

Discussion Session 6

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Today's topic: things you need for hw3 (I)

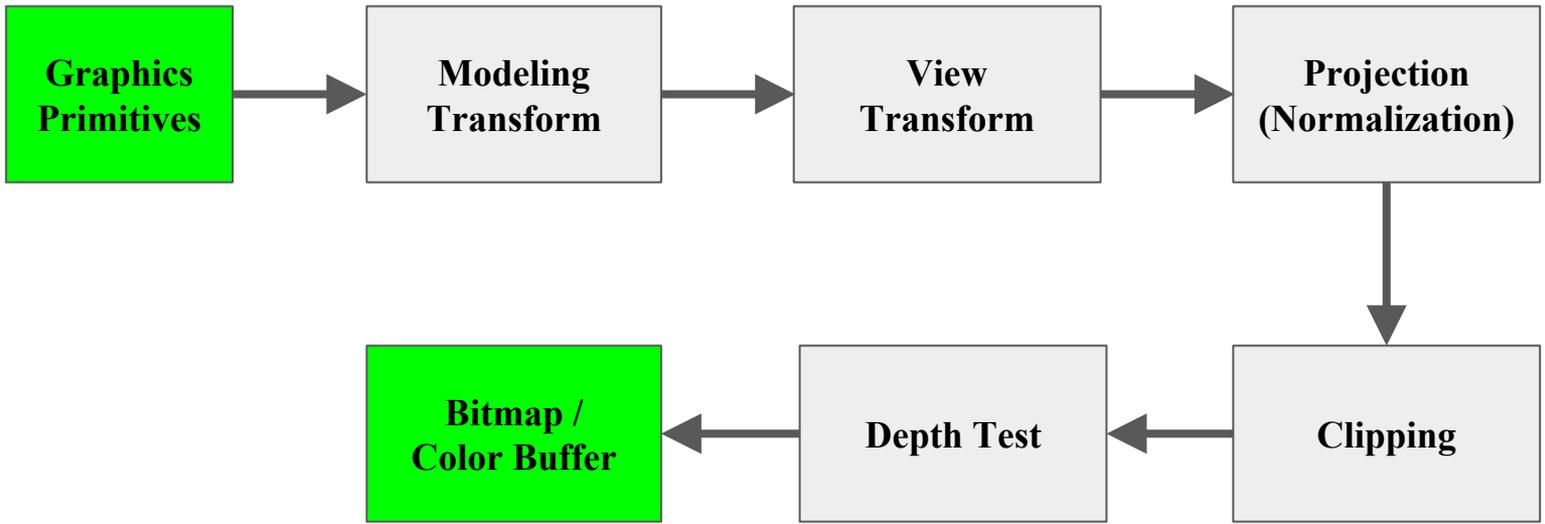
- Matrices
- Polygon clipping
- HLHSR

HW3 Requirements

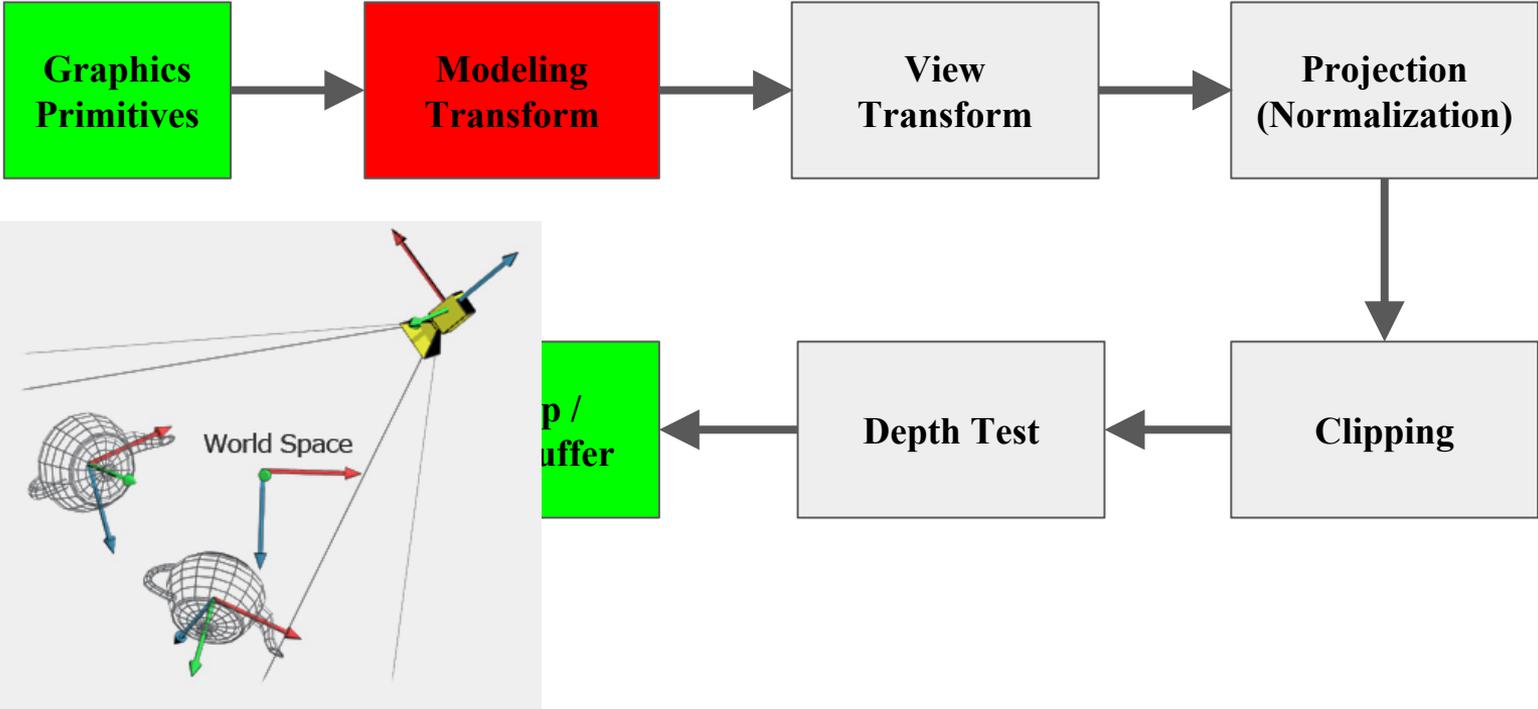
- Implement your own graphics library:
 - Object drawing in black & white
 - Animation
 - Extra credit: viewpoint change
 - No color/Lighting/shading/texture/shadow
 - **CAN'T use OpenGL or any other graphics libraries**
 - Save each frame as an image file (ex. JPG, PPM) (can use library for writing/saving image)
- What you should turn in
 - All your code
 - Makefile, which can generate a series of image files for all frames
 - A video sequence or gif showing the final display result
 - you can use any software or free-use website to concatenate those frames

