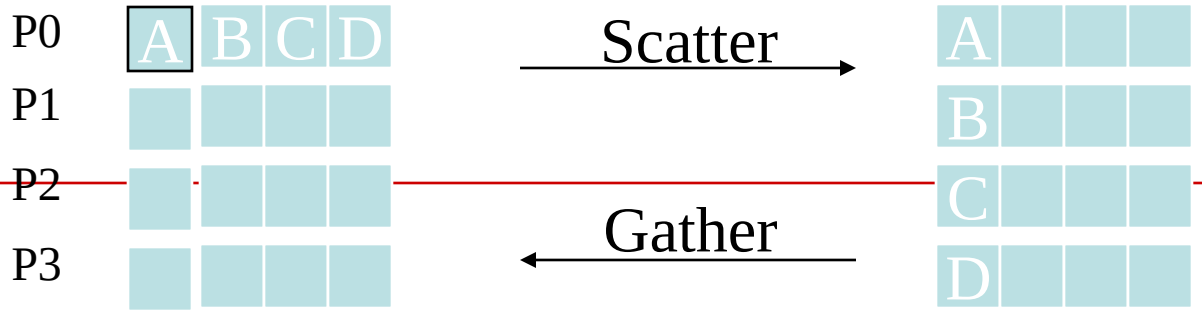
Each vector is divided into 12 subvectors and then distributed to 3 processes

Process	Components											
	Block				Cyclic				Block-cyclic Blocksize = 2			
0	0	1	2	3	0	3	6	9	0	1	6	7
1	4	5	6	7	1	4	7	10	2	3	8	9
2	8	9	10	11	2	5	8	11	4	5	10	11

Local code for subvector addition

```
void Parallel_vector_sum(
      double  local_x[]  /* in  */,
      double  local_y[]  /* in  */,
      double  local_z[]  /* out */,
      int     local_n    /* in  */) {
   int local_i;

   for (local_i = 0; local_i < local_n; local_i++)
      local_z[local_i] = local_x[local_i] + local_y[local_i];
}  /* Parallel_vector_sum */
```

Scatter for data distribution



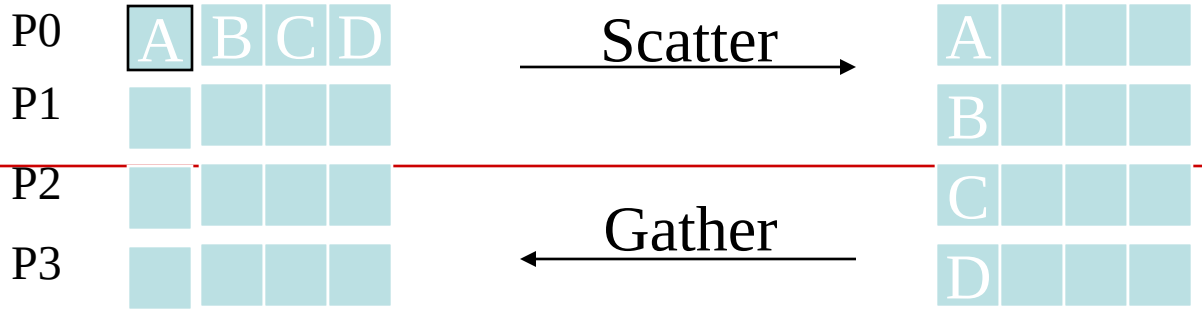
- **MPI_Scatter can be used in a function that reads in an entire vector on process 0 but only sends the needed components to each of the other processes.**

```
int MPI_Scatter(
      void*         send_buf_p  /* in  */,
      int           send_count  /* in  */,
      MPI_Datatype  send_type   /* in  */,
      void*         recv_buf_p  /* out */,
      int           recv_count  /* in  */,
      MPI_Datatype  recv_type   /* in  */,
      int           src_proc    /* in  */,
      MPI_Comm      comm        /* in  */);
```

Reading and distributing a vector

```
void Read_vector(
      double     local_a[]   /* out */,
      int        local_n     /* in  */,
      int        n           /* in  */,
      char       vec_name[]  /* in  */,
      int        my_rank     /* in  */,
      MPI_Comm   comm        /* in  */) {

   double* a = NULL;
   int i;

   if (my_rank == 0) {
      a = malloc(n*sizeof(double));
      printf("Enter the vector %s\n", vec_name);
      for (i = 0; i < n; i++)
         scanf("%lf", &a[i]);
      MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,
            0, comm);
      free(a);
   } else {
      MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,
            0, comm);
   }
}  /* Read_vector */
```

Gather data from everybody



- **Collect all of the components of the vector onto process 0, and then process 0 can process all of the components.**

```
int MPI_Gather(
      void*         send_buf_p  /* in  */,
      int           send_count  /* in  */,
      MPI_Datatype  send_type   /* in  */,
      void*         recv_buf_p  /* out */,
      int           recv_count  /* in  */,
      MPI_Datatype  recv_type   /* in  */,
      int           dest_proc   /* in  */,
      MPI_Comm      comm        /* in  */);
```

MPI code to gather and print a distributed vector

```
void Print_vector(
      double     local_b[]   /* in */,
      int        local_n     /* in */,
      int        n           /* in */,
      char       title[]     /* in */,
      int        my_rank     /* in */,
      MPI_Comm   comm        /* in */) {

   double* b = NULL;
   int i;
```

Gather and print a distributed vector (2)

```
   if (my_rank == 0) {
      b = malloc(n*sizeof(double));
      MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
            0, comm);
      printf("%s\n", title);
      for (i = 0; i < n; i++)
         printf("%f ", b[i]);
      printf("\n");
      free(b);
   } else {
      MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
            0, comm);
   }
}  /* Print_vector */
```

Parallel Matrix Vector Multiplication

Collective Communication Application

Textbook p. 113-116

Matrix-vector multiplication: y= A * x

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} * \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1*1+2*2+3*3 \\ 4*1+5*2+6*3 \\ 7*1+8*2+9*3 \end{pmatrix} = \begin{pmatrix} 14 \\ 32 \\ 50 \end{pmatrix}$$

Problem: $y = A * x$ where A is a $n \times n$ matrix and x is a column vector of dimension n.

Sequential code:

```
for i = 1 to n do
    y_i = 0;
    for j = 1 to n do
        y_i = y_i + a_{i,j} * x_j;
    endfor
endfor
```

Partitioning and Task graph for matrix-vector multiplication

Partitioned code:

for $i = 1$ **to** n **do**

S_i : $y_i = 0;$

for $j = 1$ to n **do**

$y_i = y_i + a_{i,j} * x_j;$

endfor

endfor

S_i : Read row A_i and vector x.

Write element y_i

$y_i = Row\ A_i * x$

Task graph:

Execution Schedule and Task Mapping

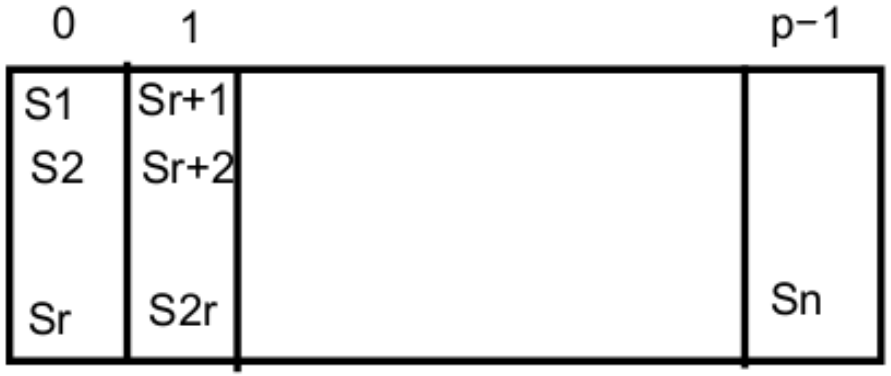
S_i : Read row A_i and vector x.

