Lecture 2: Propositional Logic and Proof Techniques

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2.1 Propositional Logic

We need to begin with some fundamental concepts from logic and then we can progress to proofs. We hinted at the notion of a proposition in the last lecture. In mathematical logic, a **proposition**, which is also called a **statement** (for example, in the Oscar Levin Discrete Mathematics textbook [Lev25]), is a sentence which is either true or false, but not both. Here are some examples of propositions/statements:

- The sum of the first 10 positive integers is 50.
- 5 * 4 * 3 * 2 * 1 = 120.
- Zip codes in USA have exactly 7 digits.
- For all integers x, $2x$ is an even number.

The first and third statements are false, and the second and fourth statements are true. They are all propositions or statements, depending on which terminology you prefer. We will simply use the term proposition from now on.

The following are not propositions:

- For integer x, $x + 1$ is even.
- 7 * 8
- $x^2 + 1 = 2$.
- Atlanta airport is the busiest airport in USA.

The first sentence is true for some x such as $x = 1$, but it is false for some x such as $x = 2$. The second sentence is not a proposition; if it said "7*8=56" then it would be a proposition which is true, or if it said "7*8=50" then it would be a proposition which is false, but at the moment it is neither true or false. The third statement depends on the value of x, for $x = 1$ it is true and for $x = 2$ it is false. The fourth sentence is vague, what does busiest mean? Whether the sentence is true or false depends on the definition of busiest.

Is the following statement a proposition or not:

$$\forall x \in \mathbb{Z},\ 2x \text{ is even.}$$

Yes, this is a proposition because it is either a true statement or a false statement. This is a proposition which is true because for all integers x, $2x$ is even because it is a multiple of 2.

Is the following statement a proposition or not:

$$\forall x \in \mathbb{Z},\ 2x \text{ is odd.}$$

This is again a proposition but in this case it is a proposition which is false.

Is the following statement a proposition or not:

$$\forall x \in \mathbb{Z},\ 2x > x.$$

This is a proposition. When $x \geq 1$ then it is true but when $x = 0$ or $x = 1$ or any $x \leq 0$ it is false. The statement is that for all integers x it holds that $2x > x$. That is a false statement because there exists an integer x where $2x \leq x$, such an x is called a **counterexample**. But earlier we looked at the expression: "For integer x, $x + 1$ is even" and said that is not a proposition because it is true for some x and false for other x – what's the difference? The latter expression does not quantify x: is it supposed to hold for all x or for specific x? It's not clear and the way it's stated there are some settings of x where it's true and some where it's false, so the statement is not true or false, it depends, so it's not a proposition.

2.2 Logical Operators: Combining Propositions

The above propositions are all atomic in the sense that they cannot be broken down into smaller propositions. We can combine propositions together using logical operators, and these combined forms are called *propositional forms.*

For a pair of propositions P and Q, here are some common operations:

Conjunction (AND): $P \wedge Q$ is true when P and Q are both true. Hence, $\wedge$ corresponds to the **and** operator.

Disjunction (OR): $P \vee Q$ is true when either P and/or Q are true. Hence, $\vee$ corresponds to the **or** operator. More precisely, this is the *inclusive or* operator, which we simply refer to it as the *or* operator.

Exclusive Disjunction (XOR): $P \oplus Q$ is true when exactly one of P and Q is true, and the other is false. Hence, $\oplus$ corresponds to the **exclusive-or** or **XOR** operator; this is often used for parity checks (odd or even number of bits which are set to 1) or in many cryptography applications.

Negation (NOT): $\neg P$ (also written as $\overline{P}$) is true when P is false. This is called the **NOT** operator.

Consider the following example propositions:

- P is the proposition: 17 is prime,
- Q is the proposition: 17 is odd,
- R is the proposition: 17 is even.

Now consider the following propositional forms:

$P \wedge Q$**:** This evaluates to true since both P and Q are true.

$P \wedge R$**:** This evaluates to false since R is false.

$P \vee R$**:** This evaluates to true since P is true.

$P \wedge \neg R$**:** This evaluates to true since P and $\neg R$ are true.

$P \oplus Q$**:** This evaluates to false since both P and Q are true.