Rendering Pipeline

Matrix value (0/1)



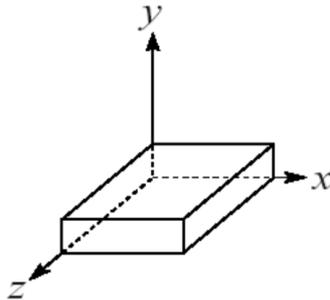
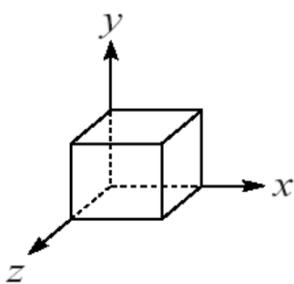
Rendering Pipeline



You need to implement

- Scaling
- Translation
- Rotation

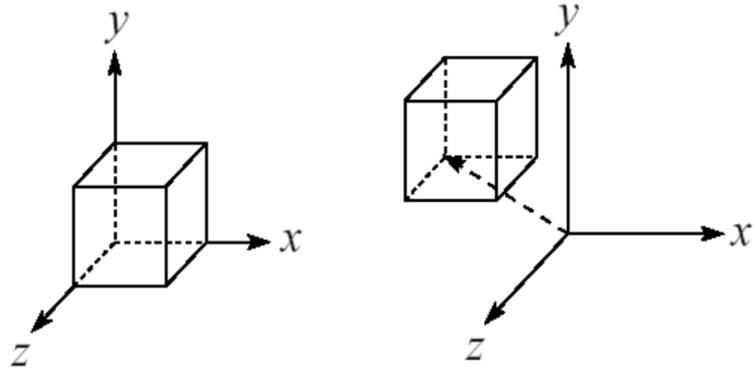
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



You need to implement

- Scaling
- Translation
- Rotation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



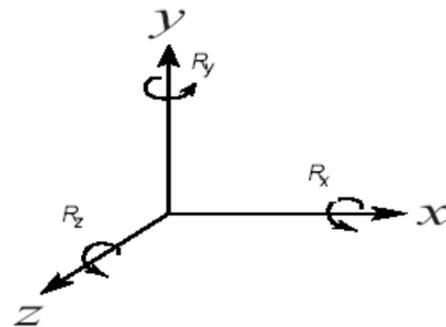
You need to implement

- Scaling
- Translation
- Rotation

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

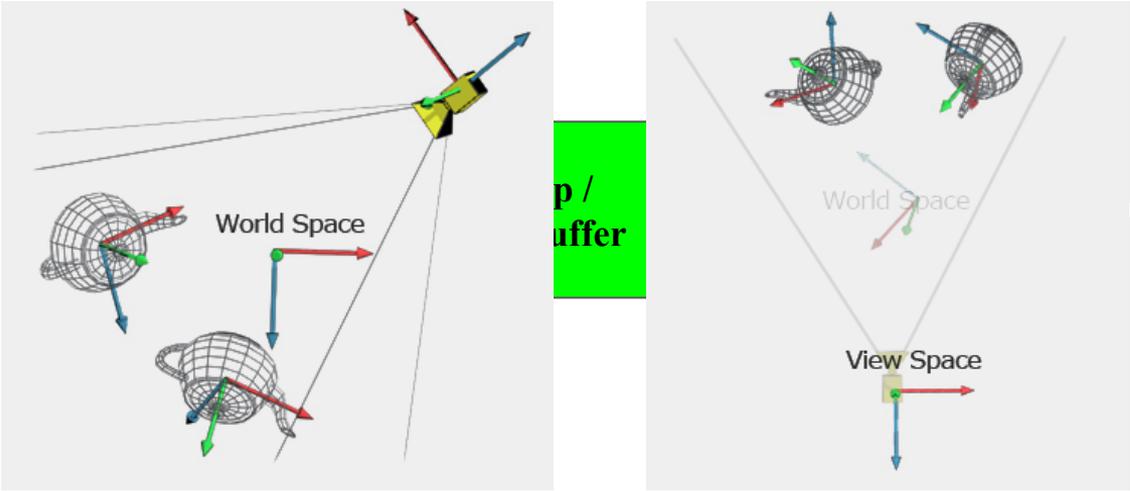
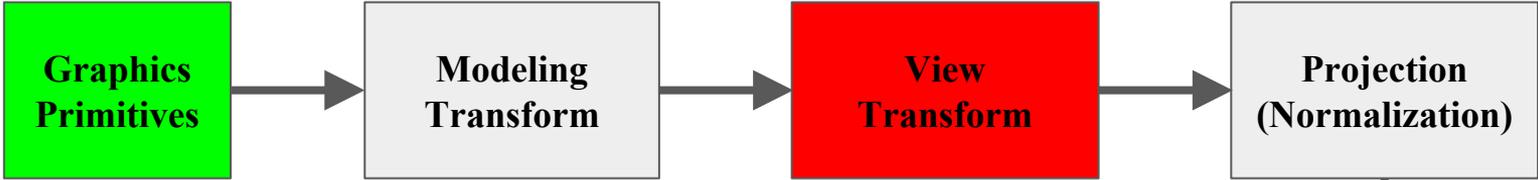
$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Use right hand rule

$$R = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$$

Rendering Pipeline



You need ...