Write element y_i

$y_i = Row\ A_i\ ^{*}\ x$

Task graph:

Schedule:



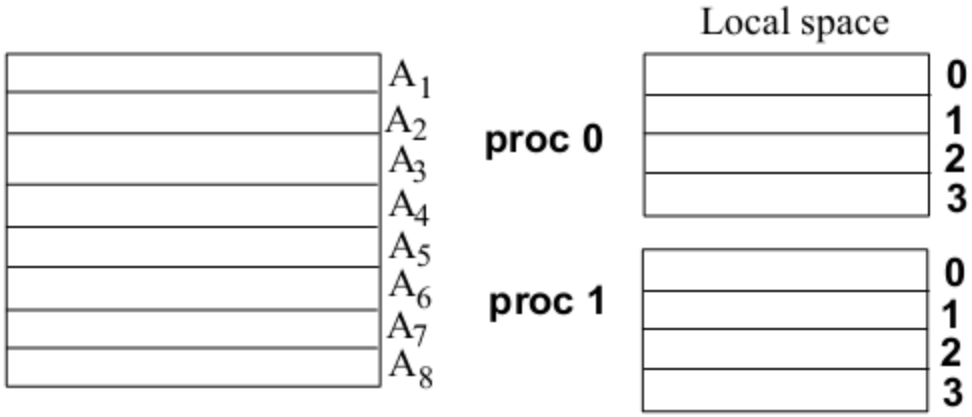
Mapping function of tasks S_i:

$proc\_map(i) = \lfloor \frac{i-1}{r} \rfloor$ where $r = \lceil \frac{n}{p} \rceil$.

Data Partitioning and Mapping for y= A*x

Data partitioning: for the above schedule:

Matrix A is divided into n rows $A_1, A_2, \cdots A_n$.

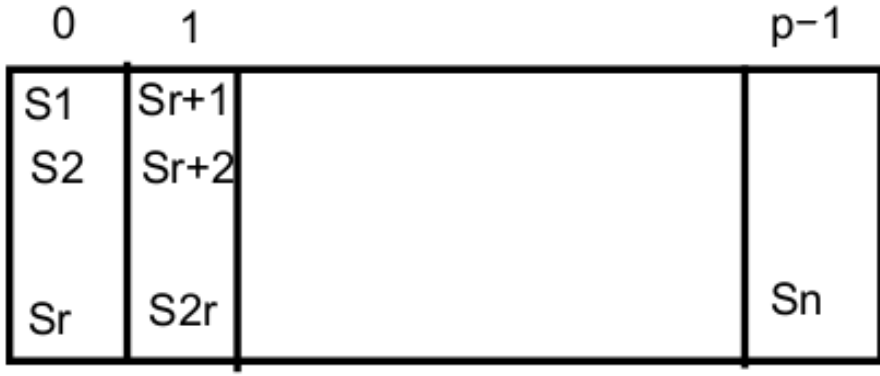


Data mapping:

Row A_i is mapped to processor $proc\_map(i)$, the same as task i. The indexing function is: $local(i) = (i-1) \bmod r$. Vectors x and y are replicated to all processors.

SPMD Code for y= A*x

Schedule:



int x[n], y[n], a[r][n];

me=mynode();

for $i = 1$ **to** n **do**

if $proc\_map(i) == me$, then do S_i:

S_i : $y[i] = 0;$

for $j = 1$ to n **do**

$y[i] = y[i] + a[local(i)][j] * x[j];$

endfor

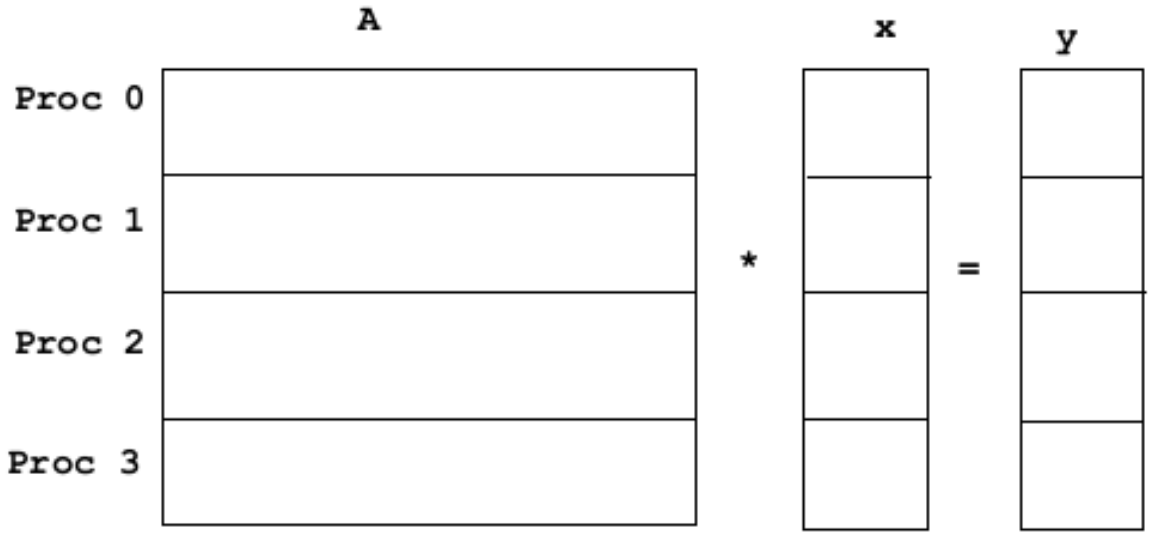
endfor

Evaluation: Parallel Time

- Ignore the cost of local address calculation.
- Each task performs n additions and n multiplications.
- Each addition/multiplication costs ω
- The parallel time is approximately $\frac{n}{p} \times 2n\omega$

How is initial data distributed?

Assume initially matrix A and vector x are distributed evenly among processes

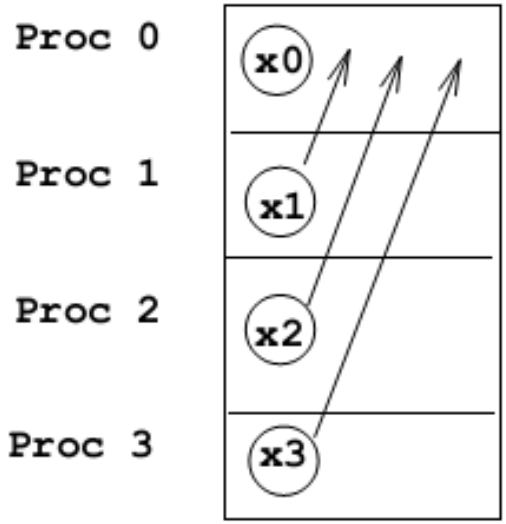


Need to redistribute vector *x* to everybody in order to perform parallel computation!

What MPI collective communication is needed?

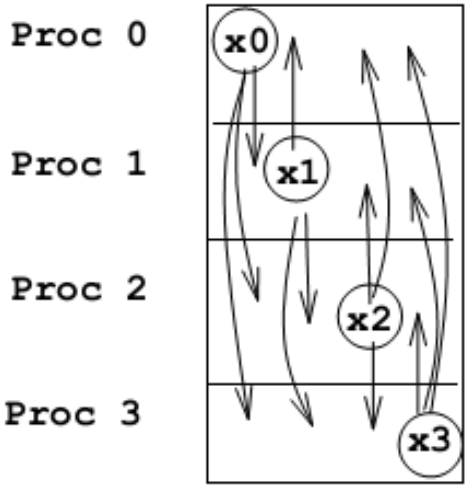
Communication Pattern for Data Redistribution