A convenient tool for evaluating propositional forms are **truth tables**.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Table 1: Truth tables for Conjunction, Disjunction, Negation, and XOR.

2.3 Implications

A more sophisticated propositional form is obtained by the **implication** operator:

$P \implies Q$**:** This says that if P is true then Q is true. Hence, this propositional form is true when, either: (P and Q are both true), or (P is false). Why the later (P is false)? Because if P is false then there are no constraints on Q (so the implication is not implying anything in this case).

Here is the truth table:

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 2: Truth table for Implication.

Let us consider the following implication:

If my house is made entirely of gold then my house is in Canada.

This implication is true, but it is what we call vacuously true because the hypothesis that my house is made of gold is false (unfortunately!) and hence the conclusion holds regardless of whether or not I live in Canada.

Note, in the implication $P \implies Q$, P is called the **hypothesis** and Q is called the **conclusion**.

Here is another example of a vacuously true implication. Let $S = \{x \in \mathbb{N} : x^2 = 5\}$. Clearly, $S = \emptyset$ since there are no natural numbers whose square is 5. Now consider the implication: if $x \in S$ then x is odd. The hypothesis that $x \in S$ is always false since $S = \emptyset$. Hence, the implication holds regardless of whether the conclusion (x is odd) is true or false.

To prove an implication is false we need to find a counterexample. That is a setting of the hypothesis (so the hypothesis is true for that setting) where the conclusion is false for that setting. Hence, here is another way to view a vacuously true implication. In the above example for the set S where $S = \emptyset$, there is no possible setting of the hypothesis (as $S = \emptyset$ there is no $x \in S$), so it is impossible to make a counterexample (because there are no examples, that's why it's vacuous).

Let us look at further examples of implications. Consider the following example propositions: P is "My university is UCSB", Q is "My university is next to the ocean", and R is "My university is surrounded by Goleta". The implication $P \implies Q$ is true since UCSB is next to the ocean.

But the implication $Q \implies P$ is false: I can prove it by constructing a **counterexample**. In particular, consider UCSD, that is a university next to the ocean so it satisfies Q but it is not UCSB so it does not satisfy Q. Therefore, the implication $Q \implies P$ is false.

Note, the implication $(R \wedge Q) \implies P$ is true since UCSB is the only university surrounded by Goleta and next to the ocean.

An important terminology note: If $P \implies Q$ is true then P is a **sufficient condition** for Q, because if P holds then that implies Q holds as well, so P holds is sufficient information to know that Q holds. In the above example, once I know my university is UCSB, then that was sufficient information to conclude that my university is next to the ocean.

If $P \implies Q$ is true, then Q is a **necessary condition** for P, because whenever P holds then Q also holds. In the above example, if my university is UCSB then it is next to the ocean, but Q is not a sufficient condition for P, whereas $Q \wedge R$ is a **sufficient condition** for P since when Q and R both hold then that is enough information to conclude that P also holds.

In the above example, $P \implies Q$ is true, but $Q \implies P$ is false. The implication $Q \implies P$ is called the **converse** of $P \implies Q$; you can think of converse as the opposite. We can look at the truth tables below to see they are not the same. But look at the truth table for $\neg Q \implies \neg P$; that is identical to $P \implies Q$. The implication $\neg Q \implies \neg P$ is the **contrapositive** of $P \implies Q$, and the contrapositive is equivalent to the original.

P	Q	$P \to Q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

P	Q	$Q \to P$
T	T	**T**
T	F	**T**
F	T	**F**
F	F	**T**

P	Q	$\neg Q$	$\neg P$	$\neg Q \to \neg P$
T	T	F	F	**T**
T	F	T	F	**F**
F	T	F	T	**T**
F	F	T	T	**T**

Table 3: Truth tables for the Implication, Converse, and Contrapositive.

Two propositions P and Q are **equivalent** is denoted by $P \iff Q$. The proposition $P \iff Q$ is the same as $(P \implies Q) \wedge (Q \implies P)$. So to prove $P \iff Q$, we need to show the forward direction: $P \implies Q$, and we also need to show the reverse direction: $Q \implies P$.

Looking at the truth table (see below), observe that $P \iff Q$ holds if P and Q are both true, or P and Q are both false. If $P \iff Q$ is true, then notice that P is a necessary and sufficient condition for Q, and also Q is a necessary and sufficient condition for P.