- Viewing matrix
- Knowns: eye position \mathbf{e} , center \mathbf{c} , up vector \mathbf{u}

$$\mathbf{f} = \mathbf{c} - \mathbf{e}$$

$$\mathbf{f}' = \frac{\mathbf{f}}{|\mathbf{f}|}$$

$$\mathbf{u}' = \frac{\mathbf{u}}{|\mathbf{u}|}$$

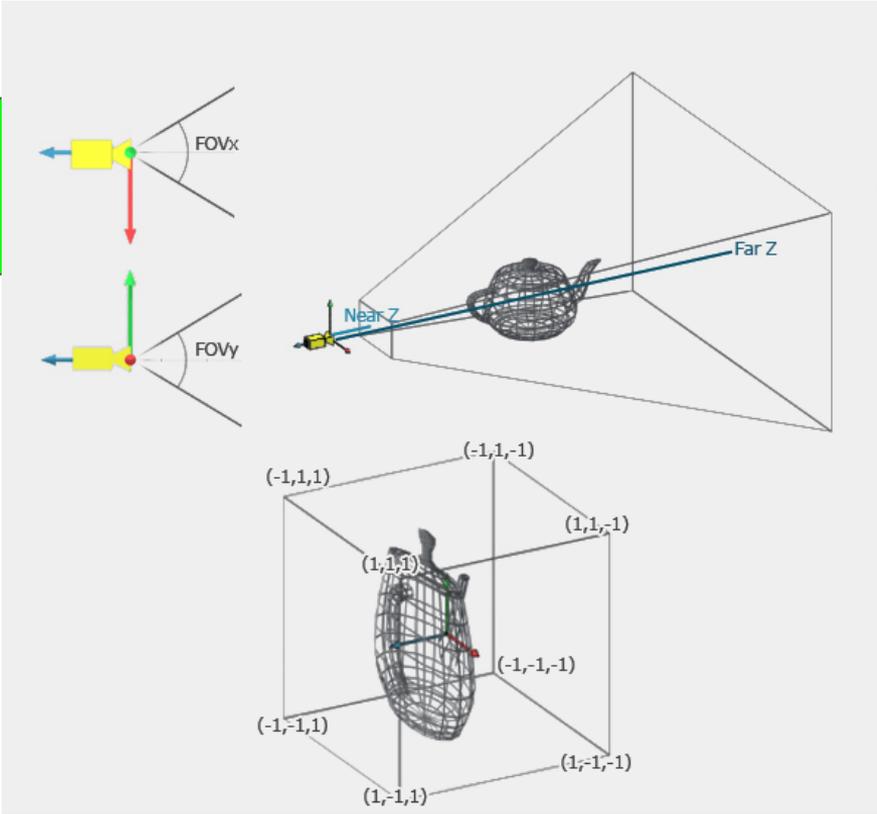
$$\mathbf{s} = \mathbf{f}' \times \mathbf{u}'$$

$$\mathbf{u}'' = \frac{\mathbf{s}}{|\mathbf{s}|} \times \mathbf{f}'$$

$$M = \begin{pmatrix} s_x & s_y & s_z & 0 \\ u''_x & u''_y & u''_z & 0 \\ -f'_x & -f'_y & -f'_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rendering Pipeline

Graphics Primitives



Projection (Normalization)

Clipping

You need ...

- Projection matrices (already transformed into canonical view volume)
- Perspective & orthographic (6 parameters for both: left right top bottom near far)

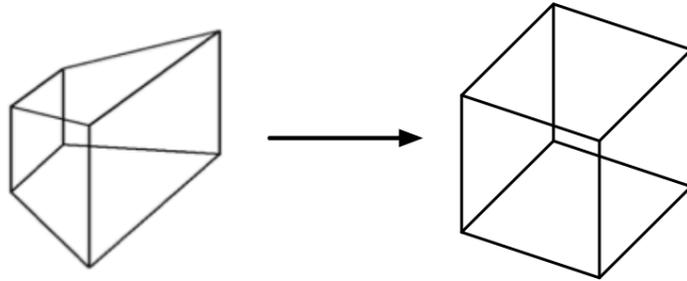
$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{(f-n)} & -\frac{f+n}{(f-n)} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Canonical view volume

It can be computationally expensive to check if a point is inside a frustum

- ▶ Instead transform the frustum into a normalised canonical view volume
- ▶ Uses the same ideas as a perspective projection
- ▶ Makes clipping and hidden surface calculation much easier

