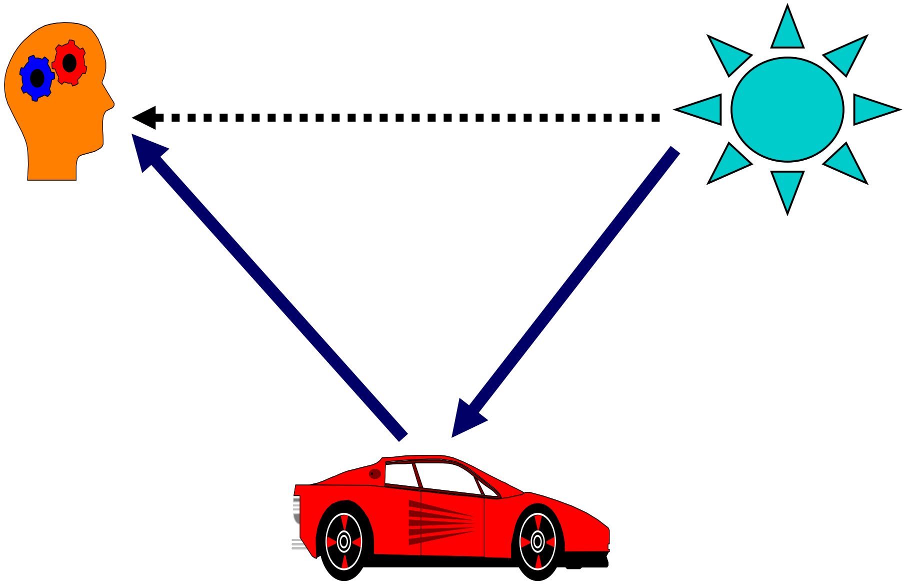
Image Formation - Radiometry

CS / ECE 181B



Geometry and Radiometry

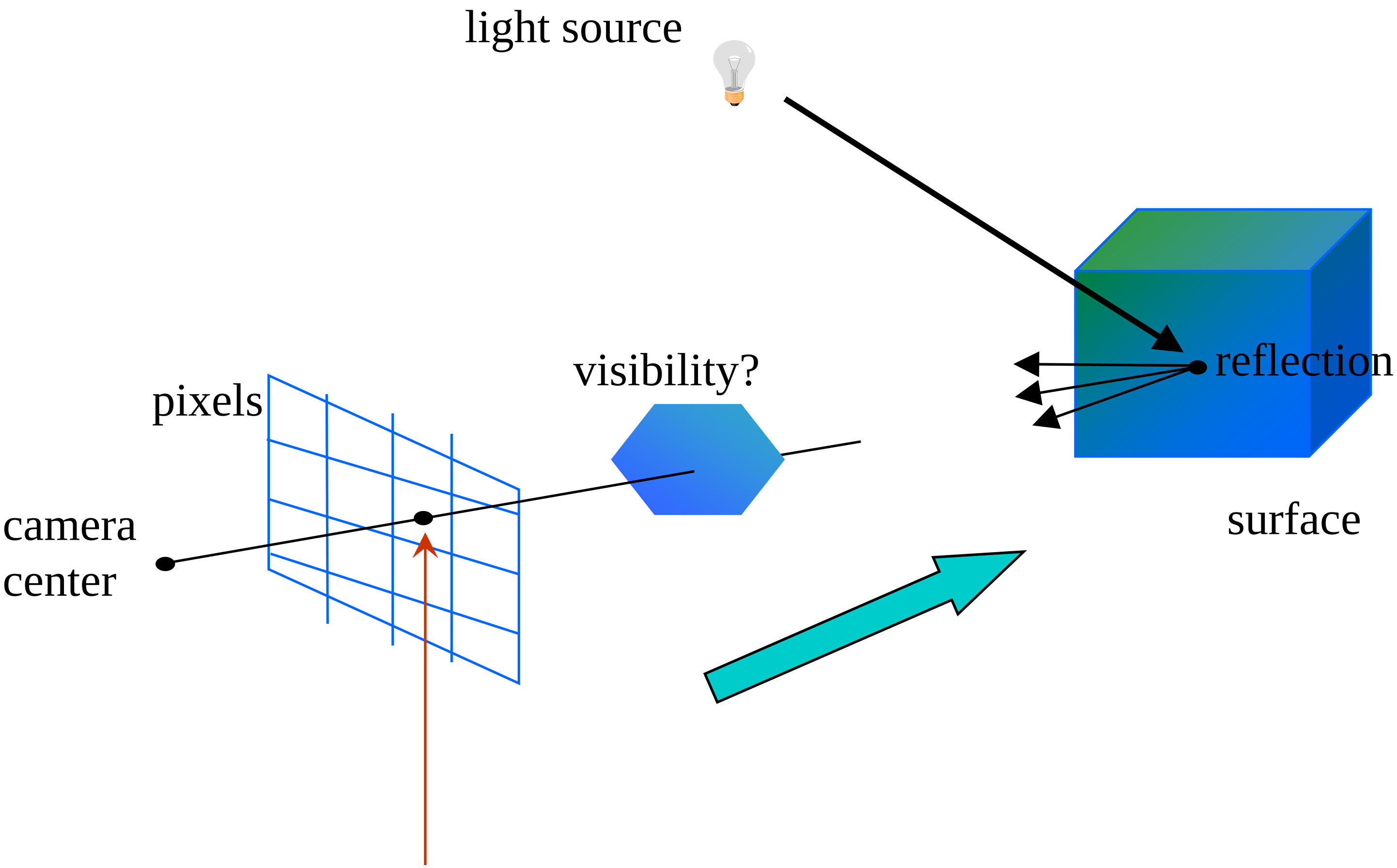
- In creating and interpreting images, we need to understand two things:
  - Geometry – Where scene points appear in the image (image locations)
  - Radiometry – How “bright” and “colorful” they are (image values)
- **Geometric** enables us to know something about the scene location of a point imaged at pixel (*u*, *v*)
- **Radiometric** enables us to know what a pixel value implies about surface lightness and illumination
- This is relevant to both *computer vision* and *computer graphics*
  - Ray tracing
  - Illumination and shading models