The symbol $\iff$ is often denoted by **iff**, which stands for "if and only if". Any of the following styles are OK (and they all mean the same thing that P and Q are equivalent statements):

- P if and only if Q.
- P iff Q.
- $P \iff Q$.

Finally, here is the associated truth table:

P	Q	$P \iff Q$
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**T**

Table 4: Truth table for equivalent propositions.

2.4 Proof by Contrapositive

The fact that an implication is equivalent to its contrapositive is a very important tool as we can use it to prove that an implication is true. From an implication $P \implies Q$ to form its contrapositive I take the negation of the conclusion as the new hypothesis, and the negation of the original hypothesis as the new conclusion. Here is an example:

Lemma 2.1. *Let n be an integer.*

If n^2 is even, then n is even.

The above example looks a bit different than earlier examples. I added the sentence "Let n be an integer" so that we restrict attention to integers. I'm framing the setting that we're working in so that it's clear to the reader.

I would like to prove the above implication but I'm stumped. Let P be the proposition "n^2 is even", and let Q be "n is even". Then $\neg P$ is "n^2 is odd", $\neg Q$ is "n is odd". Since the contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$, then in this example the contrapositive is the following:

If n is odd, then n^2 is odd.

This implication I can prove by using the fact that n is odd, and then we can show that n^2 is also odd.

Proof of Lemma 2.1. We'll establish the contrapositive: if n is odd, then n^2 is odd. Since n is odd that means there is an integer k where $n = 2k + 1$. Now consider n^2 and substitute in $n = 2k + 1$:

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$$

Since k is an integer then $2k^2 + 2k$ is also an integer, and hence $n^2 = 2\ell + 1$ for an integer $\ell = 2k^2 + 2k$. Therefore, n^2 is odd. This completes the proof of the contrapositive, which is equivalent to the lemma. □

We did our first proof in this class!

2.5 What is a Proof?

A proof is a series of logical implications, where each step is "immediately clear." Since "immediately clear" is a subjective term, you must consider your audience. In this intro to discrete math class, the level of detail required is different than it would be in a graduate-level abstract algebra course. You will notice that a good proof is not just a sequence of logical symbols. It is a narrative. We use text to explain definitions, justify implications, and provide a high-level "roadmap" so the reader knows what to expect. Ultimately, a proof should read like an essay: a cohesive piece of prose with mathematical statements interspersed to provide rigor.

Key Takeaway: Here is a "Golden Rule" for your proofs: If a classmate can't follow your logic without you standing there to explain it, the proof isn't finished yet.

A further stylistic point: do you like reading someone else's code? Of course not. So if someone wants to explain their algorithm for solving a problem, you'd prefer that they first explain it in words so you have a high-level understanding and some intuition, and only then give you the detailed code when it's necessary. A proof is similar. You don't want to read a series of mathematical statements, so give the reader some text to explain what's going on and what to expect, and most importantly to provide some intuition.

Let's explore some additional proof methods that we'll use in this class.

2.6 Direct Proofs

Consider the following proposition.

Lemma 2.2. *The sum of any two even integers is even.*

Proof. Consider two even integers i and j. Since i and j are even, there exist integers k and ℓ where $i = 2k$ and $j = 2\ell$. We need to show $i + j = 2m$ for an integer m. Substituting in $i = 2k$ and $j = 2\ell$, we can compute the sum of i and j as follows:

$$i + j = 2k + 2\ell = 2(k + \ell).$$

Since k and ℓ are integers, then $m = k + \ell$ is also an integer. Since $i + j$ is two times an integer m, therefore $i + j$ is even, which completes the proof of the implication. □

That is an example of a direct proof: we took the hypothesis and then obtained the conclusion. We started the proof by stating the hypothesis, i and j are even. Then we stated the property of being even: that they are a multiple of 2, or in other words, equal to 2 times an integer so $i = 2k$ and $j = 2\ell$ for some integers k and ℓ. Then I stated what I needed to prove: $i + j = 2m$ for an integer m. And then I stated how I was going to proceed, by plugging in $i = 2k$ and $j = 2\ell$ and then computing $i + j$. So I gave the reader a lot of guideposts.