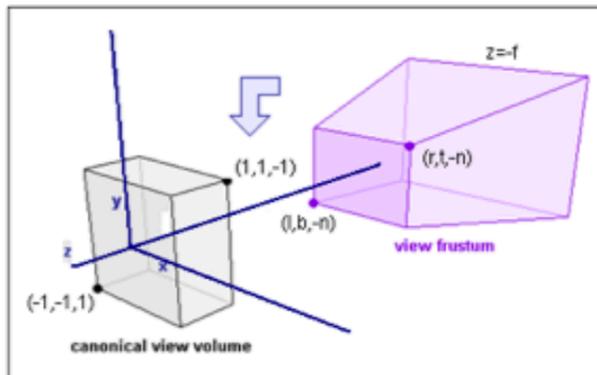


Transforming the view frustum

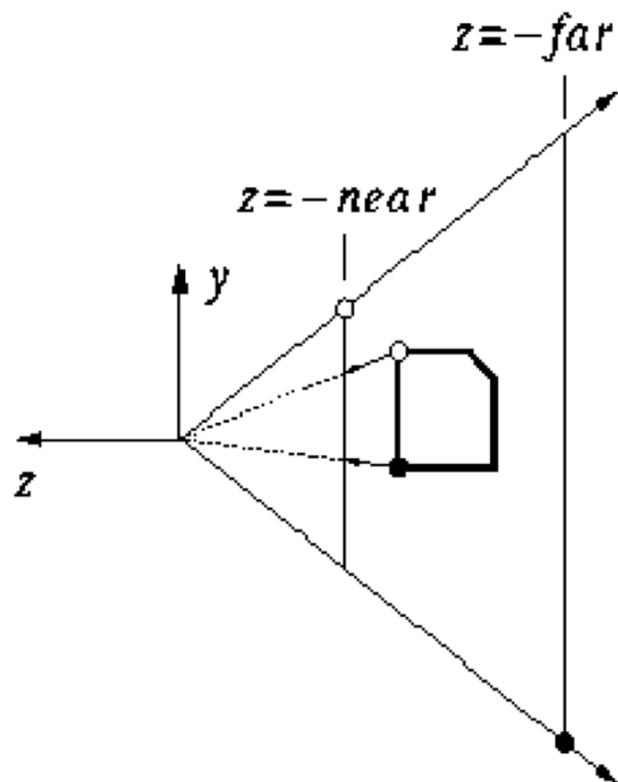
The frustum is defined by a set of parameters, l, r, b, t, n, f :

- l Left x coordinate of near plane
- r Right x coordinate of near plane
- b Bottom y coordinate of near plane
- t Top y coordinate of near plane
- n Minus z coordinate of near plane
- f Minus z coordinate of far plane

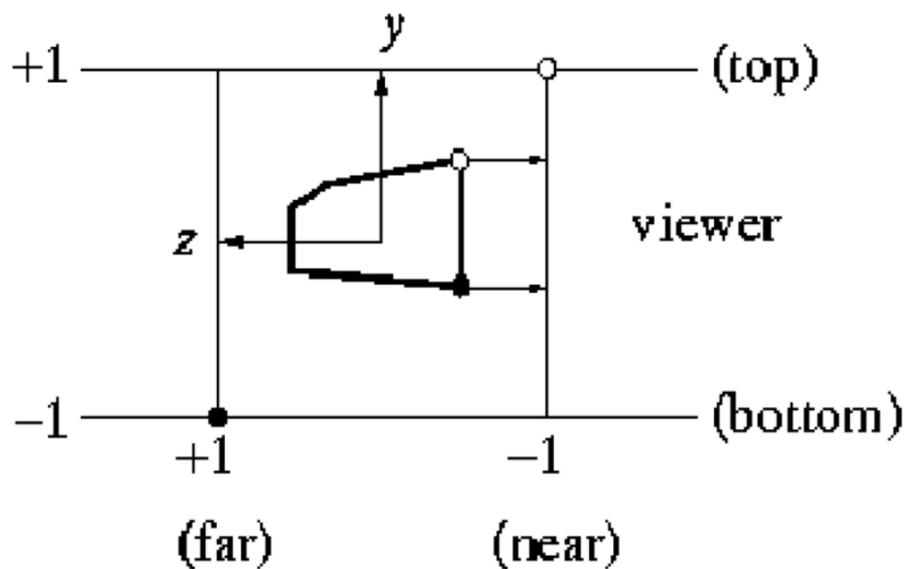
With $0 < n < f$.