Radiometry

- Radiometry is the measurement of light
  - Actually, electromagnetic energy

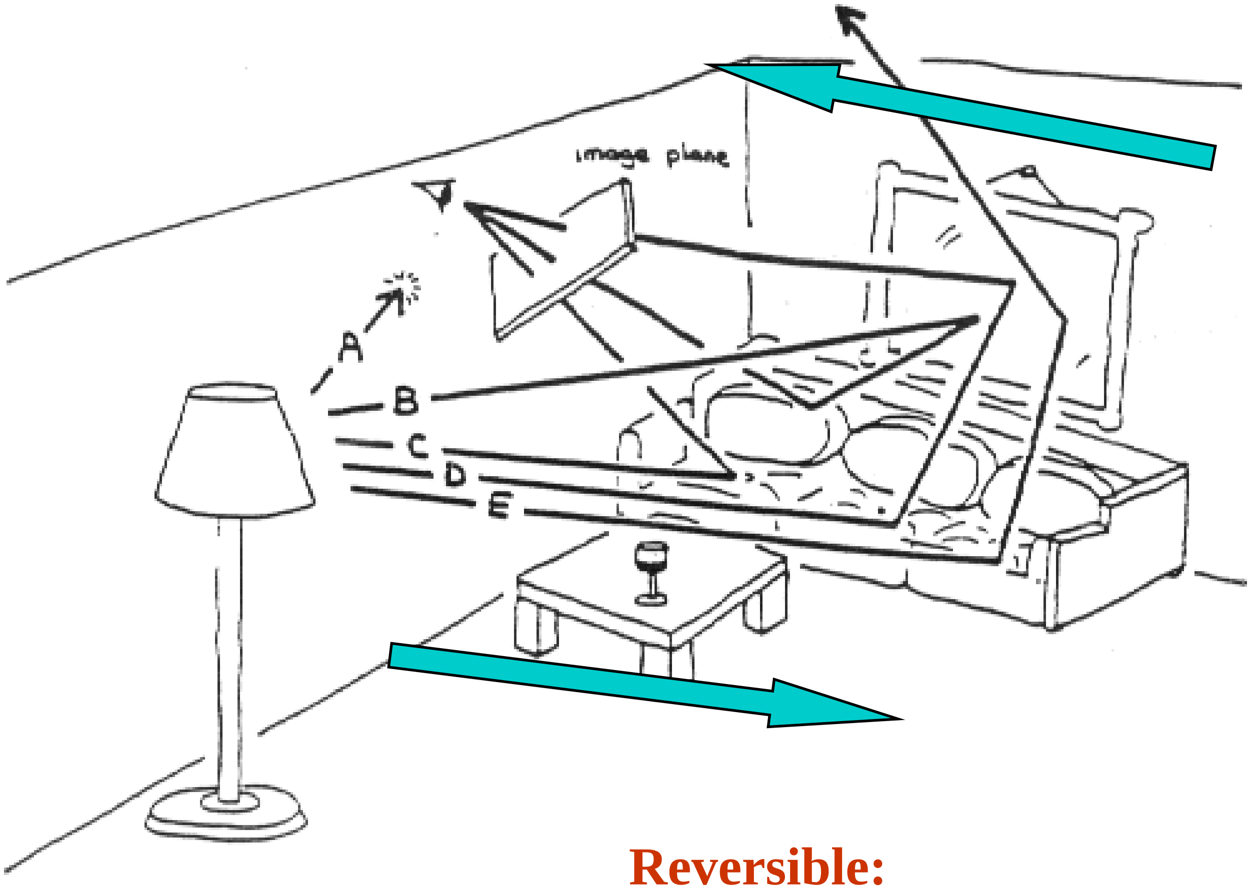
- Imaging starts with light sources
  - Emitting photons – quanta of light energy
  - The sun, artificial lighting, candles, fire, blackbody radiators …
- Light energy interact with surfaces
  - Reflection, refraction, absorption, fluorescence…
  - Also atmospheric effects (not just solid surfaces)
- Light energy from sources and surfaces gets imaged by a camera
  - Through a lens, onto a sensor array, finally to pixel values – an image!