Data requirement for Process 0



MPI_Gather

Data requirement for all processes



MPI_Allgather

MPI Code for Gathering Data

Data gather for Process 0

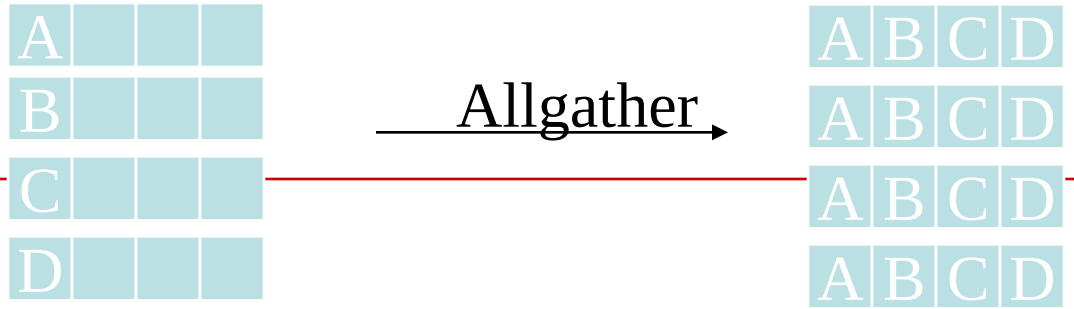
```
float local_x[]; /*local storage for x*/
float global_x[]; /*storage for all of x*/

MPI_Gather(local_x, n/p, MPI_FLOAT,
           global_x, n/p, MPI_FLOAT,
           0, MPI_COMM_WORLD);
```

Repeat for all processes

It is the same as:

```
MPI_All_gather(local_x, n/p, MPI_FLOAT,
               global_x, n/p, MPI_FLOAT,
               MPI_COMM_WORLD);
```

Allgather

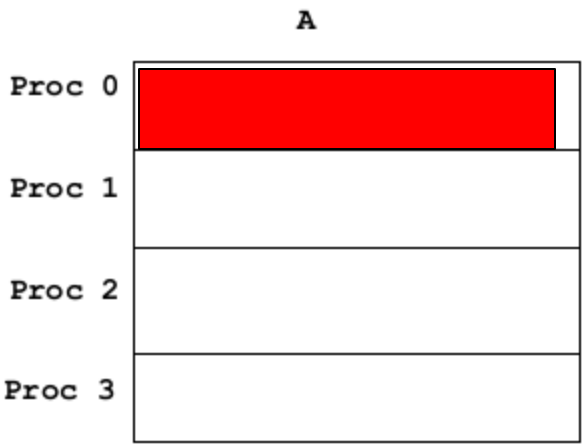


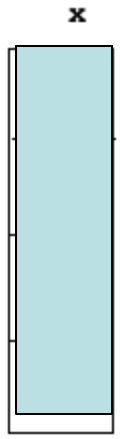
- **Concatenates the contents of each process' send_buf_p and stores this in each process' recv_buf_p.**
- **As usual, recv_count is the amount of data being received from each process.**

```
int MPI_Allgather(
      void*         send_buf_p  /* in  */,
      int           send_count  /* in  */,
      MPI_Datatype  send_type   /* in  */,
      void*         recv_buf_p  /* out */,
      int           recv_count  /* in  */,
      MPI_Datatype  recv_type   /* in  */,
      MPI_Comm      comm        /* in  */);
```

MPI SPMD Code for y=A*x

```
void Parallel_matrix_vector_prod(
        LOCAL_MATRIX_T  local_A
        int             m
        int             n
        float           local_x[]
        float           global_x[]
        float           local_y[]
        int             local_m
        int             local_n) {
 /* local_m = n/p, local_n = n/p */
 MPI_Allgather(local_x, local_n, MPI_FLOAT
               global_x, local_n, MPI_FLOAT,
                 MPI_COMM_WORLD);
```





MPI SPMD Code for y=A*x

```
    for (i = 0; i < local_m; i++) {
         local_y[i] = 0.0;
         for (j = 0; j < n; j++)
           local_y[i] = local_y[i] +
                local_A[i][j]*global_x[j];
     }
}
```

Text book solution for y=A*x. Page 114

$A = (a_{ij})$ is an $m \times n$ matrix

x is a vector with n components

a_{00}	a_{01}	$\cdots$	$a_{0,n-1}$
a_{10}	a_{11}	$\cdots$	$a_{1,n-1}$
$\vdots$	$\vdots$		$\vdots$
a_{i0}	a_{i1}	$\cdots$	$a_{i,n-1}$
$\vdots$	$\vdots$		$\vdots$
$a_{m-1,0}$	$a_{m-1,1}$	$\cdots$	$a_{m-1,n-1}$

x_0
x_1
$\vdots$
x_{n-1}

$=$

y_0
y_1
$\vdots$
$y_i = a_{i0}x_0 + a_{i1}x_1 + \cdots a_{i,n-1}x_{n-1}$
$\vdots$
y_{m-1}

$$y_i = a_{i0}x_0 + a_{i1}x_1 + a_{i2}x_2 + \cdots a_{i,n-1}x_{n-1}$$

i-th component of y

Dot product of the ith row of A with x.

Use one dimensional C array to represent 2D matrix

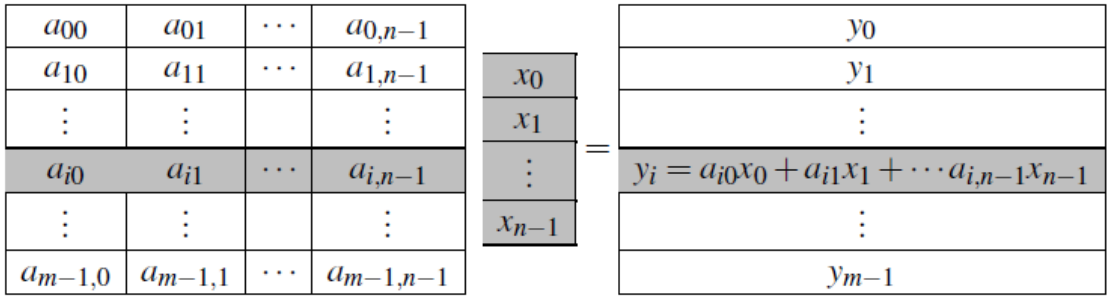
$A = (a_{ij})$ is an $m \times n$ matrix

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{pmatrix}$$

stored as

0 1 2 3 4 5 6 7 8 9 10 11

Sequential code for y=A*x



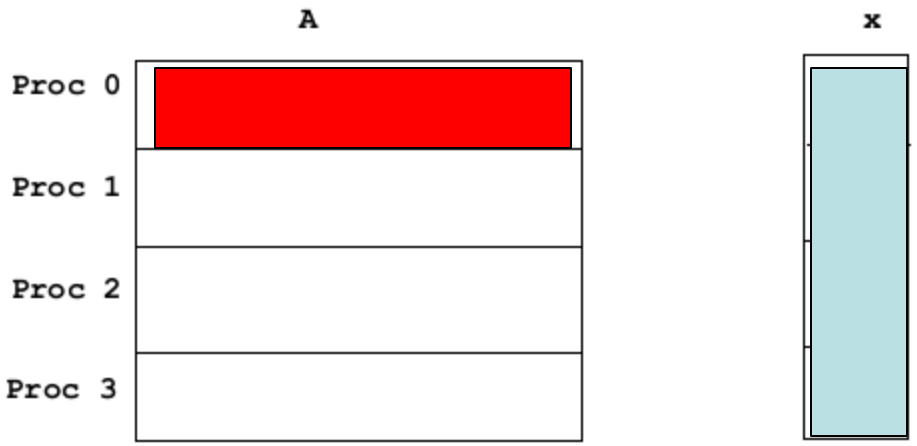

```
void Mat_vect_mult(
        double  A[]   /* in  */,
        double  x[]   /* in  */,
        double  y[]   /* out */,
        int     m     /* in  */,
        int     n     /* in  */) {
    int i, j;

    for (i = 0; i < m; i++) {
        y[i] = 0.0;
        for (j = 0; j < n; j++)
            y[i] += A[i*n+j]*x[j];
    }
}  /* Mat_vect_mult */
```

Textbook MPI code for matrix-vector multiplication

```
void Mat_vect_mult(
      double    local_A[]  /* in  */,
      double    local_x[]  /* in  */,
      double    local_y[]  /* out */,
      int       local_m    /* in  */,
      int       n          /* in  */,
      int       local_n    /* in  */,
      MPI_Comm  comm       /* in  */) {
   double* x;
   int local_i, j;
   int local_ok = 1;
```

Textbook MPI code for y=A*x

```
x = malloc(n*sizeof(double));
MPI_Allgather(local_x, local_n, MPI_DOUBLE,
      x, local_n, MPI_DOUBLE, comm);

for (local_i = 0; local_i < local_m; local_i++) {
   local_y[local_i] = 0.0;
   for (j = 0; j < n; j++)
      local_y[local_i] += local_A[local_i*n+j]*x[j];
}
free(x);
}
```



Performance Evaluation of Matrix Vector Multiplication

How to measure elapsed parallel time

- **Use MPI_Wtime() that returns the number of seconds that have elapsed since some time in the past.**

```
double MPI_Wtime(void);
```

```
double start, finish;
. . .
start = MPI_Wtime();
/* Code to be timed */
. . .
finish = MPI_Wtime();
printf("Proc %d > Elapsed time = %e seconds\n"
       my_rank, finish-start);
```

Measure elapsed sequential time in Linux

- This code works for Linux without using MPI functions
- Use GET_TIME() which returns time in microseconds elapsed from some point in the past.

