Classical Mechanics

Phys105A, Winter 2007

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Preparing for Final

- Material: all chapters 1–7, with an emphasis on 5–7.
- Set-up the same as with Midterm: no book, no electronics, 2 sided cheat sheet allowed.

- Coming Thursday: Overview and Q&A

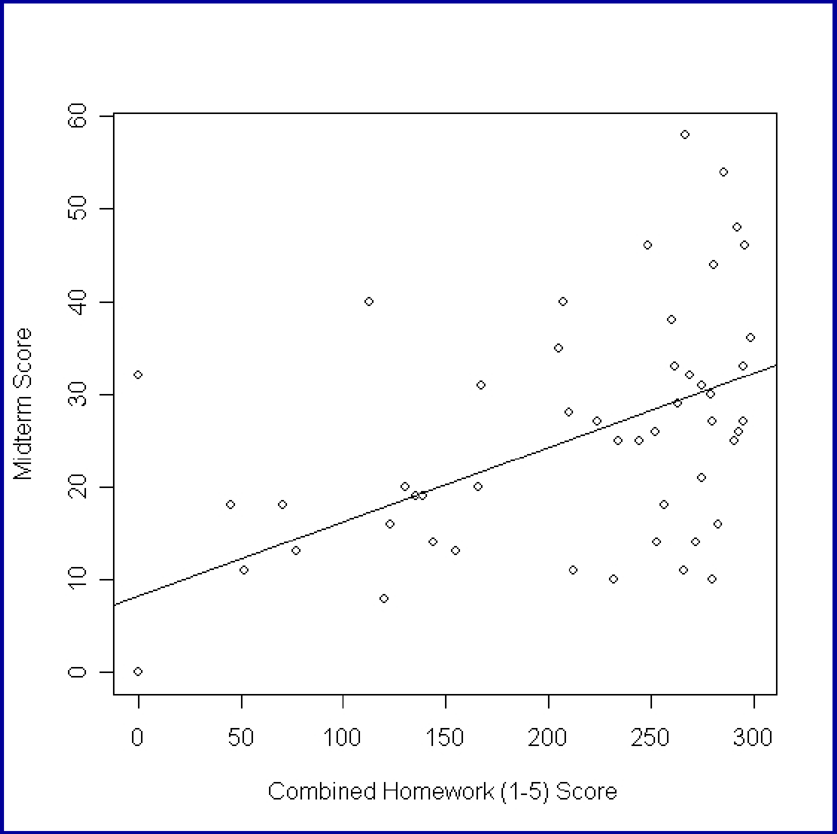
- Please email before office visits.

Midterm vs Homework

Linear regression analysis gives the HW1–5 versus Midterm predictor as

$MT = 8.25 + 0.08\ HW$

with *p*-value $\sim 10^{-5}$.



Average MT was 26 (±12), hence normalized we get

$m = -1.48 + 2h$

with $0 \leq h \leq 1$ the percentage of your homework scores and m your normalized midterm score (0=average, 1=stdev).

Solving It the Lagrangian Way

When trying to solve the evolution of a holonomic system the "Lagrangian way" you should pay special attention to **picking the right coordinate system** $(q_1,\ldots,q_n)$:

- **Do not introduce too many coordinates**

Example 7.3 shows that Atwood's machine has only one degree of freedom.

- **Pick your coordinates in an inertial system**

See the example of a pendulum hanging in an accelerating train.

- **Arrange the coordinates such that the Lagrangian $\mathcal{L}$ is independent from as many coordinates as possible.**

That is, use any kind of symmetry that you have. Example: When dealing with gravity, align the coordinates with the direction of g.

Pendulum in Train

Problem 7.30: How does a length ℓ pendulum behave in a train that accelerates with a constant rate of a m/s^2?

Figure 7.4 gives $v_y = at + \ell \cos\varphi \, d\varphi/dt$ and $v_x = \ell \sin\varphi \, d\varphi/dt$ and $U = -gm\ell \cos\varphi$, hence for the Lagrangian $\mathcal{L} = T-U$:
$\mathcal{L} = \frac{1}{2}m(a^2t^2 + 2at\ell \cos\varphi \, d\varphi/dt + \ell^2(d\varphi/dt)^2) + gm\ell \cos\varphi$.

Lagrange's equation $\partial\mathcal{L}/\partial t = d(\partial\mathcal{L}/\partial\varphi')/dt$ gives:
$g \sin\varphi + a \cos\varphi + \ell \, d^2\varphi/dt^2 = 0$.

Fixed point with $d^2\varphi/dt^2=0$ is achieved for $\varphi_0 = -\tan^{-1}(a/g)$.

Small oscillations $\varphi = \varphi_0+\delta$ around this φ_0 are of the form $d^2\delta/dt^2 = -\omega^2 \delta$ with $\omega = [(a^2+g^2)^{1/2}/\ell]^{1/2}$.

Lessons from the Pendulum

The critical step was to pick the right, inertial reference frame such that the Lagrangian contained the kinetic energy caused by the acceleration.

The frame within the train ignores the effect of the acceleration, unless one introduces a correct potential U that replaces the g by the new $\sqrt{(g^2+a^2)}$.

Example 7.6 “Bead on a spinning wire hoop”, gives another example of a system where the important point is to find the right expression for the kinetic energy.

The Proof in Section 7.4

The proof of Lagrange's equation for constrained systems in Section 7.4 has the following lay-out:

- Use Cartesian coordinates **r** so that we can express the kinetic energy

- First consider only path variations **ε** within the constraints such that we can ignore the constraining forces.

- Reconsider this Cartesian coordinate result in the any proper (=holonomic, constraint obeying) system of generalized coordinates, and you get Lagrange's equations in the general format (Equation 7.52).