Classical Mechanics

Phys105A, Winter 2007

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Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…
- Homework 3 has been posted.
  It is due Monday January 29, 11:30 am.
  Be sure to download the ‘improved’ version.
- Midterm is scheduled for Week 6, Thursday Feb 15.
  The material will be Chapters 1–4; it is not open book, but you are allowed a ‘cheat sheet’.

Help Wanted

Note Taker

You must be sensitive to the needs of students with disabilities

$75

Stop by the Disabled Students Office at 1201 SAASB and complete an application

Chapter 4:

Energy

Kinetic Energy and Work

- The kinetic energy (KE) of a single particle with velocity **v** is defined as $T = ½\, m|\mathbf{v}|^2$ (units kg m^2/s^2).
- The time derivative of kinetic energy is therefore $dT/dt = m\,(d\mathbf{v}/dt)\cdot\mathbf{v} = \mathbf{F}\cdot\mathbf{v}$, hence the change in kinetic energy is: $dT = \mathbf{F}\cdot d\mathbf{r}$ (just multiply by dt).
- This $\mathbf{F}\cdot d\mathbf{r}$ is the *work* done by **F** along d**r**.
- The change in kinetic energy/work done by **F** is expressed by the line integral $\Delta T = T_2 - T_1 = \int \mathbf{F}\cdot d\mathbf{r}$ (where the integral $\int d\mathbf{r}$ can be in >1 dim. space).
- This is also denoted as $W(1\rightarrow 2)$: the work done by **F** moving from point 1 to point 2 (path dependent).

Work-KE Theorem

The change ΔT in a particle's kinetic energy as it moves from point 1 to point 2 is expressed by

$$\Delta T = T_2 - T_1 = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = W(1 \to 2)$$

By linearity we can decompose $\mathbf{F} = \mathbf{F}_1 + \ldots + \mathbf{F}_n$ and calculate the sum of integrals if that is easier, thus giving $W(1\to 2) = W_1(1\to 2) + \ldots + W_n(1\to 2)$.

Conservative Forces

For *conservative forces* we can define definition of *potential energy.*

A force **F** on a particle is *conservative* if and only if

- **F** only depends on the position **r** of the particle
- For any two points the work $W(1 \rightarrow 2)$ does not depend on the path from point 1 to point 2.

A conservative force does not depend on time t or the velocity **v** of the particle (like drag does).

The gravitational forces is conservative.
So is the force of a spring.

Potential Energy

If all forces on an object are conservative, we can talk about the *potential energy* (PE) of the particle at position **r**, denoted by U(**r**). For a given reference point $\mathbf{r}_0$ we define the potential energy U(**r**) by:

$$U(\mathbf{r}) = -W(\mathbf{r}_0 \to \mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{F}(\mathbf{r}') \cdot d\mathbf{r}'$$

Because the force **F** is conservative, this definition is unambiguous (units kg m^2/s^2 and $U(\mathbf{r}_0)=0$).
The potential U(**r**) can be negative, what really matters is the *difference* between $U(r_1)$ and $U(r_2)$.

Total Mechanical Energy

The *total mechanical energy* E of a particle is the sum its kinetic energy T and its potential energy U: $E=T+U$.

The total mechanical energy E is *conserved*.

The work done by $\mathbf{F}$ between $\mathbf{r}_1$ and $\mathbf{r}_2$ is
$W(\mathbf{r}_1 \rightarrow \mathbf{r}_2) = -[U(\mathbf{r}_2) - U(\mathbf{r}_1)] = -\Delta U$.
As $\Delta T = W(\mathbf{r}_1 \rightarrow \mathbf{r}_2)$, we have $\Delta(T+U) = 0$.

If we have several conservative forces $\mathbf{F} = \mathbf{F}_1 + \ldots + \mathbf{F}_n$ (giving the potentials $U_1, \ldots, U_n$), the total mechanical energy $E = T + U = T + U_1 + \ldots + U_n$ is constant.

Force as a Gradient of U

Using the *gradient* we can summarize the connection between a potential U and its corresponding conservative force **F** by the statement:

$$\mathbf{F} = -\nabla U = -\frac{\partial U}{\partial x}\hat{e}_x - \frac{\partial U}{\partial y}\hat{e}_y - \frac{\partial U}{\partial z}\hat{e}_z$$

Note that each possible potential U gives rise to a conservative force **F** = – ∇U.