Exercises in Quantum Computation III

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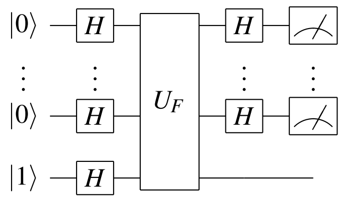
Question 1. (Deutsch-Jozsa on General Functions) Take the Boolean function $F : \{0,1\}^n \to \{0,1\}$ with the unitary implementation $U_F : |x,b\rangle \mapsto |x, b \oplus F(x)\rangle$ for all $x \in \{0,1\}^n$ and $b \in \{0,1\}$. The Deutsch-Jozsa algorithm uses the following $n+1$ qubit circuit:



where one measures the first n bits in the computational basis.

(a) Read Sections 1.4.2–1.4.4 ("Quantum parallelism", "Deutsch's algorithm", "The Deutsch-Jozsa algorithm") in Nielsen and Chuang's *Quantum Computation and Quantum Information.*
(b) Analyze how F determines the probability of observing the all zeros state $|0\rangle \otimes \cdots \otimes |0\rangle$ for the observed n qubits.

Question 2. (Preparation for Midterm) Read the following Sections in Nielsen and Chuang's *Quantum Computation and Quantum Information:* 1–1.4.4 (ignore the Bloch sphere representation and other material that we did not cover).

Question 3. (One-Out-of-Four) Define the unitary transformation that only flips a bit if is has the right two bit input $(s,r) \in \{0,1\}^2$:

$$U_{s,r}\,|x,y,b\rangle \mapsto \begin{cases} |x,y,b\oplus 1\rangle & \text{if } (x,y) = (s,r) \\ |x,y,b\rangle & \text{if } (x,y) \neq (s,r) \end{cases}$$

Consider the following sequence of operations on a 3 qubit input state $|0,0,1\rangle$:

1) Apply $H \otimes H \otimes H$
2) Apply $U_{s,r}$ (for unknown s,r)
3) Apply $H \otimes H \otimes I$
4) Apply $U_{0,0}$
5) Apply $H \otimes H \otimes H$
6) Measure the 3 qubits in the computational basis.

(a) How will the output of the measurement depend on the two unknown bits $(s,r) \in \{0,1\}^2$?
(b) Contemplate what this quantum circuit is useful for.

Question 4. (Summing Phases) Let ζ_n denote the complex number $\mathrm{e}^{2\pi\mathrm{i}/n}$ such that $\zeta_n^n = 1$.
(a) What is the value $\sum_{j=0}^{n-1} \zeta_n^j$?
(b) Let n have the decomposition $n = pq$, what is $\sum_{j=0}^{p-1} \zeta_n^{qj}$?
(c) What is $\sum_{j=0}^{m-1} \zeta_n^j$?