Computer Vision



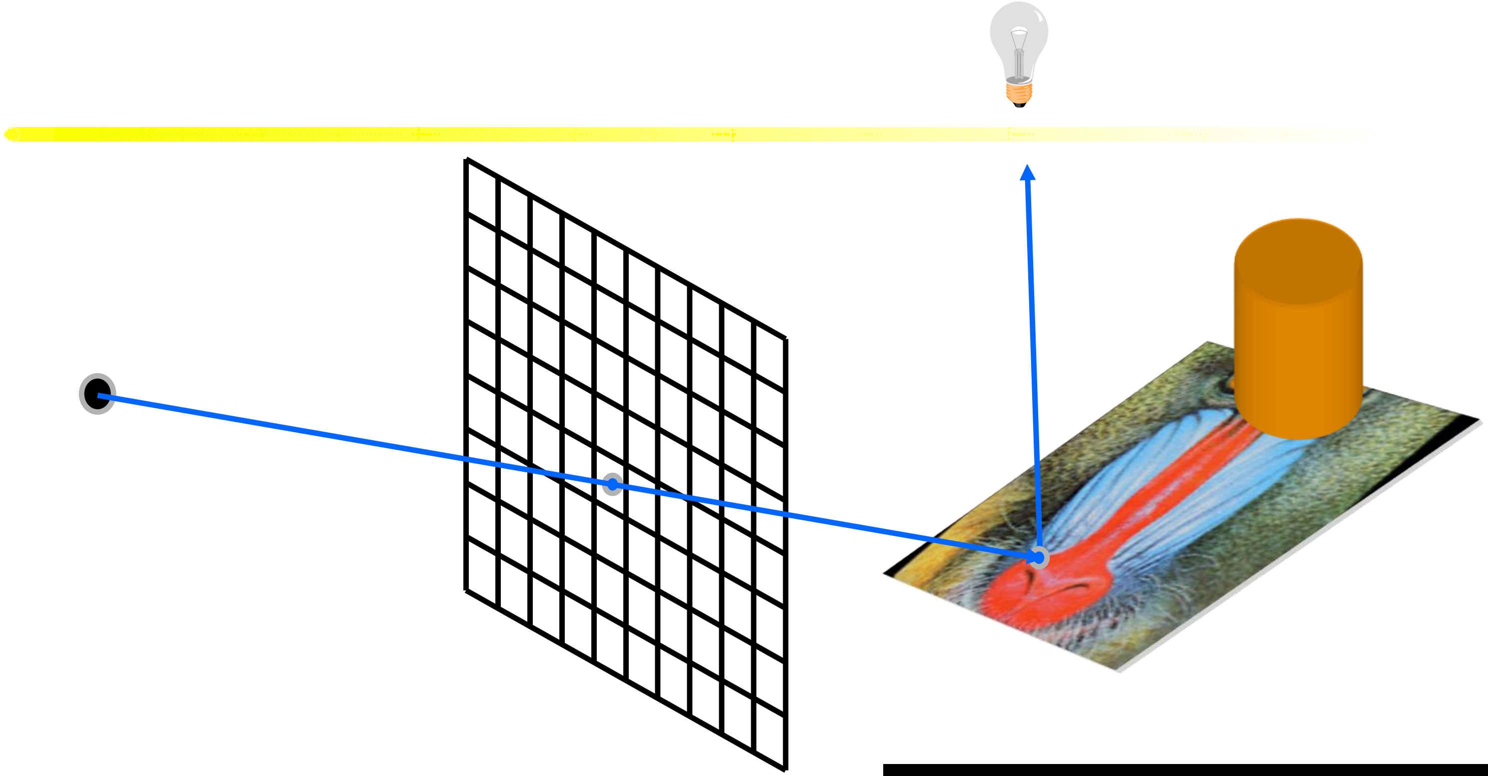
What’s the intensity value (and color) at this pixel?