$$-1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1$$



Viewing frustum

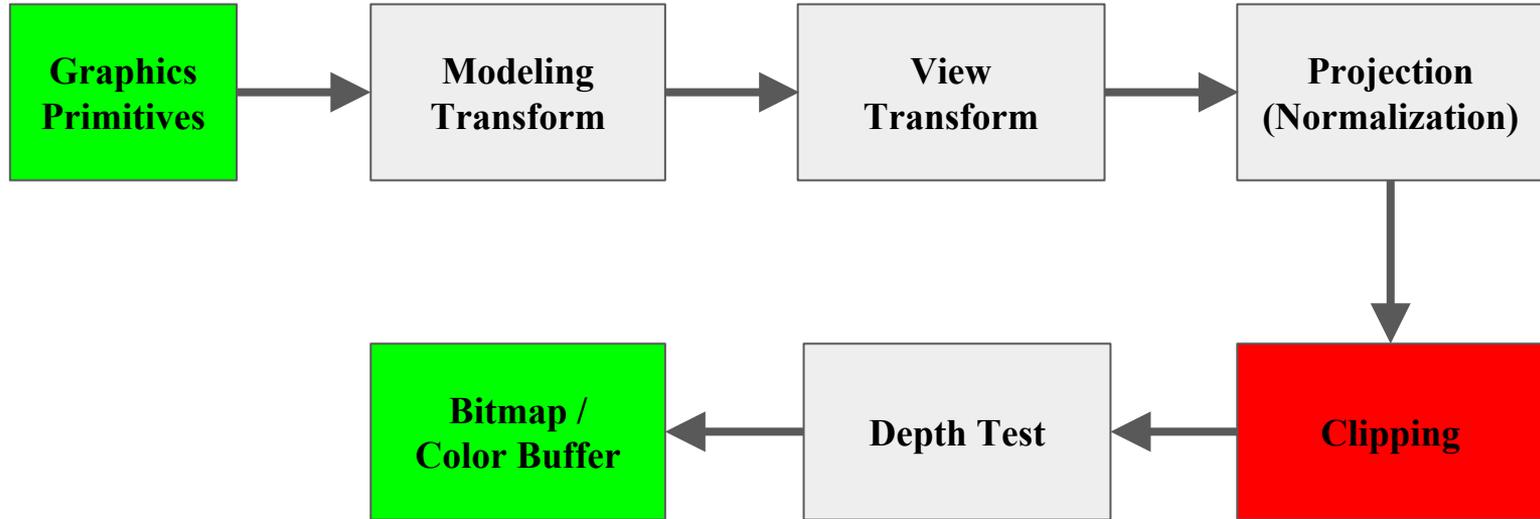


Canonical view volume

matrix.c

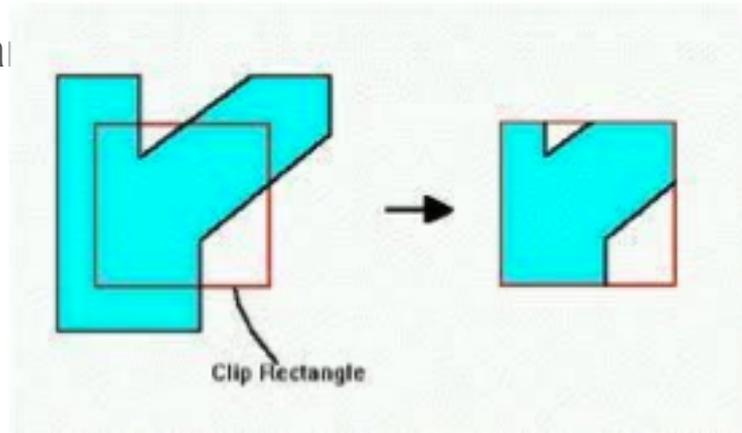
- Has all the matrix operations you need:
 - Inverse
 - Transpose
 - Addition/subtraction/multiplication
 - Inner/cross product
 - Determinant
 - ...

Rendering Pipeline



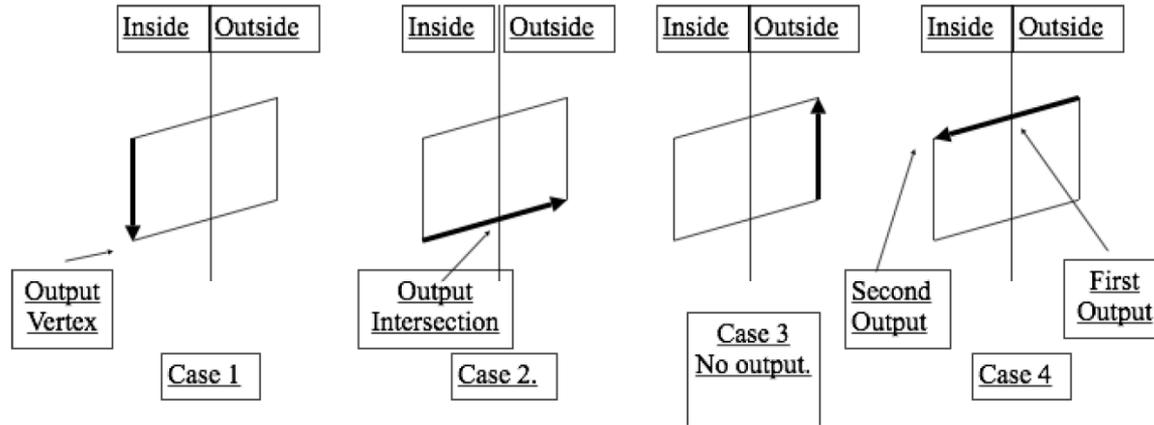
Clipping

- Perform clipping in the canonical view volume
- Polygons may intersect the canonical view volume, then we need to perform clipping:
- Sutherland-Hodgman



Sutherland-Hodgman algorithm

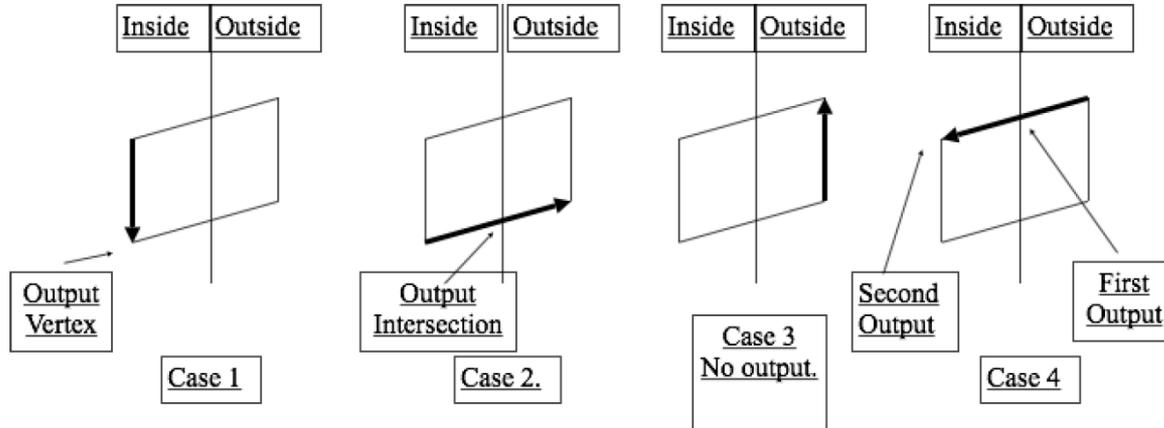
Traverse edges and divide into four types:



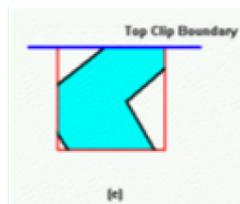
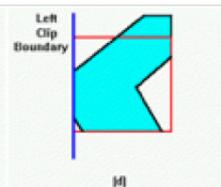
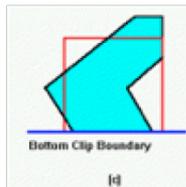
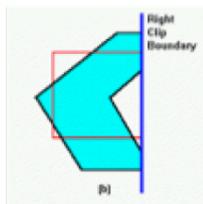
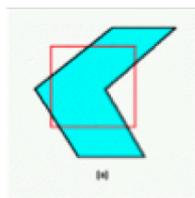
Sutherland-Hodgman algorithm

For each edge of the clipping rectangle:

- ▶ For each polygon edge between v_i and v_{i+1}

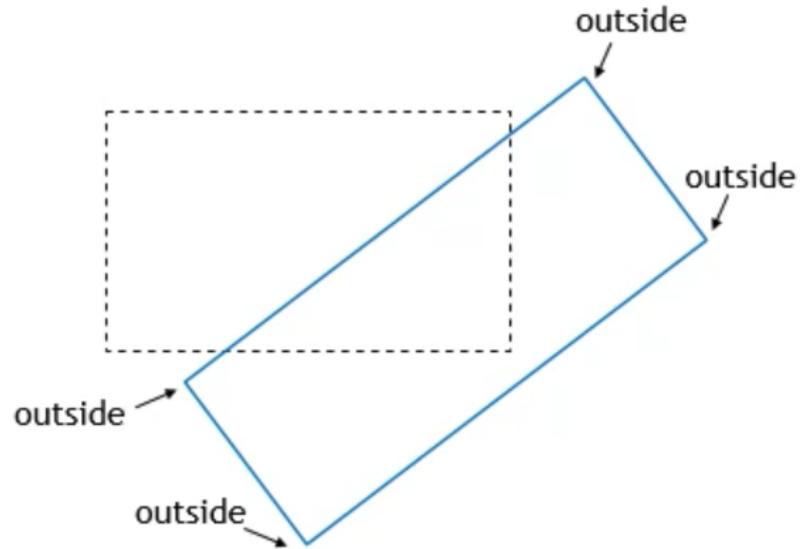


Example



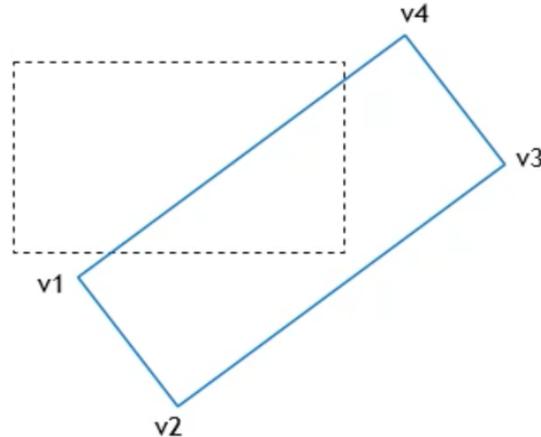
Sutherland Hodgman Polygon Clipping

- What will happen here?



Sutherland Hodgman Polygon Clipping

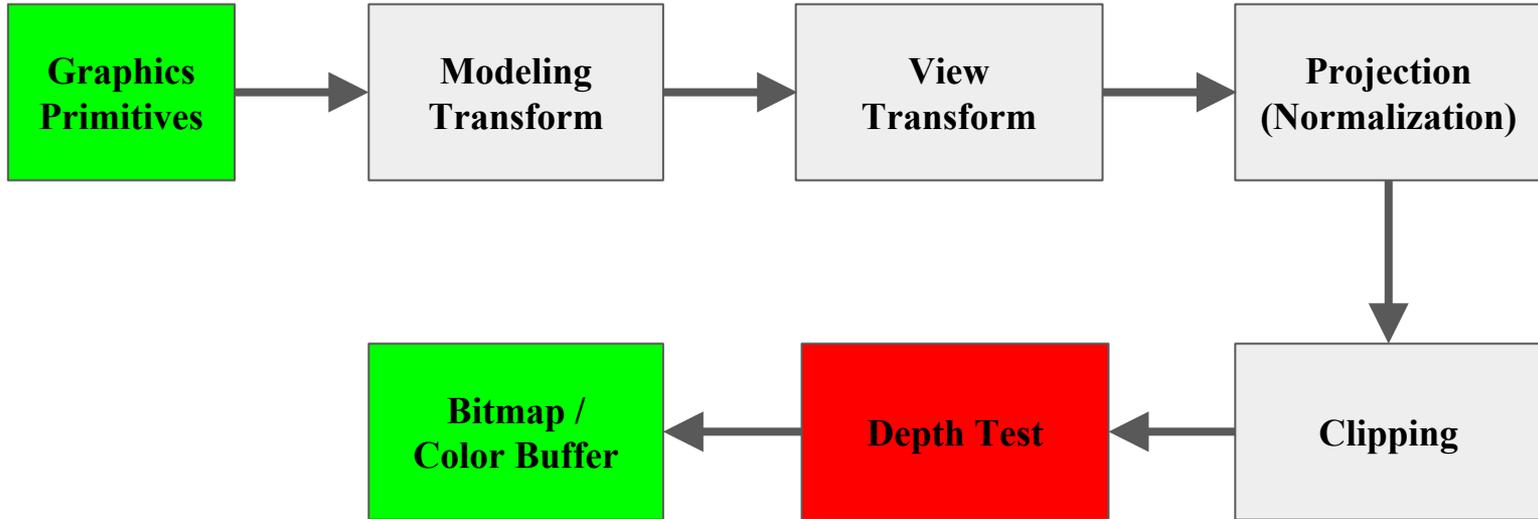
- It is **incorrect** to consider a vertex as **inside/outside of the clipping area**
- Instead, for each vertex, tell whether it is **in the inner or outer side of each edge** of the clipping area



Sutherland Hodgman Polygon Clipping

- It is incorrect to consider a vertex as inside/outside of the clipping area
- Instead, for each vertex, tell whether it is in the inner or outer side of each edge of the clipping area
- **It also works in 3D world, where the clipping area consists of 6 surfaces, instead of 4 edges.**

Rendering Pipeline



HLHSR

- Target: generate the set of pixels that form the final image.