Let's move to some other proof approaches.

2.7 Proof by Contradiction

This is perhaps the most perplexing proof approach initially, at least it's the most contradictory! (Haha!) Consider the following proposition.

Lemma 2.3.

$$\sqrt{2} \textit{ is irrational.}$$

To prove it we'll suppose it's not true and show this is impossible. Formally, a proposition is either true or false, there's no third option; this is called the *Law of the Excluded Middle*. So if I want to show that P is true, it's equivalent to show that $\neg P$ is false. And that's what we'll do now with the above example. We'll assume that $\neg P$ is true, and then we'll show this leads to an impossibility. Namely, we'll show there is a proposition R where $\neg P \implies (R \wedge \neg R)$, but since R is a proposition then it's either true or false so both cannot hold and that's our impossibility. In the below proof, the proposition R will be that a/b is in lowest terms (so no common factors).

Proof of Lemma 2.3. Suppose that $\sqrt{2}$ is rational. The definition of a rational number means that there exist integers a and $b \neq 0$ where:

$$\sqrt{2} = \frac{a}{b}.$$

There may be many such pairs a, b so let's assume a/b is in lowest terms, which means that a and b do not have any common factors (i.e., $gcd(a, b) = 1$), otherwise we can cancel out their common factors to simplify the fraction a/b and obtain a/b in lowest terms.

Now square both sides and we obtain:

$$2 = \frac{a^2}{b^2}.$$

Then multiply both sides by b^2 to obtain:

$$a^2 = 2b^2. \tag{1}$$

Since b is an integer, then b^2 is an integer. Thus, a^2 is even since it is 2 times b^2 where b^2 is an integer. By Lemma 2.1, since a^2 is even we know that a is also even. That means there exists an integer k where $a = 2k$. Substituting $a = 2k$ into Eq. (1) we obtain:

$$2b^2 = (2k)^2 = 4k^2,$$

and simplifying we have that $b^2 = 2k^2$. Thus, b^2 is even. Applying Lemma 2.1 again, we know that b is also even.

We have shown that a and b are both even, and hence they share a common factor of 2. This contradicts our original assumption that a/b was in simplest terms so that a and b had no common factors. This means that our original assumption that $\sqrt{2}$ is rational is false, and hence $\sqrt{2}$ is irrational. □

Here is another example, which is known as the **Pigeonhole Principle**.

Lemma 2.4 (Pigeonhole Principle)**.** *Assign n items (or pigeons) into m containers where $n > m$. Then, at least one container has more than one item assigned to it.*

Don't worry – the pigeons in this example are just stuffed animals, so no animal cruelty happening here.

Proof. We want to show that there exists at least one container with at least 2 items assigned to it; this is a proposition, call it P. What is $\neg P$? It is that all containers have at most 1 item assigned to it (in other words, every container has 0 or 1 item assigned to it). We will prove $\neg P$ is false, which means that P is true.

How do we prove that $\neg P$ is false? We assume $\neg P$ is true and show that this leads to a logical contradiction, which forces us to conclude that $\neg P$ is false.

Suppose all containers have at most one item assigned to it. That means that there are a total of at most m items assigned to the containers. Since $m < n$, at least $n - m$ items were not assigned to a container. Furthermore, since $n - m > 0$ and n and m are integers, thus $n - m \geq 1$. Therefore, there is at least one item that was not assigned to a container, but that contradicts our original hypothesis that all n items were assigned to containers. Hence, our assumption that all containers have at most one item assigned to it is false, and there must exist a container with at least two items assigned to it. □

The pigeonhole principle can be viewed as a proof technique. Here is an example application that we'll cover in this week's section. Prove the following proposition using the pigeonhole principle:

If there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party.

Technically in the field of mathematical logic, a proof by contradiction is known as Reductio ad Absurdum, which apparently means reduction to the absurd (it seems a bit absurd to use latin in 2026?).

Notice that proof by contradiction is different than proof by contrapositive. What's the contrapositive of the proposition: $\sqrt{2}$ is irrational? It's not clear what the contrapositive would be, but one reasonable framing is that the contrapositive is if a number is rational, then it is not $\sqrt{2}$. And it is hard to see that our proof of Lemma 2.3 is doing a direct proof of this contrapositive.