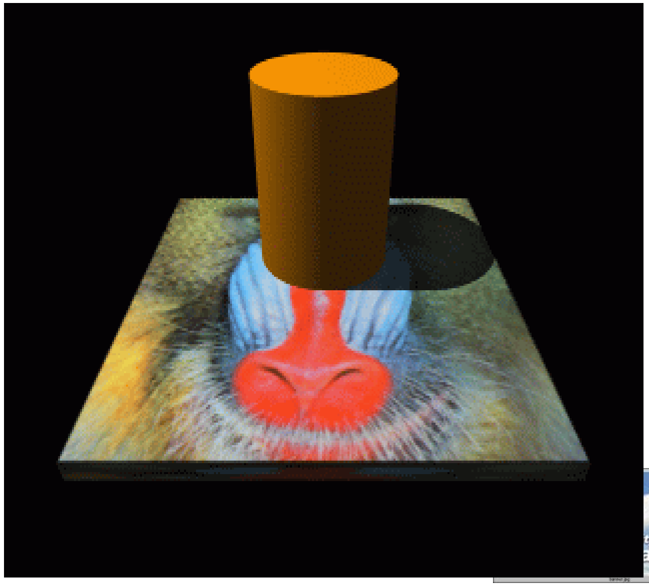
Computer Graphics



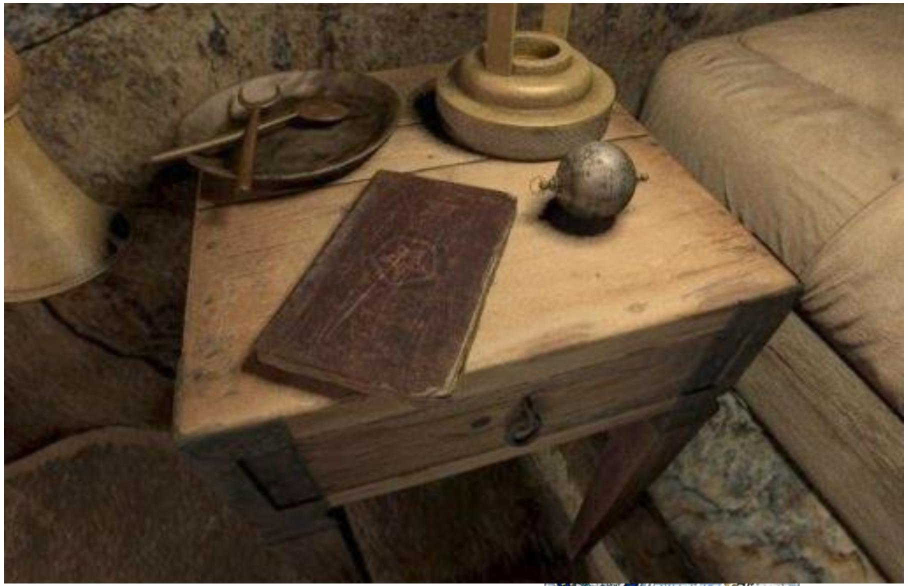
Reversible:

- **From camera to light sources**
- **From light sources to camera**





Computer graphics examples



CG example: Pixar

Geri's Game
1997 Oscar Award
Best Animated Short Film

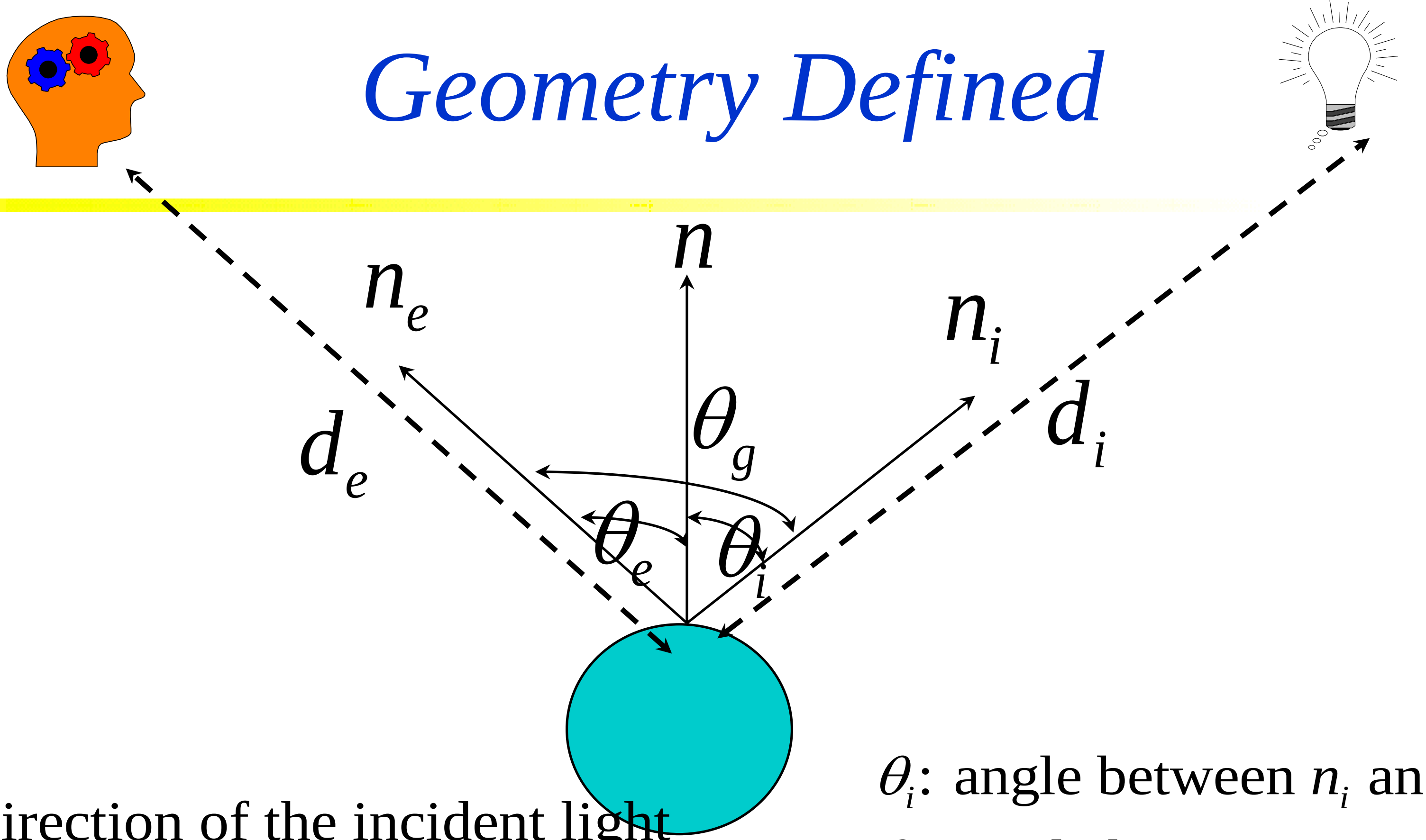
Simple Shading Models

- A jumbled collection of *ad hoc* & *heuristic* techniques, developed over the past two decades
- Concerned mostly with the *primary* ray (light source *to* surface *to* viewer)
- Secondary, tertiary, etc. reflection *not* considered
- Shading individual points and polygons
- Shadow, texture, etc.