Algorithms (we'll cover 2 this time)

- **Scan-line Algorithm**
- **z-buffer Algorithm** (*neat, simple and fast!*)
- Depth-Sort Algorithm
- Binary Space Partition (BSP) Trees
- Area-subdivision Algorithm

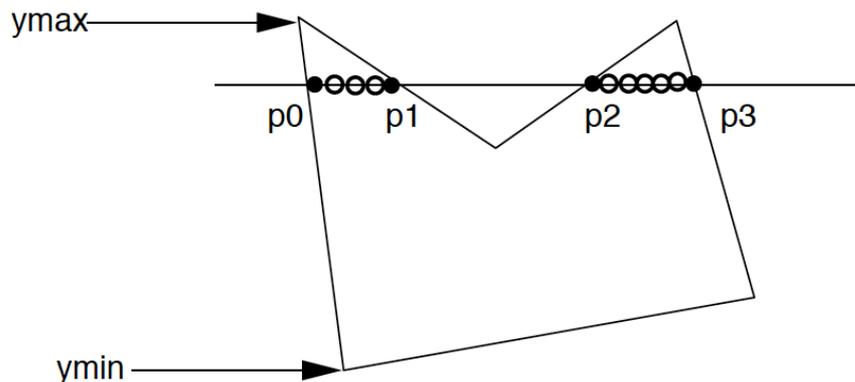
Scan-line Algorithm

- Intersect scanline with polygon edges
- Fill between pairs of intersections

- Basic algorithm:

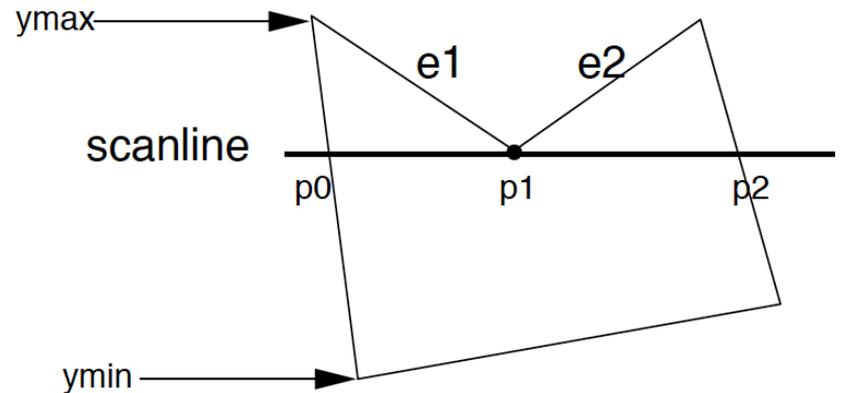
For $y = y_{\min}$ to y_{\max}

- 1) intersect scanline y with each edge
- 2) sort intersections by increasing x
[p_0, p_1, p_2, p_3]
- 3) fill pairwise ($p_0 \rightarrow p_1, p_2 \rightarrow p_3, \dots$)



Scan-line Algorithm: Special handling

- Intersection is an edge end point



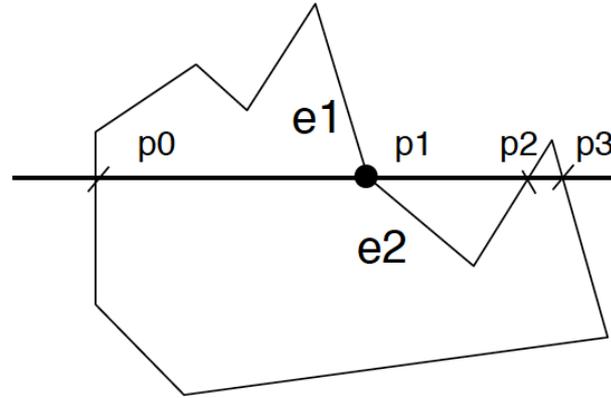
- Intersection points: (p_0, p_1, p_2) ???

-> (p_0, p_1, p_1, p_2) so we can still fill pairwise

-> In fact, if we compute the intersection of the scanline with edge e_1 and e_2 separately, we will get the intersection point p_1 twice. Keep both of the p_1 .

Scan-line Algorithm: Special handling (cont.)

