Computer Science 130B
Spring 2017
Homework #3

Due: **4pm** May 19th, Friday

**Problem 1** We show in written assignment # 2 that the greedy algorithm can solve the coin change problem where the denominations of coins are 1 (penny), 5 (nickel), 10 (dime), and 25 (quarter). Do you think that the greedy algorithm will work for all possible coin denominations? If yes, prove that the greedy algorithm always uses the smallest number of coins to make change for any coin denomination. If not, given a counterexample that the greedy algorithm may not use the smallest number of coins to make change, and then design a dynamic programming algorithm that solves the coin change problem using the smallest number of coins. Given the recurrence equation. What is the complexity of your algorithm?

**Problem 2** There are two warehouses $W_1$ and $W_2$ from which supplies are to be shipped to destinations $D_i$, $1 \leq i \leq n$. Let $d_i$ be the demand at $D_i$ and let $r_i$ be the inventory at $W_i$ (all integer numbers). Assume that $r_1 + r_2 = \sum d_i$. Let $c_{ij}(x_{ij})$ be the cost of shipping $x_{ij}$ units from warehouse $W_i$ to destination $D_j$. The warehouse problem is to find nonnegative integers $x_{ij}$, $1 \leq i \leq 2$ and $1 \leq j \leq n$ such that $x_{1j} + x_{2j} = d_j$, $1 \leq j \leq n$ and $\sum_{i,j} c_{ij}(x_{ij})$ is minimized. Let $g_i(x)$ be the cost incurred when $W_1$ has an inventory of $x$ and supplies are sent to $D_j$, $1 \leq j \leq i$, in an optimal manner (the inventory at $W_2$ is $\sum_{1 \leq j \leq i} d_j - x$). The cost of an optimal solution to the warehouse problem is $g_n(r_1)$.
**a.** Use the optimality principle to obtain a recurrence relation for $g_i(x)$.
**b.** Write an algorithm to solve the recurrence and obtain an optimal sequence of values for $x_{ij}$, $1 \leq i \leq 2$ and $1 \leq j \leq n$.
**c.** Show steps and tables of your DP algorithm on this particular problem instance: two warehouses with supply of 10 and 7 units, five destinations with demands of 4, 3, 2, 5, and 3 units. The cost of supplying these destinations from warehouse one is 1, 2, 3, 1, 2/unit and from warehouse two is 2, 3, 3, 1, 1/unit.

**Problem 3** Consider a variation of problem#2 in homework #2. You are given $n$ intervals on the $x$-axis. You are to find a collection of non-overlapping intervals such that *the sum of the lengths* of all selected intervals is maximized (not *the number of intervals* is maximized).
**a.** Will your greedy algorithm in homework #2 produces the optimal solution? If so, prove it. If not, give a counter example.
**b.** Develop a DP algorithm to solve this problem. Be specific and clear of what optimality and feasibility constraints are used in enumerating your solutions and trim the solution space.