Answers to Exercises in Quantum Computation VI

Wim van Dam
Department of Computer Science, University of California at Santa Barbara, Santa Barbara, CA 93106-5110, USA

Question 1. (Reading) —

Question 2. (Classical Impossibilities)
(a) The requirement states that $x_a \oplus y_b = ab$ for all $(a,b) \in \{0,1\}^2$ with $x_a, y_b \in \{0,1\}$. As $x \oplus y = 0$ implies $x = y$, it must hold that $x_0 = y_0$, $x_1 = y_0$ and $x_0 = y_1$, or in short $x_0 = x_1 = y_0 = y_1$. This contradicts the fourth requirement $x_1 \oplus y_1 = 1$.
(b) The assignment $(x_0, x_1, y_0, y_1) = (0,0,0,0)$ satisfies all equalities except the case $x_1 \oplus y_1 = 1$. More generally, we have the following assignments that satisfy three out of four equalities

x_0	x_1	y_0	y_1	unsatisfied equality
0	0	0	0	$x_1 \oplus y_1 = 1$
0	0	0	1	$x_0 \oplus y_1 = 0$
0	1	0	0	$x_1 \oplus y_0 = 0$
0	1	1	0	$x_0 \oplus y_0 = 0$

If A and B employ randomly one of these four assignments, each case $(a,b) \in \{0,1\}^2$ will have a success rate of 0.75.
(c) The answer of (a) shows that of the four cases always at least one will get contradicted, hence a success probability of 0.75 is optimal.

Question 3. (Towards Quantum Error Correction)
(a) If the two measurements have output “1” and “1”, then it must hold that $b = b''$ and $b, b'' \neq b'$.
(b) If the first measurement outcome is “zero”, then $b = b'$ and if the outcome is “one” then $b \neq b'$. If the second measurement outcome is “zero”, then $b' = b''$ and if the outcome is “one” then $b' \neq b''$.
(c) By the previous answer, for each of the four possible measurement outcomes there are two corresponding bit strings $(b, b', b'') \in \{0,1\}^3$. Hence, a general superposition of these two strings will be unaffected by the measurement of the two outcome bits. These ‘stable state’ superpositions are as follows (with $\alpha, \beta \in \mathbb{C}$):

1st outcome	2nd outcome	stable state
0	0	$\alpha\|000\rangle + \beta\|111\rangle$
0	1	$\alpha\|001\rangle + \beta\|110\rangle$
1	0	$\alpha\|100\rangle + \beta\|011\rangle$
1	1	$\alpha\|010\rangle + \beta\|101\rangle$