Classical Mechanics, Phys105A, Wim van Dam, UC Santa Barbara
Exercises Week 5; due *Friday February 23, 11:30 am*

Question 1 (10+10 points). A NONCONSERVATIVE FORCE
Consider a nonconservative force defined over the plane with the following (topological) property for the work done over the closed paths from 1 back to 1: $W(1 \to 1) = 0$ if the loop does not go around the origin O, $W(1 \to 1) = c$ if the loop goes around the origin O once in a clockwise fashion, $W(1 \to 1) = -c$ if the loop goes around the origin O once in an anti-clockwise fashion, and so on. In other words $W(1 \to 1)/c$ counts how many times the path went around O clockwise.

▷ **(a)** Write down a force $\mathbf{F}$ that has this property. Give arguments why your answer is correct.

▷ **(b)** Locally, in small patches that does not involve the origin, this force is conservative, and we can indeed give a local potential like function $V(r, \phi)$ with all the right properties. Yet globally no such potential should exist. What is going on here?

Write the answers to the questions below on a separate set of pages.

Question 2 (15 points). TIME OF IMPACT UNDER INVERSE QUADRATIC FORCE
We drop a particle with mass m at distance $r = d$ from the origin under the influence of a central potential $U(r) = -km/r$. Let s be the time required for the particle to reach the origin $r = 0$. As a function of m and d, it holds that $s = \gamma m^{\alpha} d^{\beta}$.

▷ **(a)** Determine these powers α and β.

Question 3 (10+15 points). ORBITS AND CENTRAL FORCES
A particle with mass m moves in the plane under influence of a central force $f(r)\hat{e}_r$. The trajectory of the particle is described by $r(t) = r_0 e^{k \cdot \phi(t)}$ where $\phi(t)$ is the time dependent angle in the polar coordinate system that we are using.

▷ **(a)** Prove that $\phi(t)$ has to change logarithmically in time t.

▷ **(b)** Prove that $f(r)$ has to depend in an inverse cube way on r.