- Sample code for GET_TIME()

```
#include <sys/time.h>
/* The argument now should be a double (not a pointer to a
   double) */
#define GET_TIME(now) {
      struct timeval t;
      gettimeofday(&t, NULL);
     now = t.tv_sec + t.tv_usec/1000000.0;
}
```

Measure elapsed sequential time

```
#include "timer.h"
. . .
double start, finish;
. . .
GET_TIME(start);
/* Code to be timed */
. . .
GET_TIME(finish);
printf("Elapsed time = %e seconds\n", finish-start);
```

Use MPI_Barrier() before time measurement

Start timing until every process in the communicator has reached the same time stamp

```
double local_start, local_finish, local_elapsed, elapsed;
. . .
MPI_Barrier(comm);
local_start = MPI_Wtime();
/* Code to be timed */
. . .
local_finish = MPI_Wtime();
local_elapsed = local_finish - local_start;
MPI_Reduce(&local_elapsed, &elapsed, 1, MPI_DOUBLE,
    MPI_MAX, 0, comm);

if (my_rank == 0)
    printf("Elapsed time = %e seconds\n", elapsed);
```

Run-times of serial and parallel matrix-vector multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	4.1	16.0	64.0	270	1100
2	2.3	8.5	33.0	140	560
4	2.0	5.1	18.0	70	280
8	1.7	3.3	9.8	36	140
16	1.7	2.6	5.9	19	71

(Seconds)

Speedup and Efficiency

$$S(n,p) = \frac{T_{\text{serial}}(n)}{T_{\text{parallel}}(n,p)}$$

$$E(n,p) = \frac{S(n,p)}{p} = \frac{T_{\text{serial}}(n)}{p \times T_{\text{parallel}}(n,p)}$$

Speedups of Parallel Matrix-Vector Multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.0	1.0	1.0	1.0	1.0
2	1.8	1.9	1.9	1.9	2.0
4	2.1	3.1	3.6	3.9	3.9
8	2.4	4.8	6.5	7.5	7.9
16	2.4	6.2	10.8	14.2	15.5

Efficiencies of Parallel Matrix-Vector Multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.00	1.00	1.00	1.00	1.00
2	0.89	0.94	0.97	0.96	0.98
4	0.51	0.78	0.89	0.96	0.98
8	0.30	0.61	0.82	0.94	0.98
16	0.15	0.39	0.68	0.89	0.97

Scalability

- A program is scalable if the problem size can be increased at a rate so that the efficiency doesn't decrease as the number of processes increase.

- Programs that can maintain a constant efficiency without increasing the problem size are sometimes said to be strongly scalable.

- Programs that can maintain a constant efficiency if the problem size increases at the same rate as the number of processes are sometimes said to be weakly scalable.

-

A PARALLEL SORTING ALGORITHM

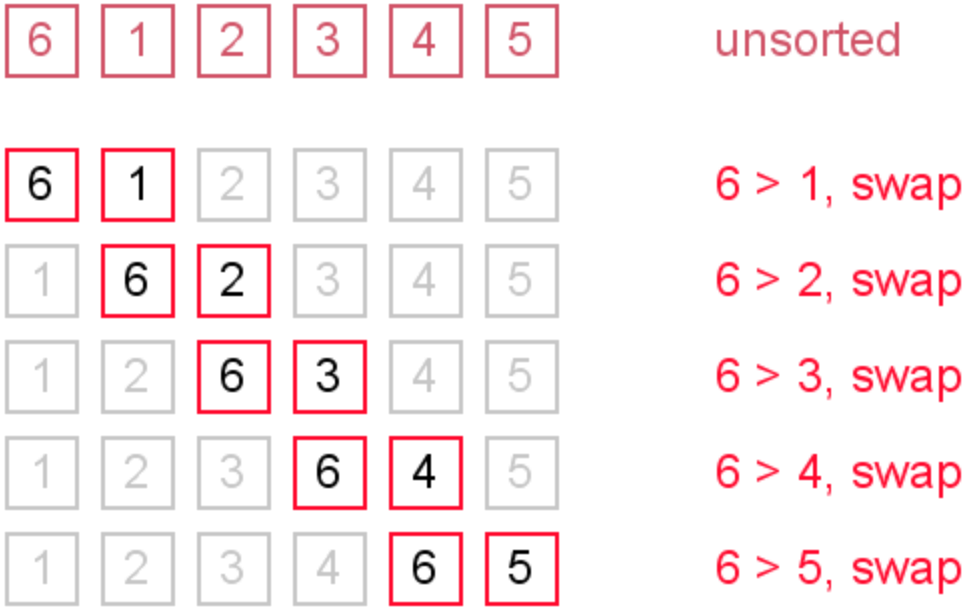
Textbook p. 127-136

Sorting

- **n keys and p = # of processes.**
- **n/p keys assigned to each process.**
- **When the algorithm terminates:**
  - The keys assigned to each process should be sorted in (say) increasing order.
  - If $0 \leq q < r < p$, then each key assigned to process q should be less than or equal to every key assigned to process r.

Simple bubble sort with pairwise swaps

```
void Bubble_sort(
        int  a[]  /* in/out */,
        int  n    /* in     */) {
    int list_length, i, temp;

    for (list_length = n; list_length >= 2; list_length--)
        for (i = 0; i < list_length-1; i++)
            if (a[i] > a[i+1]) {
                temp = a[i];
                a[i] = a[i+1];
                a[i+1] = temp;
            }

}   /* Bubble_sort */
```



Odd-even sort

- **Expose more parallelism with pairwise swaps**
  - Also called Odd-even transposition sort or brick sort.
- **Algorithm: Repeat at most n phases**
  - Even phases, compare swaps:

$$(a[0], a[1]), (a[2], a[3]), (a[4], a[5]), \ldots$$

  - Odd phases, compare swaps:

