Classical Mechanics, Phys105A, Wim van Dam, UC Santa Barbara
Homework 6; due Monday March 5, 11:30 am

Question 1 (Trigonometry, 4 points).
For fixed $\alpha, \beta, \omega \in \mathbb{R}$, let γ, δ be such that $\alpha\cos(\omega t) + \beta\sin(\omega t) = \gamma\cos(\omega t - \delta)$ for all t.

▷ **(a)** Provide a simple, elegant proof that gives the values of γ and δ in terms of α and β.

Question 2 (Taylor, Question 5.19, 8+8 points).
Consider a mass attached to four identical springs, as shown in Figure 5.7(b) in the textbook. Each spring has force constant k and unstretched length ℓ_0, and the length of each spring when the mass is at its equilibrium at the origin $(0, 0)$ is a (which is not necessarily the same as ℓ_0).

▷ **(a)** When the mass is displaced a small distance to the point (x, y), show that its potential energy has the form $1/2k'r^2$, just like an isotropic harmonic oscillator.

▷ **(b)** Show that $k' = 2k(2 - \ell_0/a)$.

Write the answers to the questions below on a separate set of pages.

Question 3 (Ratio between maxima, 8 points). Consider a weakly damped, harmonic oscillation and look at its local maxima.

▷ **(a)** Show that the ratio between two consecutive maxima in the displacement is constant.

Question 4 (Taylor, Question 5.45, 8+6+6 points).
Energy flow in damped, driven oscillation:

▷ **(a)** Taylor, Question 5.45(a)

▷ **(b)** Taylor, Question 5.45(b)

▷ **(c)** Taylor, Question 5.45(c)

Question 5 (Taylor, Question 5.26, 12 points).
What happens with just a little bit of damping?

▷ **(a)** Taylor, Question 5.26.