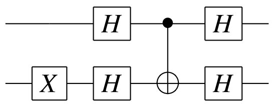
Exercises in Quantum Computation I

Wim van Dam
Department of Computer Science, University of California at Santa Barbara, Santa Barbara, CA 93106-5110, USA

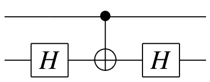
Exercises for the course "Quantum Computation and Quantum Information" (290A), Spring 2005. v1

Question 1 (Analyzing Small Circuits) *Consider the following small circuits. Describe their behavior by analyzing their effect on classical basis states $\{0,1\}^n$. Next, calculate the (unitary) $2^n \times 2^n$ matrices of these circuits. Compare your these answers for consistency.*
(a)



(1)

(b)



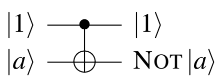
(2)

Question 2 (Designing Small Circuits)
(a) *Construct a two qubit SWAP gate defined by $|a,b\rangle \mapsto |b,a\rangle$ for all $a,b \in \{0,1\}$, using only CNOT gates and Hadamard gates.*
(b) *Create the Controlled-Phase Rotation*

$$C\text{-}R_z(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix} \quad (3)$$

for a given θ, using only CNOTs and single qubit Phase Rotations $R_z(\alpha) = \left(\begin{smallmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{smallmatrix}\right)$. You are allowed to use several gates $R_z(\alpha)$ with diferent angles α.

Question 3 (Controlled NOTs) *Obviously one can implement a NOT gate (on $a \in \{0,1\}$) with a CNOT gate using the following circuit*



(4)

On the other hand, it is impossible to implement a CNOT gate, using only NOT gates.
(a) *Is it possible to implement a CCNOT gate using only CNOT gates?*
(b) *Is it possible to implement a CCCNOT using only CCNOT gates?*

Try to prove the impossibility, or give an example of the implementation.

Question 4 (Unitarity) *Let T be a linear transformation on $\mathbb{C}^N$ such that $T|\psi\rangle = T|\phi\rangle = |0,\ldots,0\rangle$ with $|\psi\rangle \neq |\phi\rangle$. Prove that T is not norm-preserving.*

Acknowledgement: The circuits in these exercises were drawn using the Q-circuit LaTeXpackage of Bryan Eastin and Steven T. Flammia.