$$(a[1], a[2]), (a[3], a[4]), (a[5], a[6]), \ldots.$$

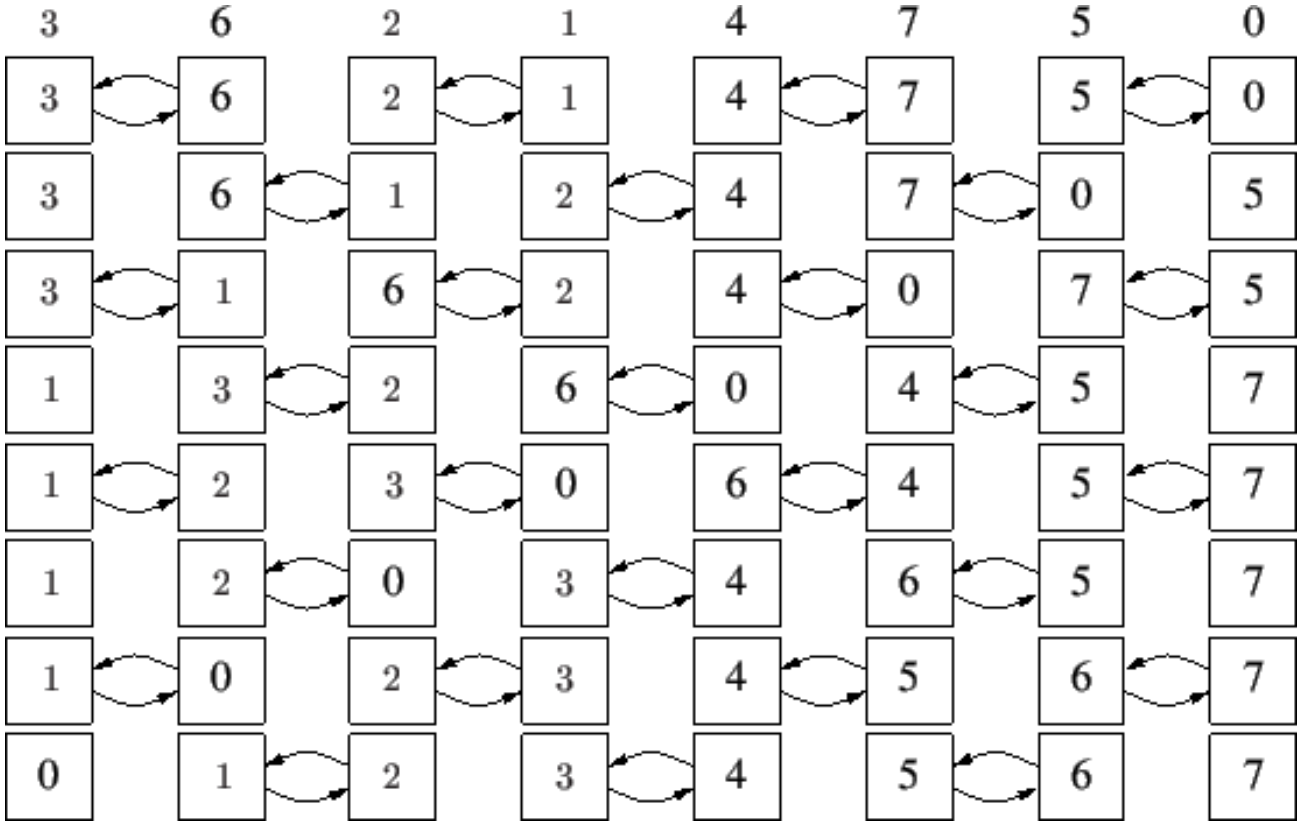
  - Complexity: best case O(n). Worst case $O(n^2)$

Example of odd-even sort

Start: 3, 6, 2, 1, 4, 7, 5, 0

Even phase: compare-swap (3,6), (2,1), (4,7), (5,0)
getting the list 3, 6, 1, 2, 4, 7, 0, 5

Odd phase: compare-swap (6,1), (2,4), (7,0)



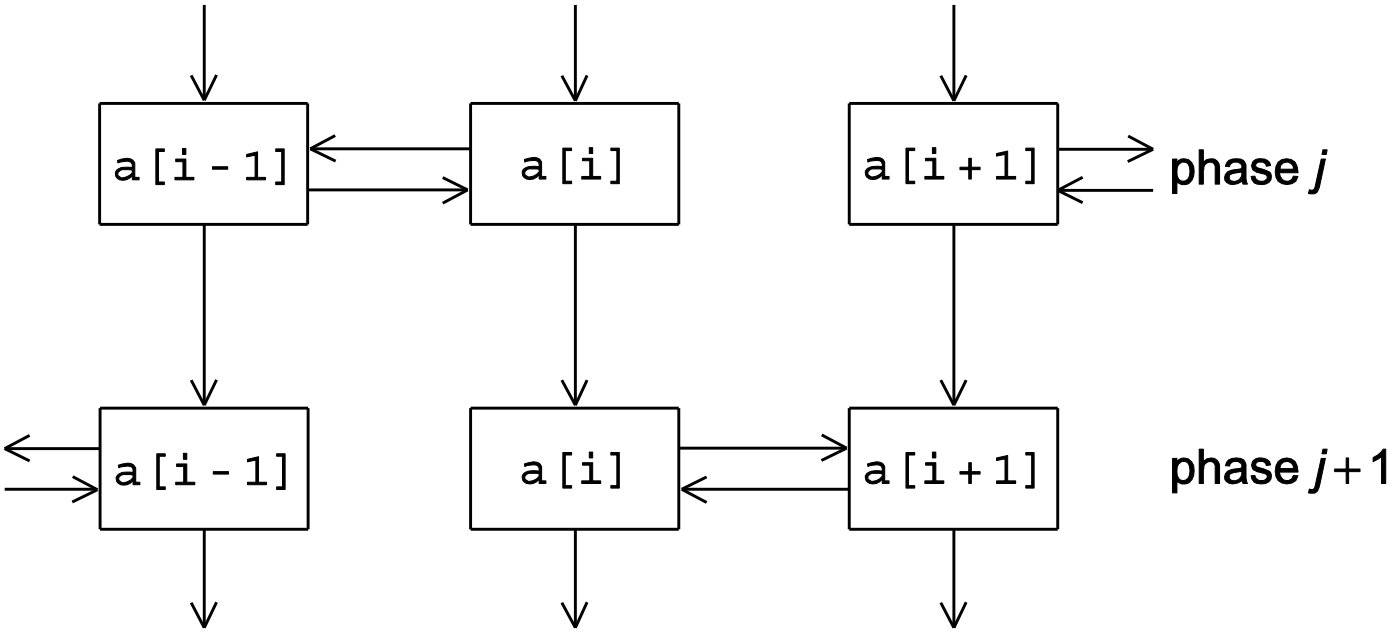
Sequential odd-even sort

```
void Odd_even_sort(
      int  a[]  /* in/out */,
      int  n    /* in     */) {
   int phase, i, temp;

   for (phase = 0; phase < n; phase++)
      if (phase % 2 == 0) { /* Even phase */
         for (i = 1; i < n; i += 2)
            if (a[i-1] > a[i]) {
               temp = a[i];
               a[i] = a[i-1];
               a[i-1] = temp;
            }
      } else { /* Odd phase */
         for (i = 1; i < n-1; i += 2)
            if (a[i] > a[i+1]) {
               temp = a[i];
               a[i] = a[i+1];
               a[i+1] = temp;
            }
      }
}  /* Odd_even_sort */
```