To be precise, a proof by contrapositive compares to a proof by contradiction in the following manner. Suppose our goal is to show: $P \implies Q$. In both approaches (proof by contrapositive or proof by contradiction) we start by assuming $\neg Q$ is true. In a proof by contrapositive we then try to prove that $\neg P$ holds, so we are proving $\neg Q \implies \neg P$. Whereas in a proof by contradiction we try to prove that $R \wedge \neg R$ holds for some proposition R, so we are proving $\neg Q \implies (R \wedge \neg R)$; that is impossible since either R holds or $\neg R$ holds because R is a proposition so it is either true or false but not both. I personally prefer a proof by contrapositive as it's more clear what I'm trying to show, namely $\neg P$, but in some cases, such as the above example for $\sqrt{2}$ is irrational, it's more intuitive to do a proof by contradiction.

2.8 Proof by Induction

The most common proof technique that we'll see many times in this course (and hopefully in your life!) is an inductive proof. Here is an example looking at the sum of the first n positive integers. Recall, $\sum_{i=0}^{n} i = 0 + 1 + 2 + \cdots + n$.

Lemma 2.5. *For all integers $n \geq 0$,*

$$\sum_{i=0}^{n} i \;=\; \frac{(n) \times (n+1)}{2}.$$

In other words, $1 + 2 + 3 + \cdots + n = n \times (n+1)/2$. In the above lemma, n is a variable like in a computer program. The above proposition can be broken down into an atomic proposition for each particular n. For each integer $n \geq 0$, let $P(n)$ denote the proposition that $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$. The lemma is then asserting that $P(n)$ is true for all $n \geq 0$.

We will prove that $P(n)$ is true for all $n \geq 0$ in the following manner.

Base Case: We first prove that $P(0)$ is true.

Induction hypothesis: For arbitrary $k \geq 0$, assume that $P(k)$ is true.

Inductive step: We will prove that $P(k) \implies P(k+1)$. In other words, we assume that $P(k)$ is true, and then we show, using this assumption about $P(k)$, that $P(k+1)$ is true.

Notice that we'll establish $P(0)$ in the base case. Then the inductive step for $k = 0$ shows that $P(0) \implies P(1)$, and since we showed $P(0)$ is true in the base case, we then have that $P(1)$ is true. Applying again the inductive step we then conclude that $P(2)$ is true. Continuing on in this manner, we conclude that $P(n)$ holds for all $n \geq 0$. In summary: the base case gives the initial statement $P(0)$, then applying the inductive step with $k = 0$ we obtain $P(1)$, then applying the inductive step with $k = 1$ we obtain $P(2)$, etc.

This chain of implications is often explained metaphorically as a chain of dominoes. The i-th domino represents $P(i)$ and when the i-th domino is knocked over that represents that $P(i)$ is true. We start by knocking over the 0-th domino by proving the base case. The inductive step shows that when the 0-th domino falls over, then it knocks over the next domino, and so on, eventually all of the dominoes fall over in succession.

Proof of Lemma 2.5. We will prove the lemma by induction on n.
Base case: Consider the case $n = 0$. Then $\sum_{i=0}^{0} i = 0$, and also $\frac{0 \times 1}{2} = 0$. Therefore, $P(0)$ is true.
Inductive hypothesis: For an integer $k \geq 0$, assume that $P(k)$ is true and thus we know that:

$$\sum_{i=0}^{k} i = \frac{k(k+1)}{2}. \qquad (2)$$

Inductive step: Now let's prove that $P(k+1)$ holds. Thus, we need to show that:

$$\sum_{i=0}^{k+1} i = \frac{(k+1)(k+2)}{2}, \tag{3}$$

and we can use our assumption that Eq. (2) is true to aid our proof.

Beginning with the left-hand-side of Eq. (3) we have the following:

$$\begin{aligned}
\sum_{i=0}^{k+1} i &= \left(\sum_{i=0}^{k} i\right) + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) && \text{by the inductive hypothesis Eq. (2)} \\
&= \frac{k(k+1)+2(k+1)}{2} && \text{finding a common denominator} \\
&= \frac{(k+1)(k+2)}{2} && \text{factoring out } (k+1).
\end{aligned}$$

which proves the inductive step Eq. (3), and completes the proof of the lemma. □

Here is another example of a proof by induction.