Simple Shading Models

- Color (Shading) = f (light source, surface material, geometry, viewer perception model, etc.)
    - light sources: color (spectrum distribution), position, orientation, spatial extent, etc.
    - surface material: orientation, reflectivity, transparency, roughness, etc.
    - geometry: distance, relative orientation, etc.
    - viewer perception model: color model, sensitivity, etc.

Geometry Defined



n_i: direction of the incident light

n: surface normal direction

n_e: direction to the observer (camera)

θ_i: angle between n_i and n

θ_g: angle between n_i and n_e

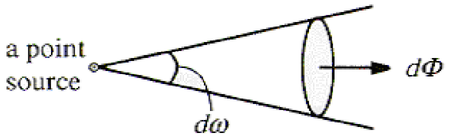
θ_e: angle between n_e and n

d_i: distance from the light source to the object

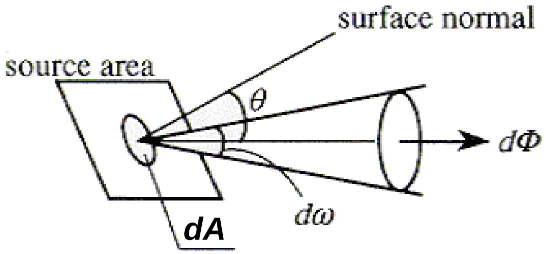
d_e: distance from the object to the camera

Quantities Defined

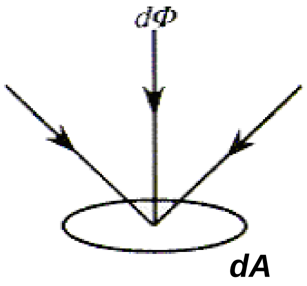
- Radiant energy – photons
- Radiant flux – rate of radiant energy (rate of photons)
- Radiant intensity



- Radiance



- Irradiance – total amount of radiant flux falling on a unit surface from all directions (a.k.a. *radiant flux density*) (*incident* energy)



Radiometric Measurements/Quantities

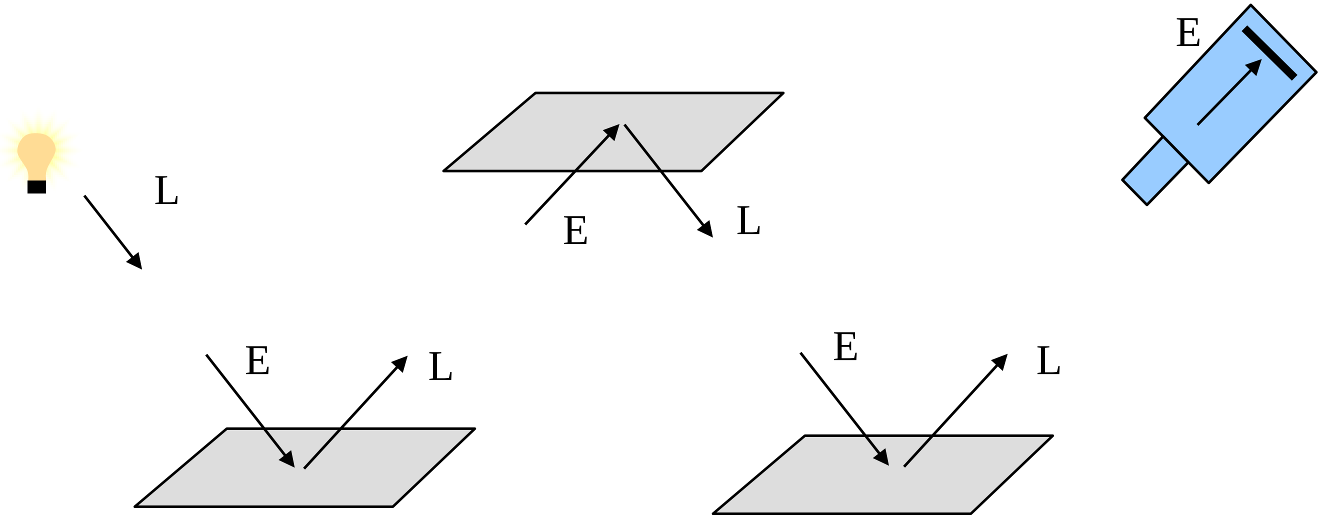
Radiant energy	Q_e		Energy	J
Radiant flux	Φ_e	$\Phi_e = \frac{\Delta Q_e}{\Delta t}$	Energy per unit time (power)	J/s or W
Radiant intensity	I_e	$I_e = \frac{\Delta \Phi_e}{\Delta \omega}$	Source power radiated per unit solid angle	W/sr
Radiance	L_e	$L_e = \frac{\Delta I_e}{\Delta A}$	Source power radiated per unit area per unit solid angle	W/m^2-sr
Irradiance	E_e	$E_e = \frac{\Delta \Phi_e}{\Delta A}$	Power falling on unit area of target	W/m^2

These are all functions of wavelength

Photons = Energy

Radiance and irradiance

- Radiance (L) – energy exiting a source or surface
- Irradiance (E)– incoming energy



Which (E or L) does a camera sensor array directly measure?