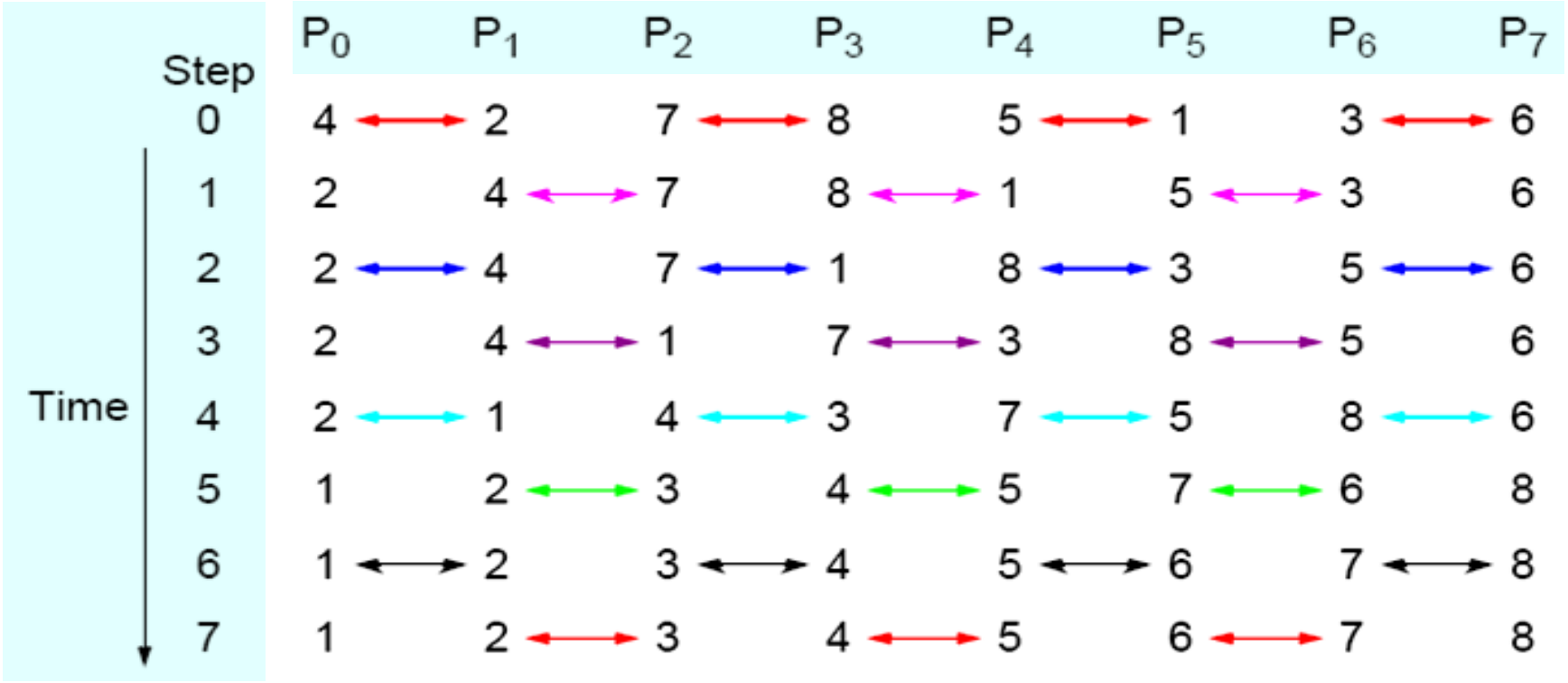
Communications among tasks in odd-even sort

Computation contains a set of tasks

Each task handling a[i] is labeled with “”a[i]”.

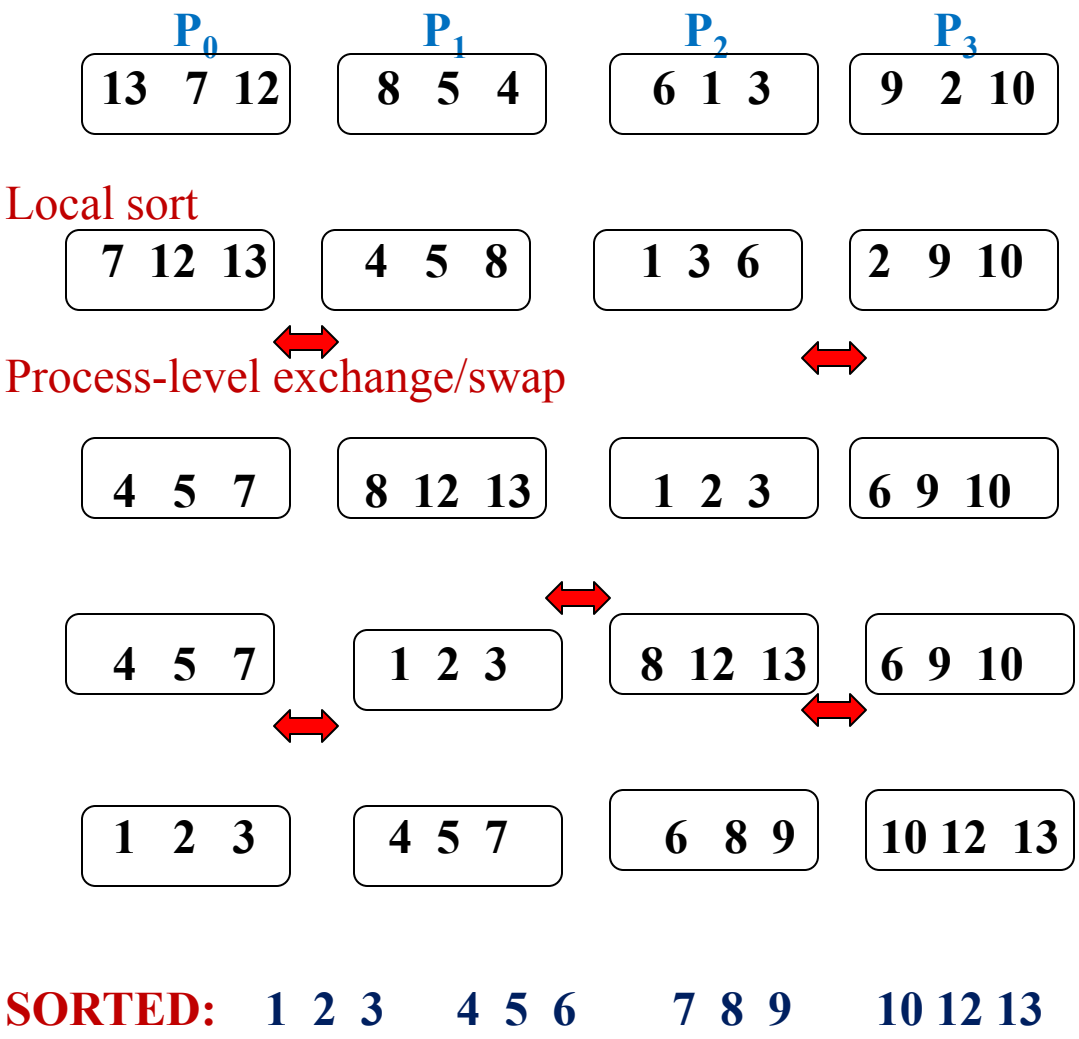


Parallel odd-even sort for n keys and p processes (n=p)



- **Weakness**
  - When n>>p, modify to let each process handle n/p keys
  - Too much communication overhead with key-level fine-grain data exchange/swap

Parallel odd-even sort for n keys and p processes (n >> p)



SORTED: 1 2 3 4 5 6 7 8 9 10 12 13

Parallel odd-even sort of n keys with p processes

- **Each process owns n/p keys.**
- **First each process sorts its keys locally in parallel.**
  - E.g. call C library qsort for quick sorting
- **Repeat at most p phases**
  - Even phases, process with even ID exchanges data with odd ID and swaps keys
    - (P0, P1), (P2, P3), (P4, P5) …
  - Odd phases, compare swaps:
    - (P1, P2), (P3, P4), (P5, P6) …

Textbook example of parallel odd-even sort

Time	Process			
	0	1	2	3
Start	15, 11, 9, 16	3, 14, 8, 7	4, 6, 12, 10	5, 2, 13, 1
After Local Sort	9, 11, 15, 16	3, 7, 8, 14	4, 6, 10, 12	1, 2, 5, 13
After Phase 0	3, 7, 8, 9	11, 14, 15, 16	1, 2, 4, 5	6, 10, 12, 13
After Phase 1	3, 7, 8, 9	1, 2, 4, 5	11, 14, 15, 16	6, 10, 12, 13
After Phase 2	1, 2, 3, 4	5, 7, 8, 9	6, 10, 11, 12	13, 14, 15, 16
After Phase 3	1, 2, 3, 4	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16

Parallel time of odd-even sort

- **Total cost**
  - Local sorting using the best algorithm.
  - At most p phases
    - Neighbor process data exchanges of n/p keys
    - Merge and split two n/p key lists
- **T_{par} = (local sort) + (p data exchanges) + (p merges/splits)**

  = O((n/p)log(n/p)) + p*O(n/p) + p*O(n/p)

  = O((n/p)log(n/p)) + O(2n)

Pseudo-code

Comm_sz= # of processes

```
Sort local keys;
for (phase = 0; phase < comm_sz; phase++) {
   partner = Compute_partner(phase, my_rank);
   if (I'm not idle) {
      Send my keys to partner;
      Receive keys from partner;
      if (my_rank < partner)
         Keep smaller keys;
      else
         Keep larger keys;
   }
}
```

