Classical Mechanics, Phys105A, Wim van Dam, UC Santa Barbara
Exercises Week 1; due Monday January 22, 11:30 am

Question 1 (Scalar products, vector products, and their meaning, 5+5 points).

▷ **(a)** Prove the following equality: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$.

▷ **(b)** Let the above vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ define a three dimensional *parallelepiped* with vertices $\pm\mathbf{a}\pm\mathbf{b}\pm\mathbf{c}$ (where each $\pm\pm\pm$ combination defines one of the eight vertices). With this in mind, give a geometric interpretation of the above equality, including an interpretation of the vector products $\mathbf{b}\times\mathbf{c}$ and $\mathbf{c}\times\mathbf{a}$.

Question 2 (Basis independence of vector product, 10 points).

▷ **(a)** Prove that the vector product $\mathbf{r}\times\mathbf{s}$ is independent of the basis in which you express $\mathbf{r}$, $\mathbf{s}$ and $\mathbf{r}\times\mathbf{s}$ (cf. Problem 1.16 for hints how to attack a similar question for the scalar product).

Write the answers to the questions below on a separate set of pages.

Question 3 (Derivative of scalar product, 10 points).

▷ **(a)** Prove that $\frac{d}{dt}(\mathbf{r}\cdot\mathbf{s}) = \mathbf{r}\cdot\frac{d\mathbf{s}}{dt} + \frac{d\mathbf{r}}{dt}\cdot\mathbf{s}$.

Question 4 (Some differential equations, 5+5+5 points). Find the time dependent position $\mathbf{x}$ and velocity $\dot{\mathbf{x}}$ of a particle of mass m and with initial conditions $\mathbf{x}(t) = \dot{\mathbf{x}}(t) = \mathbf{0}$ at time $t = 0$, subject to the following force functions

▷ **(a)** $\mathbf{F} = (F_0 + \alpha t)\hat{\mathbf{x}}$

▷ **(b)** $\mathbf{F} = (F_0 + \beta x)\hat{\mathbf{x}}$ (with $\beta \geq 0$)

▷ **(c)** $\mathbf{F} = (F_0 + \beta x)\hat{\mathbf{x}}$ (with $\beta \leq 0$)

Question 5 (A question from Taylor, 5+5+5 points).

▷ **(a)** Problem 1.36a

▷ **(b)** Problem 1.36b

▷ **(c)** Problem 1.36c