Photometric Measurements/Quantities

- Photometry is the measurement of visible light, weighted by the spectral response of the human visual system

Luminous energy	Q_v	Energy in visible spectrum	Talbot
Luminous flux	Φ_v	Luminous energy per unit time (power)	Talbot /s or Lumen
Luminous intensity	I_v	Luminous power radiated per unit solid angle	Lumen/sr or candela
Luminance	L_v	Luminous power radiated per unit area per unit solid angle	Lumen/m^2-sr
Illuminance	E_v	Luminous power falling on unit area of target	Lumen/m^2 or Lux

Example

- Luminance of common sources:
  - surface of the sun: 2,000,000,000 cd/m2
  - sunlit clouds: 30,000 cd/m2
  - clear day: 3000 cd/m2
  - overcast day: 300 cd/m2
  - moonlight: 0.03 cd/m2
  - moonless sky: 0.00003 cd/m2

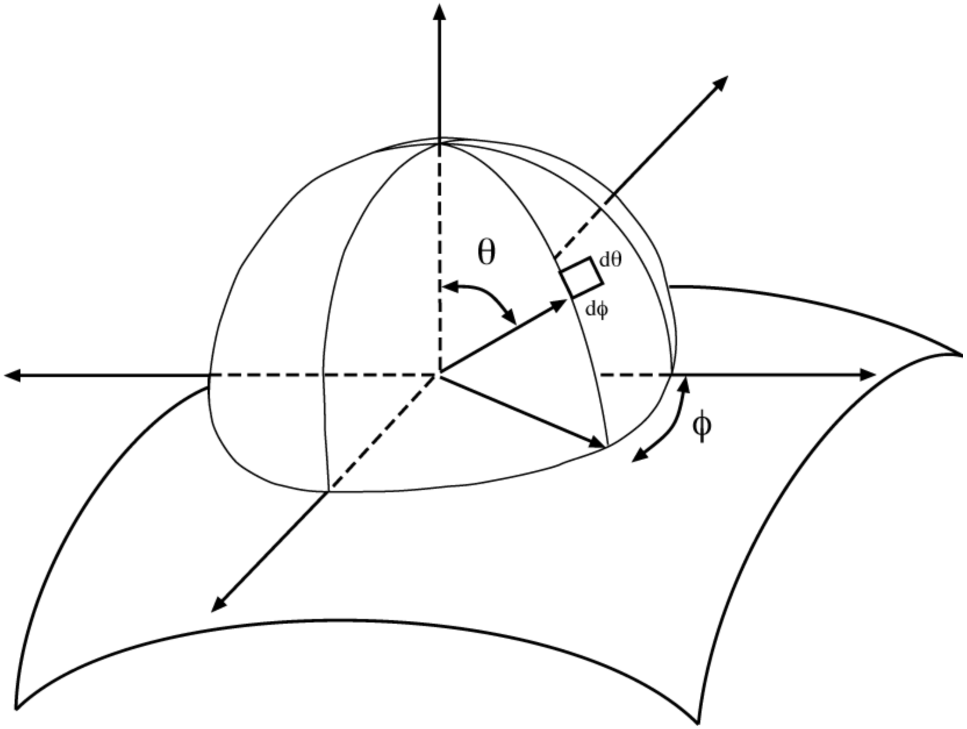
Photometric term

- Luminance = commonly called “brightness”
  - Density of radiated power
- Radiance = “scene brightness”
- Irradiance = “image brightness”

