Classical Mechanics

Phys105A, Winter 2007

Wim van Dam
Room 5109, Harold Frank Hall
vandam@cs.ucsb.edu
http://www.cs.ucsb.edu/~vandam/

Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…

- There will no discussion session this Friday.
- Homework will be announced coming weekend.

- Formula for final score:
  40% homework + 20% midterm + 40% final.

Reading ‘Around’ 105A

- More introductory textbook: “An Introduction to Mechanics” by Kleppner and Kolenkow.
- More advanced textbook: “Classical mechanics” by Goldstein et al. (a classic that prepares well for QM).
- Paving the way for relativity theory: “Science of Mechanics”, Ernst Mach.
- Always a good idea: anything by Feynman.

Answers to Exercises

- Exercise: Prove that $\mathbf{r}\times\mathbf{s}$ is orthogonal to **r** and **s**.
- Proof: Evaluate the dot product $\mathbf{r}\cdot(\mathbf{r}\times\mathbf{s})$ in terms of r_x,r_y,r_z,s_x,s_y,s_z and observe that it is always zero.

- Problem 1.10: Given $\mathbf{r}(t) = R\cos(\omega t)\mathbf{e}_x + R\sin(\omega t)\mathbf{e}_y$ What is the speed and acceleration of this particle?
- Answer: Because $d(\cos\omega t)/dt = -\omega\sin\omega t$ and so on, we get $\mathbf{v}(t) = -R\omega\sin(\omega t)\mathbf{e}_x + R\omega\cos(\omega t)\mathbf{e}_y$ and $\mathbf{a}(t) = -R\omega^2\cos(\omega t)\mathbf{e}_x - R\omega^2\sin(\omega t)\mathbf{e}_y$.

Continuing Chapter 1

- From now on we are comfortable dealing with vector calculus and differential equations.
- Let’s do some physics now: how do space, time, position, velocity, acceleration, mass and force relate?
- First: how to measure these quantities?
- Given a reference frame (= orthogonal basis for our 3 dimensional space, an origin and a t=0 moment), measuring position, velocity and acceleration in meters and seconds (always SI units in Phys105A!) is easy.
- What about mass and force?

Mass

- SI Unit of mass: **kilogram**
- Ultimately defined in terms of the "International prototype of the kilogram", in Sèvres in Paris, France (1880)

- "Weight" refers to the gravitational force that act on an object of certain mass, is measured by a weighing scale and depends on where on earth you measure it.
- At the moment we are concerned with *inertial mass*: the resistance of an object to change its velocity.

Force

- SI unit of force: **Newton** = kilogram×meter / second^2
- Understanding what force is, is a tricky business.
- Is **F**=m**a** a physical law, or the definition of force?
- To try to avoid this issue and get down to work, you can think of force as a way of comparing the different m**a** quantities with each other.

Newton's Laws of Motion

- First law, "Law of Inertia": In the absence of forces, a particle moves with constant velocity **v**

- Second law, "Law of acceleration": For any particle, the net force F is equal to its mass m times acceleration **a**: **F** = m**a** or **F** = d**p**/dt with **p** = m**v** the *momentum*.

- Third law, "Law of reciprocal actions": When two objects 1 and 2 interact, the forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ they exert on each other are equal and opposite: $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

(Zeroth law, "Law of preservation of mass")

Newton's 1st Law and Frames

- The Law of Inertia can be used to make a crucial distinction between reference frames.
- Each reference frame in which the law of inertia holds is an *inertial frame* (those that we think of as moving with a constant speed).
- Reference frames in which the law of inertia does not hold are *noninertial frames* (think rotating frames).

The 3rd Law and Momentum

- Given two particles 1 and 2 with no external force, we have for the change in total momentum $\mathbf{P} = \mathbf{p}_1+\mathbf{p}_2$:
  $d\mathbf{P}/dt = d\mathbf{p}_1/dt + d\mathbf{p}_2/dt = \mathbf{F}_{12} + \mathbf{F}_{21} = \mathbf{0}$.
  This means that the total momentum stays constant.

- We can generalize this to multipartite systems with total momentum $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \ldots + \mathbf{p}_N$.

  "Principle of Conservation of Momentum": If the net external force on an N patricle system is **0**, the total momentum **P** of the system remains constant.

Cartesian Coordinates

- Rewriting the law of acceleration as **F** = m d²**r**/dt² in Cartesian (x,y,z) coordinates gives (of course):

$$\mathbf{F} = m\ddot{\mathbf{r}} \quad \Leftrightarrow \quad \begin{cases} F_x = m\ddot{x} \\ F_y = m\ddot{y} \\ F_z = m\ddot{z} \end{cases}$$

- This allows us to decompose our calculations into three independent, 1 dimensional problems.

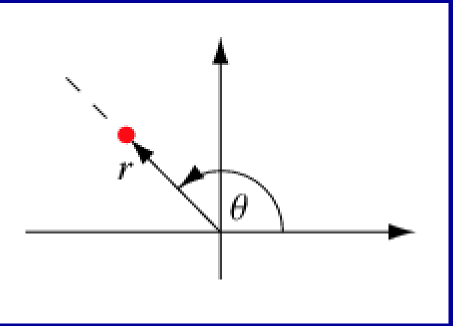
Example 1.1

- Example 1.1 “A block sliding down an incline”
- **Note:**
  - **N** is the unknown normal force of the incline preventing the block from fall through the incline.
  - The weight **w** is the weight force m**g** with **g** determined by the laws of gravity.
  - The friction **f** is a reactive force (in –x direction) when sliding down a surface, proportional to **N**, hence $f=\mu N$.

- Many systems though have non-Cartesian symmetries…

2D-Polar Coordinates

- Many 2 dimensional systems have symmetries that are best captured by *polar coordinates*:



$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} r &= \sqrt{x^2+y^2} \\ \varphi &= \arctan(y/x) \end{aligned}\right.$$

- Given a position **r**, we can define basis with unit vectors:

$$\mathbf{e}_r = \hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|} \quad \text{and} \quad \mathbf{e}_\varphi = \hat{\boldsymbol{\varphi}} \perp \hat{\mathbf{r}}$$

- Decomposing **F** into $\mathbf{e}_r$ and $\mathbf{e}_\varphi$ can have great benefits but $d\mathbf{e}_r/dt$ is no longer **0**…