Compute_partner(phase,my_rank)

```
if (phase % 2 == 0)          /* Even phase */
   if (my_rank % 2 != 0)        /* Odd rank */
      partner = my_rank - 1;
   else                         /* Even rank */
      partner = my_rank + 1;
else                         /* Odd phase */
   if (my_rank % 2 != 0)        /* Odd rank */
      partner = my_rank + 1;
   else                         /* Even rank */
      partner = my_rank - 1;
if (partner == -1 || partner == comm_sz)
   partner = MPI_PROC_NULL;
```

Merge/split in parallel odd-even sort

```
void Merge_low(
      int  my_keys[],      /* in/out     */
      int  recv_keys[],    /* in         */
      int  temp_keys[],    /* scratch    */
      int  local_n         /* = n/p, in  */) {
   int m_i, r_i, t_i;

   m_i = r_i = t_i = 0;
   while (t_i < local_n) {
      if (my_keys[m_i] <= recv_keys[r_i]) {
         temp_keys[t_i] = my_keys[m_i];
         t_i++; m_i++;
      } else {
         temp_keys[t_i] = recv_keys[r_i];
         t_i++; r_i++;
      }
   }

   for (m_i = 0; m_i < local_n; m_i++)
      my_keys[m_i] = temp_keys[m_i];
}  /* Merge_low */
```

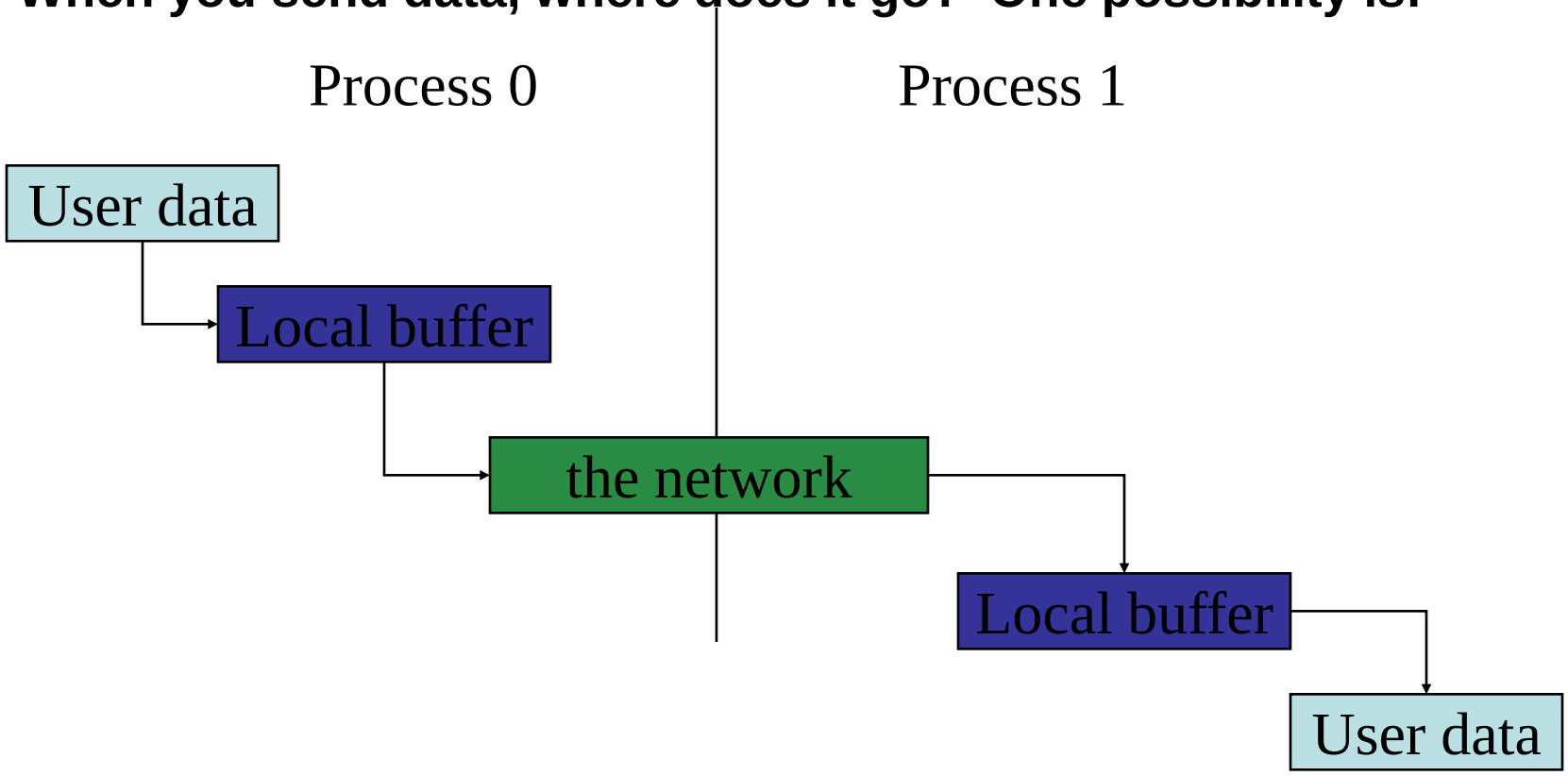
Safety Issues in MPI programs

Safety in MPI programs

- **The MPI standard allows MPI_Send to behave in two different ways:**
  - it can simply copy the message into an MPI managed buffer and return,
  - or it can block until the matching call to MPI_Recv starts.

Buffer a message implicitly during MPI_Send()

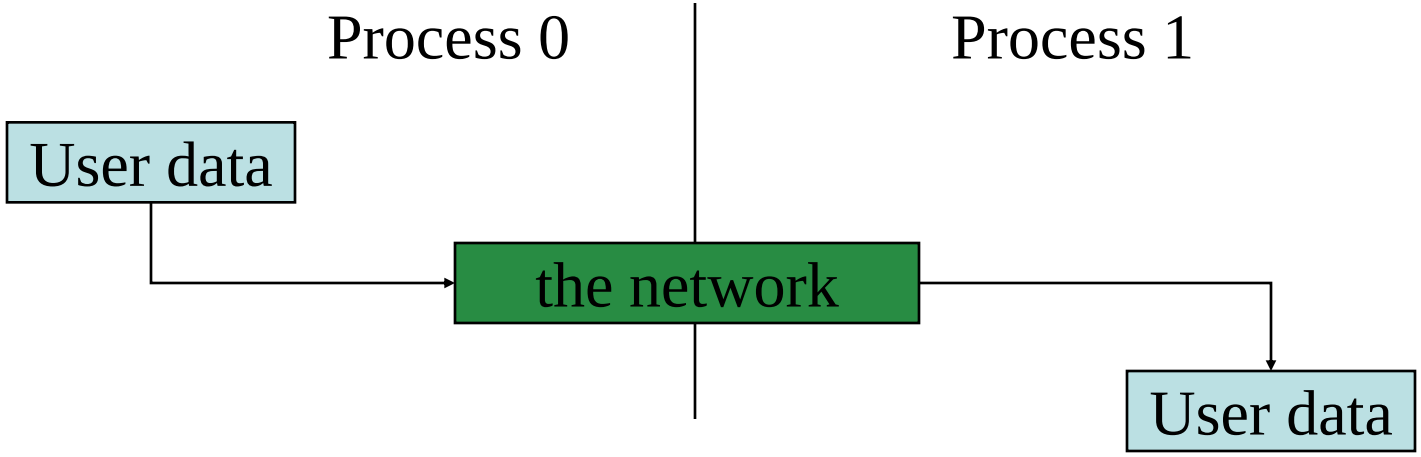
- **When you send data, where does it go? One possibility is:**



Slide source: Bill Gropp, ANL

Avoiding Buffering

- **Avoiding copies uses less memory**
- **May use more or less time**

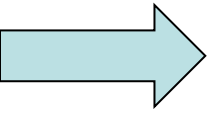


`MPI_Send`() waits until a matching receive is executed.

Slide source: Bill Gropp, ANL

Safety in MPI programs

- Many implementations of MPI set a threshold at which the system switches from buffering to blocking.
  - Relatively small messages will be buffered by MPI_Send.
  - Larger messages, will cause it to block.