Radiometric terms

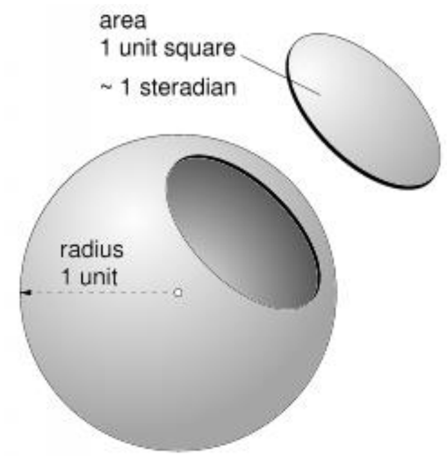
Solid angle

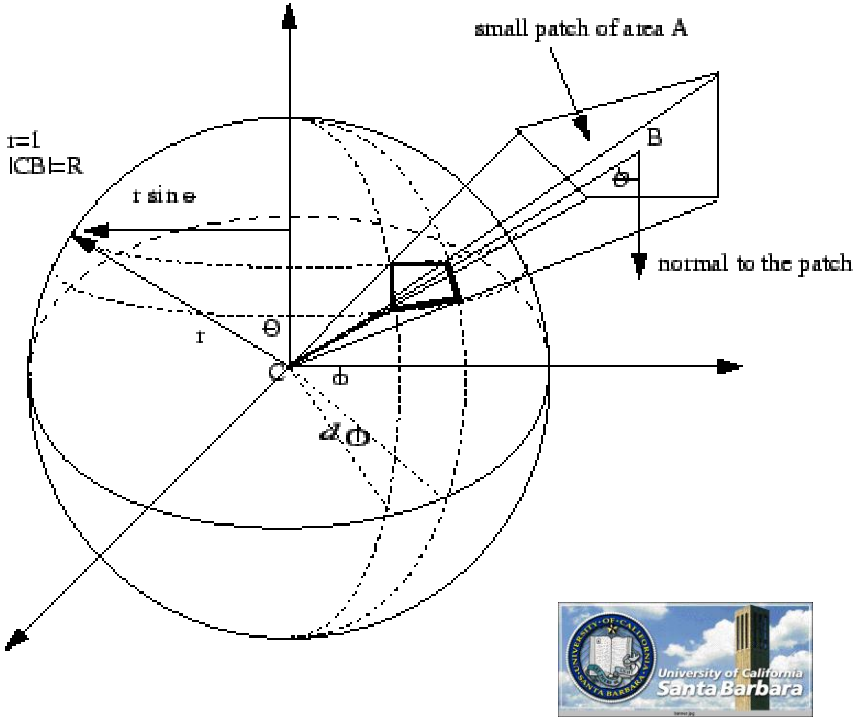
- Around any point on a surface is a hemisphere of directions
  - Parameterized by two angles, θ and ϕ
- We’ll be considering light entering and exiting such hemispheres



Solid angle

- Solid angle of a cone of directions – the area cut out by the cone on the unit sphere
  - Solid angle (ω) = surface area at $r=1$
  - $\omega = A/r^2$
- Solid angle of a complete sphere = 4π steradians (sr)





What’s the area of a circle?
What’s the surface area of a sphere?