Lemma 2.6. *Let n be a positive integer. Then, $1+3+5+7+\cdots+(2n-1)$ is a perfect square.*

An equivalent form of the above lemma is that

$$(\forall n \geq 1)\,(\exists r \in \mathbb{Z})\left(\sum_{i=1}^{n}(2i-1) = r^2\right).$$

We'll try to prove this by induction as we did in our proof of Lemma 2.5. Suppose we've established that:

$$1+3+5+7+\cdots+(2k-1) = r^2,$$

for some integer r. Now we need to establish that:

$$1+3+5+7+\cdots+(2k-1)+(2(k+1)-1) = s^2,$$

for some integer s. Working as in the proof of Lemma 2.5, we'll consider the first k terms and use that they sum to r^2 to obtain:

$$1+3+5+7+\cdots+(2k-1)+(2(k+1)-1) = r^2 + (2k+1).$$

We then need to show that $r^2+(2k+1) = s^2$ for some integer s. But that's not necessarily true, it depends on the value of r, so we're stuck! To prove the lemma we need more information, namely the value of r.

How do we find r? By considering small examples (namely, small values of k or equivalently n) we hope to get a guess (which will be our conjecture) then we'll try to prove that conjecture.

Let $f(n) = \sum_{i=1}^{n}(2i-1)$. Then, $f(1) = 1$, $f(2) = 1+3 = 4 = 2^2$, and $f(3) = 1+3+5 = 9 = 3^2$. So let's conjecture that $f(k) = k^2$. If we wanted to be more sure we could write a quick computer program to check it for all $k \leq 1000$. That would not be a proof, but it would give us more confidence.

Proof of Lemma 2.6. We will prove the lemma by induction on n. In fact, we will prove the stronger statement that for all integers $n \geq 1$,

$$\sum_{i=1}^{n}(2i-1) = n^2. \tag{4}$$

Base case: For $n = 1$, we have that $\sum_{i=1}^{1}(2i-1) = 1 = 1^2$ so this establishes Eq. (4) for $n = 1$.
Inductive hypothesis: For an integer $k \geq 1$, assume that Eq. (4) holds for $n = k$.

Inductive step: Now let's prove that Eq. (4) holds for $n = k + 1$:

$$\begin{aligned}
\sum_{i=1}^{k+1}(2i-1) &= \left(\sum_{i=1}^{k}(2i-1)\right) + (2(k+1)-1) \\
&= k^2 + (2k+1) && \text{by the inductive hypothesis} \\
&= (k+1)^2,
\end{aligned}$$

which proves Eq. (4) for $n = k + 1$, and thus establishes Eq. (4) for all $n \geq 1$ by induction. Note, Eq. (4) implies Lemma 2.6 since n^2 is a perfect square. □

Key Takeaway: In an inductive proof, we often need more information, or more properties, in order to complete the proof of the inductive step. Hence, we "strengthen" the inductive proof in the sense that the new statement which we prove implies the original statement, as we did in the proof of Lemma 2.6 where we quantified the perfect square that the summation equaled.

2.9 Strong Induction (optional)

Recall, in our "recipe" for an inductive proof, the key step is the inductive step in which we prove that

$$P(k) \implies P(k+1).$$

In fact, we have established $P(0), \ldots, P(k)$ by that point (or $P(1), \ldots, P(k)$ depending on the base case). So the inductive proof would still hold if we proved:

$$(P(0), P(1), \ldots, P(k)) \implies P(k+1).$$

That is termed **strong induction** as we are making the hypothesis stronger. This distinction is typically not needed so we are not going to cover it in this class. Oftentimes, when I'm writing a proof by induction I state it as a strong induction proof as I state the induction hypothesis as: For arbitrary $k \geq 0$, assume that for all $i \leq k$, $P(i)$ is true. But, in fact, in the proof of the induction hypothesis typically I'm not using the stronger hypothesis, it's simply my habit to write the stronger hypothesis.

References

[Lev25] Oscar Levin. *Discrete Mathematics: An Open Introduction.* CRC press, 4th edition, 2025. http://discrete.openmathbooks.org/.