Classical Mechanics

Phys105A, Winter 2007

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Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…
- Homework 2 has been posted.
  It is due Monday January 29, 11:30 am.
- Questions?

Motion & Magnetic Force

- In a uniform magnetic field **B**, a q-charged particle with speed **v** undergoes a force **F** = q**v** × **B**.
- Think "orthogonal linear drag".
- For a B-field in the z-direction and no **g** force this means that the *transverse* motion in the xy plane of the particle moves in a circle ~ $c \cdot e^{-i\omega t}$ with:
- radius: $c = vm/qB$
- angular velocity: $\omega = qB/m$

Chapter 3: Momentum and Angular Momentum

3.1 Conservation of Momentum

Consider N particles, with **linear momentum** $\mathbf{P} = \Sigma_\alpha \mathbf{p}_\alpha$.
By Newton's 3rd Law, we have d $\mathbf{P}$/dt = $\mathbf{F}^{ext}$.

"Principle of Conservation of Momentum"
If for the net external force $\mathbf{F}^{ext} = \mathbf{0}$, then the total momentum of the multipartite system $\mathbf{P} = \Sigma_\alpha m_\alpha \mathbf{v}_\alpha$ remains constant.

Some examples:

- Inelastic collisions of several bodies
- Rocket propulsion

3.3 Center of Mass

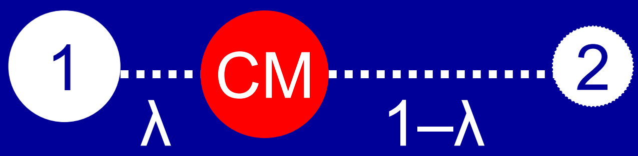
For N particles with masses m_α and positions $\mathbf{r}_\alpha$ from the origin O, its **center of mass CM** (relative to the origin O) is defined as the position $\mathbf{R} = (\Sigma_\alpha m_\alpha \mathbf{r}_\alpha)/\Sigma_\alpha m_\alpha$.

Note that the position of **R** with respect to the particles does not depend on the specific origin O.

With total mass $M = \Sigma_\alpha m_\alpha$, the total momentum of the system can be expressed as $\mathbf{P} = M\, d\mathbf{R}/dt$ and so we have $\mathbf{F}^{ext} = M\, d^2\mathbf{R}/dt^2$. Hence, If we are only interested in the total momentum, we can view the N particle system as a single particle with mass M and position **R**.

Calculating CMs

For 2 particles with masses m_1 and m_2 the *barycenter* (=center of mass) lies on the line between r_1 and r_2:



with $\lambda = m_2/(m_1+m_2)$.

For solid bodies with density ρ over its volume we have $\mathbf{R} = (\int \rho\cdot\mathbf{r}\, dV) / (\int \rho\, dV)$; see Example 3.2.

Besides Total Momentum…

Consider a multipartite system with total momentum **P** and without external forces acting on it: $\mathbf{F}^{ext} = \mathbf{0}$.

We know that **P** will stay constant but that is not all…

Example: For a solar system it is impossible to revert the directions of the movements of the planets, although that would not change the total momentum **P**.

There are other conservation principles that we have to include to better capture the predictions of Newton’s laws.

3.4 Angular Momentum (N=1)

For a particle with momentum **p** and position **r** (relative to the origin O), the **angular momentum ℓ** *relative to* O is defined as **ℓ** = **r**×**p** (hence **ℓ** is orthogonal to **r** and **p**).

Note that

- d**ℓ**/dt = d**r**/dt×**p** + **r**×d**p**/dt
- d**r**/dt and **p** have the same direction
- **F** = d**p**/dt

As a result we have d**ℓ**/dt = **r**×d**p**/dt = **r**×**F** = **Γ**,
with **Γ** = **r**×**F** the **net torque about O** on the particle.