If the MPI_Send() executed by each process blocks, no process will be able to start executing a call to MPI_Recv, and the program will hang or deadlock.

  - Each process is blocked waiting for an event that will never happen.

Will there be a deadlock?

- **Assume tag/process ID is assigne properly.**

Process 0	Process 1
`Send(1)`	`Send(0)`
`Recv(1)`	`Recv(0)`

Slide source: Bill Gropp, ANL

Example of unsafe MPI code with possible deadlocks

- **Send a large message from process 0 to process 1**
  - If there is insufficient storage at the destination, the send must wait for the user to provide the memory space (through a receive)
- **What happens with this code?**

Process 0	Process 1
`Send(1)`	`Send(0)`
`Recv(1)`	`Recv(0)`

- This is called "unsafe" because it depends on the availability of system buffers in which to store the data sent until it can be received

Slide source: Bill Gropp, ANL

Safety in MPI programs

- A program that relies on MPI provided buffering is said to be unsafe.
- Such a program may run without problems for various sets of input, but it may hang or crash with other sets.

How can we tell if a program is unsafe

- Replace MPI_Send() with MPI_Ssend()
- The extra “s” stands for synchronous and MPI_Ssend is guaranteed to block until the matching receive starts.
- **If the new program does not hang/crash, the original program is safe.**
- MPI_Send() and MPI_Ssend() have the same arguments

```
int MPI_Ssend(
      void*          msg_buf_p      /* in */,
      int            msg_size       /* in */,
      MPI_Datatype   msg_type       /* in */,
      int            dest           /* in */,
      int            tag            /* in */,
      MPI_Comm       communicator   /* in */);
```

Some Solutions to the "unsafe" Problem

- Order the operations more carefully:

Process 0	Process 1
`Send(1)`	`Recv(0)`
`Recv(1)`	`Send(0)`

- Simultaneous send and receive in one call

Process 0	Process 1
`Sendrecv(1)`	`Sendrecv(0)`

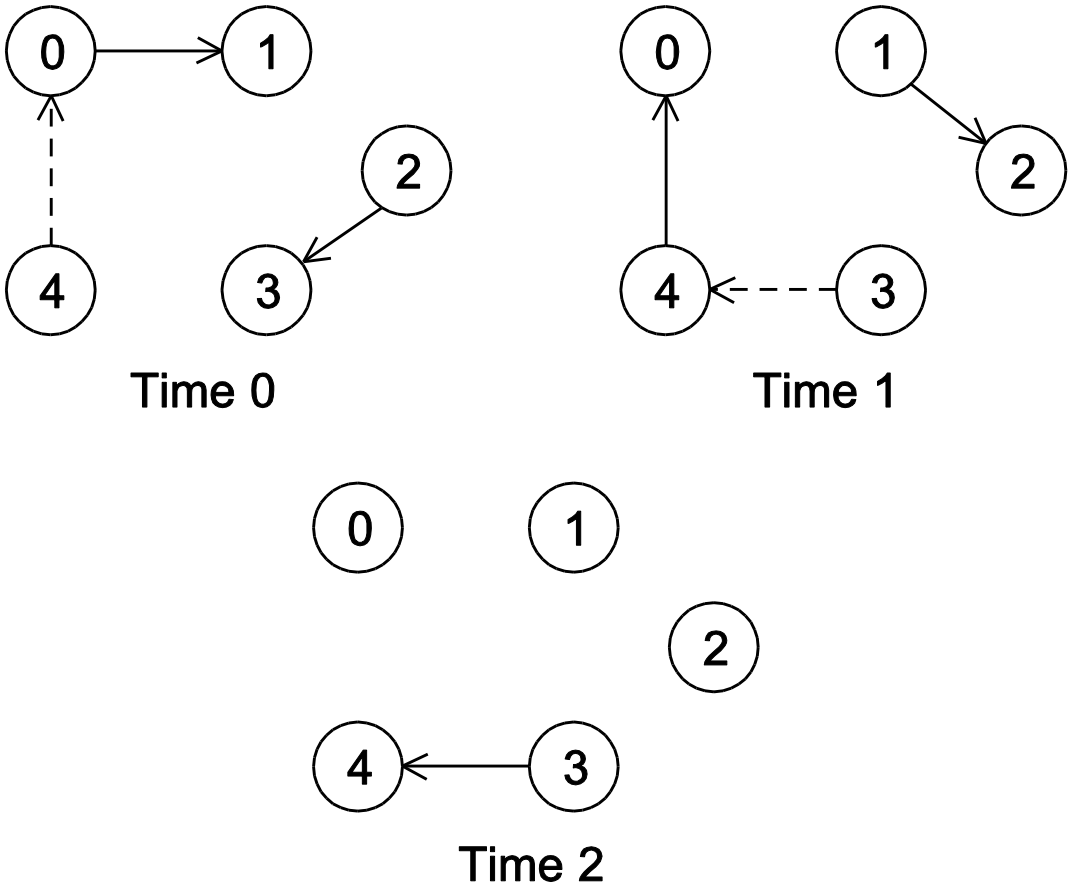
Slide source: Bill Gropp, ANL

Restructuring communication in odd-even sort

```
MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);
MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,
      0, comm, MPI_STATUS_IGNORE.
```

```
if (my_rank % 2 == 0) {
   MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);
   MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,
         0, comm, MPI_STATUS_IGNORE.
} else {
   MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,
         0, comm, MPI_STATUS_IGNORE.
   MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);
}
```

Uncertainty with five processes



Use MPI_Sendrecv()

to conduct a blocking send and a receive in a single call.

```
int MPI_Sendrecv(
      void*          send_buf_p      /* in  */,
      int            send_buf_size   /* in  */,
      MPI_Datatype   send_buf_type   /* in  */,
      int            dest            /* in  */,
      int            send_tag        /* in  */,
      void*          recv_buf_p      /* out */,
      int            recv_buf_size   /* in  */,
      MPI_Datatype   recv_buf_type   /* in  */,
      int            source          /* in  */,
      int            recv_tag        /* in  */,
      MPI_Comm       communicator    /* in  */,
      MPI_Status*    status_p        /* in  */);
```

Use MPI_Sendrecv() in odd-even sort

- An alternative to scheduling determinstic communications
  - The dest and the source can be the same or different.
  - Send and receive datatypes may be different
  - Can use Sendrecv with plain Send or Recv (or Irecv or Ssend_init, …)
- Ensure safer communication behavior so that the program won't hang or crash.

```
MPI_Sendrecv(
mykeys, n/comm_sz, MPI_INT, partner,0,
recvkeys,n/comm_sz, MPI_INT, partner, 0,
comm, MPI_Status_ignore)
```

More Solutions to the "unsafe" Problem

- Supply own space as buffer for send

Process 0	Process 1
`Bsend(1)`	`Bsend(0)`
`Recv(1)`	`Recv(0)`

- Use non-blocking operations:

Process 0	Process 1
`Isend(1)`	`Isend(0)`
`Irecv(1)`	`Irecv(0)`
`Waitall`	`Waitall`

Run-times of parallel odd-even sort

Processes	Number of Keys (in thousands)				
	200	400	800	1600	3200
1	88	190	390	830	1800
2	43	91	190	410	860
4	22	46	96	200	430
8	12	24	51	110	220
16	7.5	14	29	60	130

(times are in milliseconds)

Concluding Remarks (1)

- **MPI works in C, C++, or Fortran.**
- **A communicator is a collection of processes that can send messages to each other.**
- **Many parallel programs use the SPMD approach.**
- **Most serial programs are deterministic:** if we run the same program with the same input we'll get the same output.
  - Parallel programs often don't possess this property.
- **Collective communications involve all the processes in a communicator.**

Concluding Remarks (2)

- **Performance evaluation**
  - Use elapsed time or “wall clock time”.
  - Speedup = sequential/parallel time
  - Efficiency = Speedup/ p
  - If it’s possible to increase the problem size (n) so that the efficiency doesn’t decrease as p is increased, a parallel program is said to be scalable.
- **An MPI program is unsafe if its correct behavior depends on the fact that MPI_Send is buffering its input.**