Homework 3

Due: Wednesday, October 20, 2021 before the start of class at 11am.

Turn-in via Gradescope.

Let me know if you're having difficulties and I'll give you help/hints.

Zoom office hours are Tuesdays/Thursdays at 8pm.

Problem 1:

Implement coupling from the past for the Glauber dynamics of the ferromagnetic Ising model on a $\sqrt{n} \times \sqrt{n}$ grid (with no boundary which is called free boundary or with the toroidal boundary which is called periodic boundary, your choice). Do it for large n, do many trials and look at the median coupling time (by coupling time I mean the coupling from the past convergence time M) or look at it as a function of n. How does the coupling time behave as you increase β? The mixing time is $O(n \log n)$ for all $\beta < \beta_c$ but when does your coupling from the past convergence time start to blow up? Submit a figure to demonstrate your results, you decide what exactly is a useful figure to clearly explain your results. Be sure to let me know details of your experiment, e.g., what's the n you used, how many experiments did you do, what language did you use (any is fine), did you have to do any non-trivial implementation ideas to get it to run for a larger n?

Part b: Try the same experiment with all + boundary condition. Does it slow down? Also try for very large β (i.e., small temperature). It's conjectured that the mixing time of the Glauber dynamics is polynomial in n for all β when there is the all + boundary condition.