Exercises in Quantum Computation V

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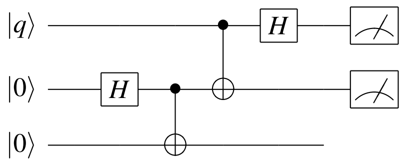
Question 1. (Reading) Read Sections 12.6, 12.6.1 and 12.6.3 in Nielsen and Chuang's *Quantum Computation and Quantum Information.*

Question 2. (Towards Teleportation) (See Handout III if you have problems answering this question.) Consider the following three qubit circuit that has as input an unknown qubit $|q\rangle$ and two zero states:



(a) With $|q\rangle = \alpha|0\rangle + \beta|1\rangle$, what is the output state before the measurements?
(b) What is the probability of measuring the outcome "00" on the first two qubits?
(c) What is the quantum state of the third qubit after the outcome "00" has been measured?
(d) Answer the previous two questions for the other possible outcomes "01", "10" and "11".

Question 3. (Rewriting Entanglement) In class we noted that $|\text{EPR}\rangle = (|00\rangle + |11\rangle)/\sqrt{2} = (|{+}{+}\rangle + |{-}{-}\rangle)/\sqrt{2}$. We want to understand for which other qubits $|q\rangle$ (with orthogonal state $|q^\perp\rangle$) we have $|\text{EPR}\rangle = (|q,q\rangle + |q^\perp, q^\perp\rangle)/\sqrt{2}$.
(a) For $|q\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$, is there an orthogonal qubit state $|q^\perp\rangle$ such that $|\text{EPR}\rangle = (|q,q\rangle + |q^\perp, q^\perp\rangle)/\sqrt{2}$? Prove it.
(b) Same question as above, now for the qubit state $|q\rangle = (|0\rangle + \mathrm{i}|1\rangle)/\sqrt{2}$.
(c) For which states $|q\rangle = \alpha|0\rangle + \beta|1\rangle$ can we have $|q^\perp\rangle$ such that $|\text{EPR}\rangle = (|q,q\rangle + |q^\perp, q^\perp\rangle)/\sqrt{2}$?
(d) For general $|q\rangle = \alpha|0\rangle + \beta|1\rangle$ and its orthogonal dual $|q^\perp\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$ define a qubit state $|s\rangle$ and its orthogonal dual $|s^\perp\rangle$ such that $|\text{EPR}\rangle = (|q,s\rangle + |q^\perp, s^\perp\rangle)/\sqrt{2}$.