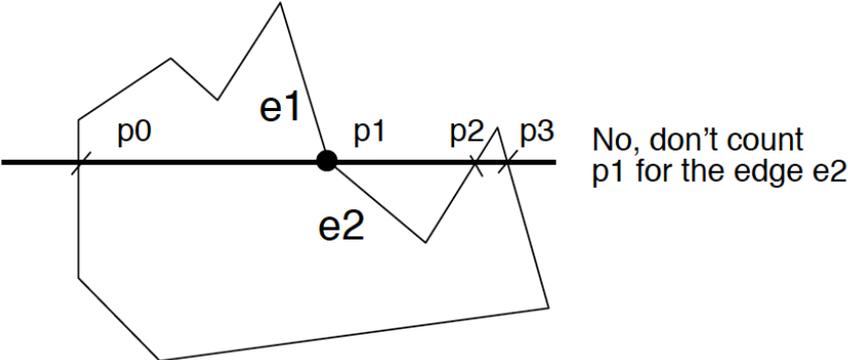
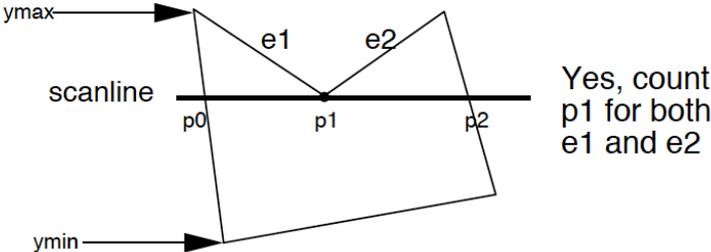
- Intersection is an edge end point



- However, in this case we don't want to count p1 twice (p0,p1,p1,p2,p3), otherwise we will fill pixels between p1 and p2, which is wrong

Scan-line Algorithm: Special handling (cont.)

- Rule: If the intersection is the y_{min} of the edge's endpoint, count it. Otherwise, don't.



z-buffer (a.k.a. depth-buffer) algorithm

- Initialization (2 buffers)

z buffer (set to a value > 1) \mathbf{z}

color buffer (set to BG color) \mathbf{c}

- For each polygon

Scan convert

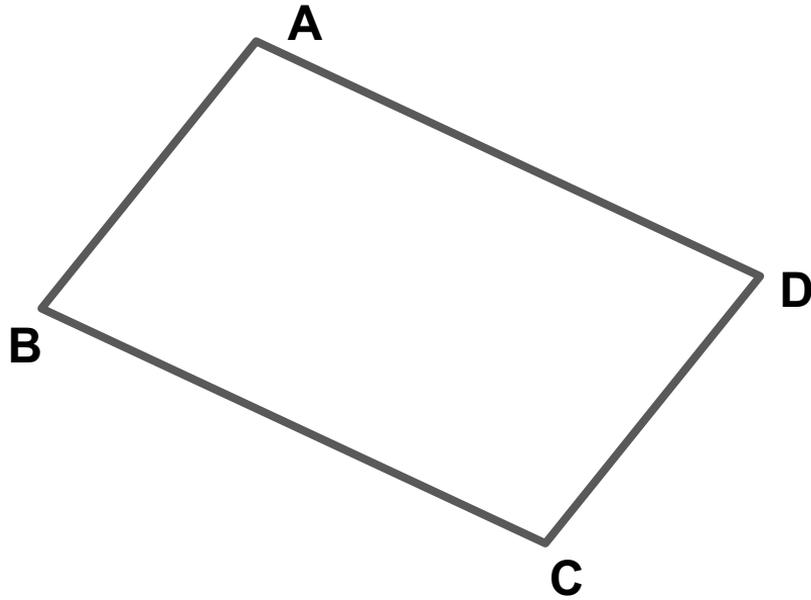
For each pixel in the polygon

if($z_poly(x,y) < \mathbf{z}(x,y)$)

$\mathbf{z}(x,y) = z_poly(x,y)$

$\mathbf{c}(x,y) = polygon_color$

Obtain the Plain Equation from Polygon Vertices



z-buffer combined with scanline

- Calculating z_{poly} :
- Plane equation: $0 = A x + B y + C z + D$

$$\text{Solve for } z: z = (-A x - B y - D) / C$$

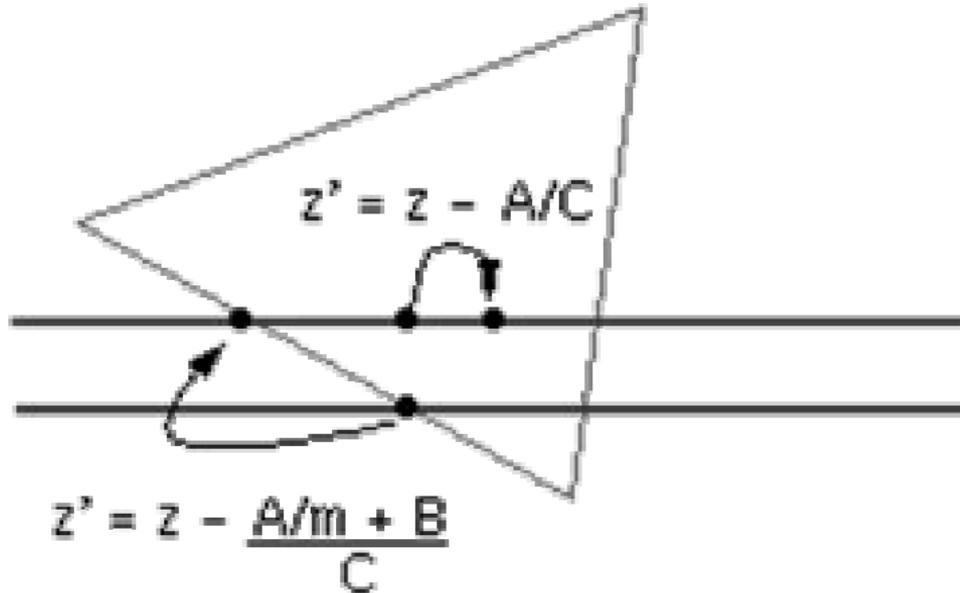
- Moving along a scanline, so want z at next value of x

$$Z' = (-A (x+1) - B y - D) / C$$

$$Z' = z - A/C$$

z-buffer combined with scanline

- For moving between scanlines, know $x' = x + 1/m$
- The new left edge of the polygon is $(x+1/m, y+1)$, giving $z' = z - (A/m + B)/C$



Q & A