Classical Mechanics

Phys105A, Winter 2007

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Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…
- Homework 2 has been posted.
  It is due Monday January 29, 11:30 am.
- Midterm is tentatively scheduled for Week 6, Tuesday Feb 13, or Thursday Feb 15.
  The material will be Chapters 1–4; it is not open book, but you are allowed a ‘cheat sheet’.

Help Wanted

Note Taker

You must be sensitive to the needs of students with disabilities

$75

Stop by the Disabled Students Office at 1201 SAASB and complete an application

Chapter 3: Momentum and Angular Momentum

3.4 Angular Momentum (N=1)

For a particle with momentum **p** and position **r** (relative to the origin O), the **angular momentum ℓ** *relative to* O is defined as **ℓ** = **r**×**p** (hence **ℓ** is orthogonal to **r** and **p**).

Note that

- d**ℓ**/dt = d**r**/dt×**p** + **r**×d**p**/dt
- d**r**/dt and **p** have the same direction
- **F** = d**p**/dt

As a result we have d**ℓ**/dt = **r**×d**p**/dt = **r**×**F** = **Γ**,
with **Γ** = **r**×**F** the **net torque about O** on the particle.

3.5 Angular Momentum (N>1)

Generalizing to several particles with their angular momenta $\boldsymbol{\ell}_\alpha = \mathbf{r}_\alpha \times \mathbf{p}_\alpha$ we define the **total angular momentum** as $\mathbf{L} = \Sigma_\alpha \boldsymbol{\ell}_\alpha = \Sigma_\alpha \mathbf{r}_\alpha \times \mathbf{p}_\alpha$.

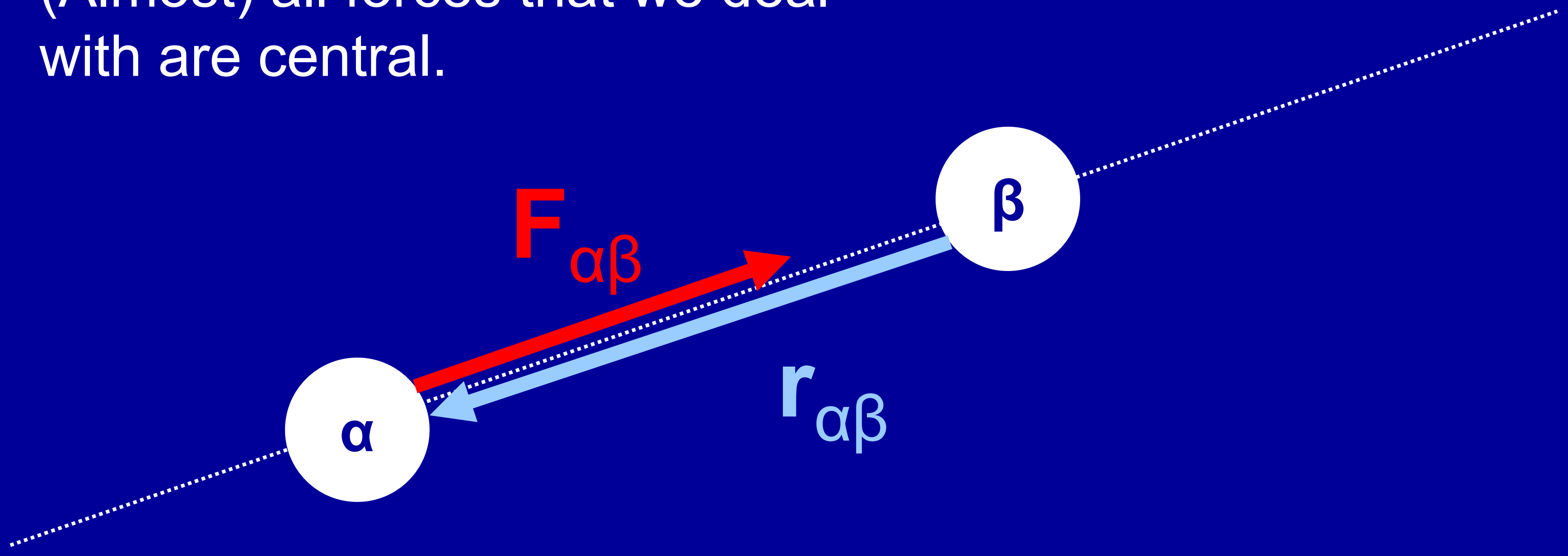
Its rate of change obeys $d\mathbf{L}/dt = \Sigma_\alpha d\boldsymbol{\ell}_\alpha/dt = \Sigma_\alpha \mathbf{r}_\alpha \times \mathbf{F}_\alpha$.

Assuming that the internal forces are *central*, i.e. that $\mathbf{F}_{\alpha\beta}$ and $\mathbf{r}_{\alpha\beta}$ 'lie on the same line', we can show that $d\mathbf{L}/dt = \boldsymbol{\Gamma}^{ext}$ with $\boldsymbol{\Gamma}^{ext}$ the **net external torque**.

"Principle of Conservation of Angular Momentum"
If for the net external torque $\boldsymbol{\Gamma}^{ext} = \mathbf{0}$, then the system's total angular momentum $\mathbf{L} = \Sigma_\alpha \mathbf{r}_\alpha \times \mathbf{p}_\alpha$ remains constant.

Central Forces

(Almost) all forces that we deal with are central.



Moment of Inertia

The **moment of inertia** of an object is to angular momentum what mass is to linear momentum.

A (rigid) object that rotates around the z-axis will have an angular momentum **L** (measured from O on the z-axis) where its z-component L_z is proportional to the angular velocity ω of the rotation: $L_z = I_z \omega$. Here I_z is the *moment of inertia* of the object for the given axis.

It can be calculated by $I_z = \Sigma_\alpha m_\alpha (r_{\alpha x}^2 + r_{\alpha y}^2)$ or the integral variant $I_z = \int_\alpha \ldots$ of this.

See Chapter 10 for more on this.

Role of the CM

The center of mass of a multiparticle system plays a special role in the theory of angular momentum.

Later (again in Chapter 10) you will see/prove the following.

- The conservation of momentum $d\mathbf{L}/dt = \mathbf{\Gamma}^{ext}$ also holds in the (accelerating, hence non inertial) frame that has CM as its origin at any given moment.
- For any origin O, we can decompose the total angular momentum $\mathbf{L}$ as $\mathbf{L}_{total} = \mathbf{R}\times\mathbf{P} + \mathbf{L}_{around\ CM}$.
- Moral lesson: the origin dependency of $\mathbf{L}$ is described completely by the $\mathbf{R}\times\mathbf{P} = \mathbf{L}_{CM}$ of the total mass M with its center of mass moving at velocity $d\mathbf{R}/dt$.