

# Classical Mechanics

**Phys105A, Winter 2007**

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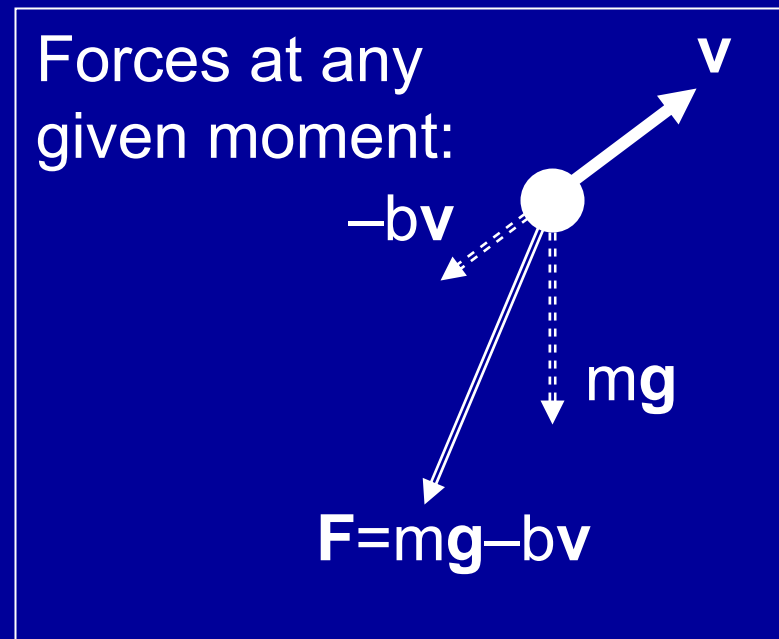
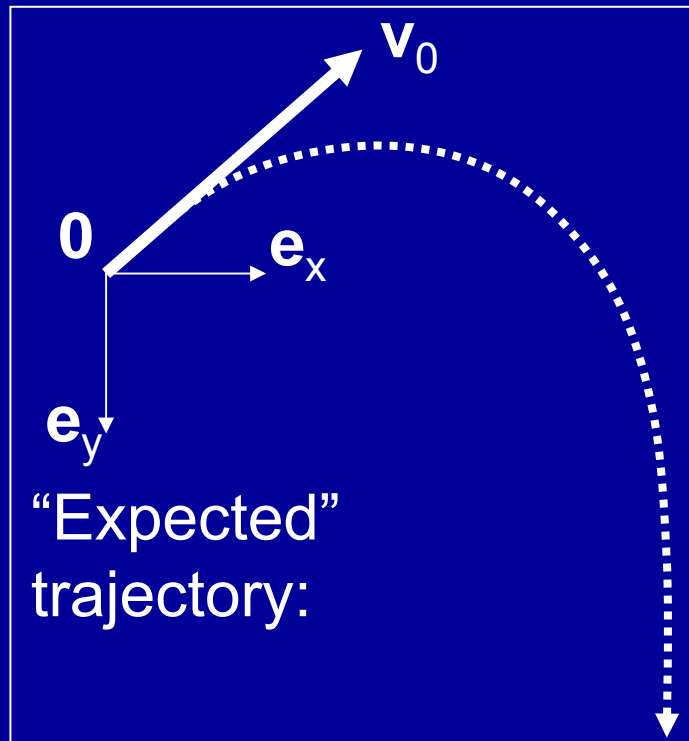
<http://www.cs.ucsb.edu/~vandam/>

# Formalities

- Latest news and course slides always found on the Phys105A site at [http://www.cs.ucsb.edu/~vandam/...](http://www.cs.ucsb.edu/~vandam/)
- Homework 1 has been posted.  
It is due Monday January 22, 11:30 am.
- You have to hand in two separate sets of answers so that the two TAs can grade the different questions.
- Slides of last Tuesday are now online for real.
- Questions?

# Linear Air Resistance

- Consider a flying projectile of mass  $m$  for which the quadratic drag force is negligible:  $\mathbf{f} = -b \mathbf{v}$ .
- Summing the forces gives  $\mathbf{F} = m\mathbf{g} - b\mathbf{v} = m \, d^2\mathbf{r}/dt^2$ .
- With  $\mathbf{r}(0) = \mathbf{0}$  and  $\mathbf{v}(0) = \mathbf{v}_0$  we expect something like:



# Differential Equations

- You may have to overcome your fear of solving differential equations with pen and paper.



Tips:

1. Use your pen
2. Use lots of empty paper
3. When in doubt, try “separation of variables”
4. Know your complex valued functions

$$F = mg - bv$$

- Take  $x_0=y_0=0$  and initial speeds  $v_{x0}$ ,  $v_{y0}$ .
- After some calculations [pp. 48–56] we get that

$$x = v_{x0} (m/b) (1 - e^{-tb/m})$$

$$y = v_{\text{ter}} t + (v_{y0} - v_{\text{ter}}) (m/b) (1 - e^{-tb/m})$$

where  $v_{\text{ter}}$  is the *terminal y-speed*  $mg/b$

Note that as  $t \rightarrow \infty$  we have

$$x_t \rightarrow x_\infty = v_{x0} m/b \text{ and } v_{yt} \rightarrow v_{y\infty} = mg/b.$$

# Quadratic Air Resistance

- Consider a flying projectile of mass  $m$  with quadratic drag force:  $\mathbf{f} = -c|\mathbf{v}| \mathbf{v}$ .
- Summing the forces gives  $\mathbf{F} = m\mathbf{g} - c|\mathbf{v}|\mathbf{v} = m \, d^2\mathbf{r}/dt^2$ .
- Unlike the linear case, the  $x$  and  $y$  motion can now not be separated: the general case 'can not be solved'.
- For  $x$ -motion with  $x_0=0$  and  $v_0$  we get:  
 $v = v_0/(1+t c v_0/m)$  and  $x = (m/c) \ln(1+t c v_0/m)$   
Note that as  $t \rightarrow \infty$  we now have  $x_\infty = \infty$ .
- For  $y$ -motion with  $y_0=0$  and  $v_0=0$  we get  
 $v = v_\infty \tanh(gt/v_\infty)$  and  $y = v_\infty^2/g \ln[\cosh(gt/v_\infty)]$   
with terminal speed  $v_\infty = \sqrt{(mg/c)}$