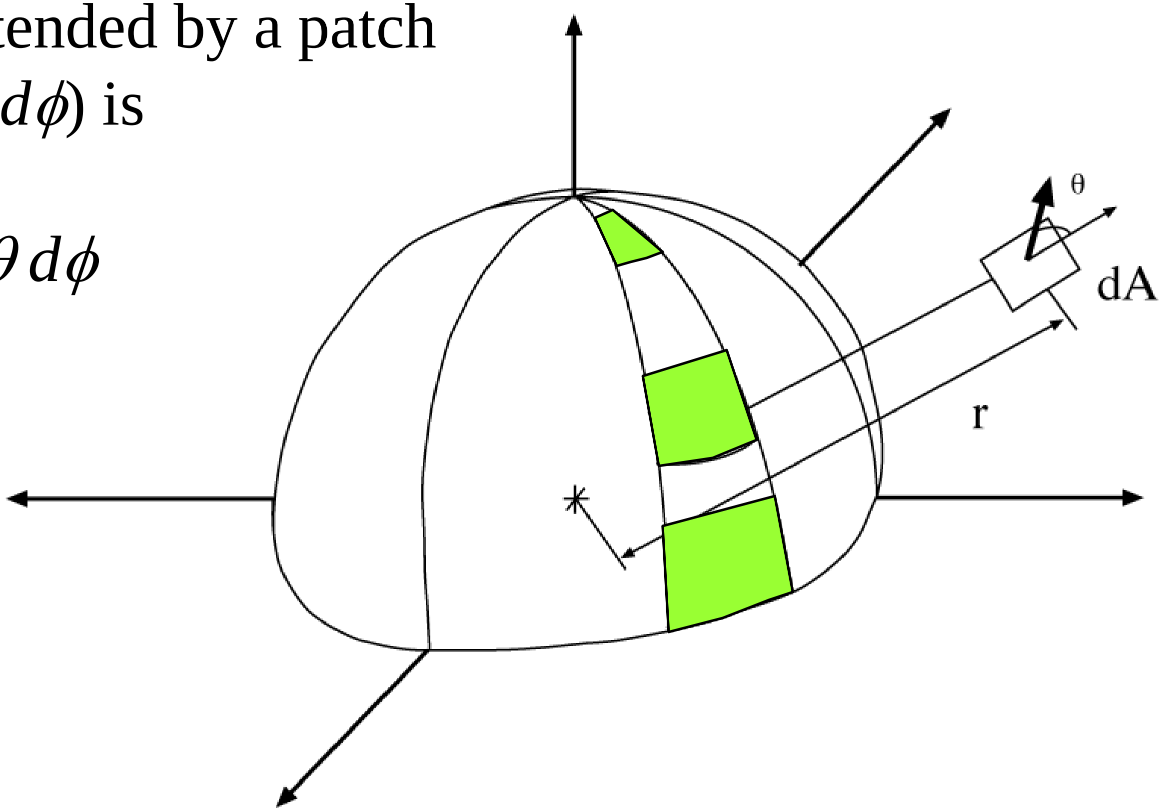
Solid angle

- The solid angle subtended by a patch of area *dA* is given by

$$d\omega = \frac{dA\cos\theta}{r^2}$$

- The solid angle subtended by a patch of angular size ($d\theta$, $d\phi$) is

$$d\omega = \sin\theta \, d\theta \, d\phi$$



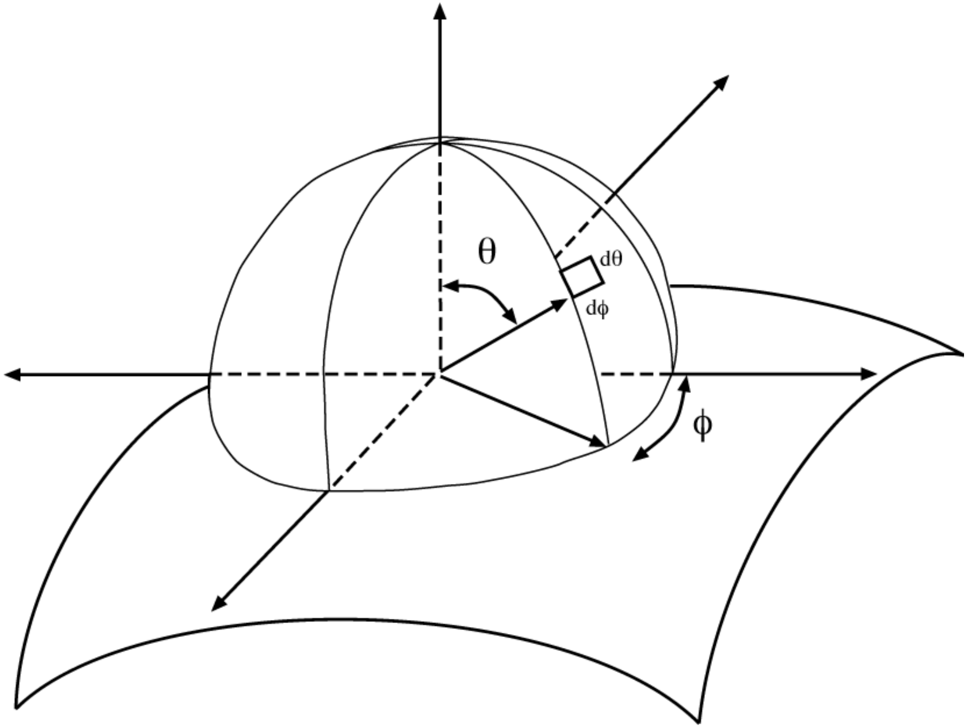
Total solid angle

- To calculate the total solid angle, integrate over the unit hemisphere:

$$\int_{\Omega} d\omega = \int_{0}^{2\pi} \int_{0}^{\frac{\pi}{2}} \sin\theta \, d\theta \, d\phi$$

$$= \int_{0}^{2\pi} \left[-\cos\theta\right]_{0}^{\frac{\pi}{2}} d\phi$$

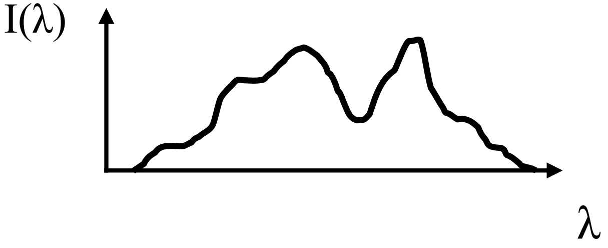
$$= \int_{0}^{2\pi} d\phi = 2\pi$$

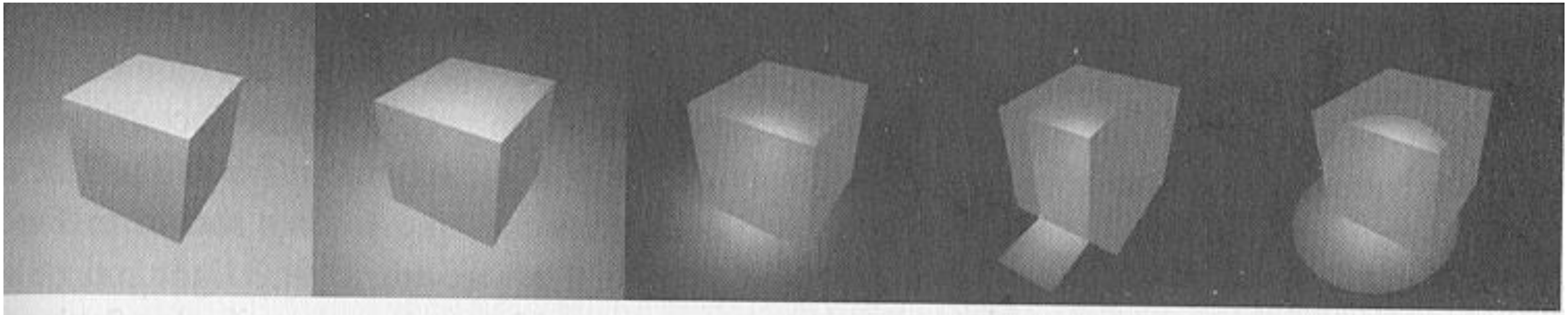


So there are 4π steradians (sr) in a sphere

Light sources

- Spectral properties: R-G-B, H-S-V, etc.
- Strength: characterized by its radiance (joules/sec m^2 sr, watts/m^2 sr, energy/unit-time-area-solid-angle)
- Geometry:
  - Point source (location only, e.g. bulb)
  - Directional source (orientation only, e.g. Sun)
  - Ambient source (no location nor orientation)
  - Spot light (point source + spread angle)
  - Flap, barn-door (directional source + spatial extent)



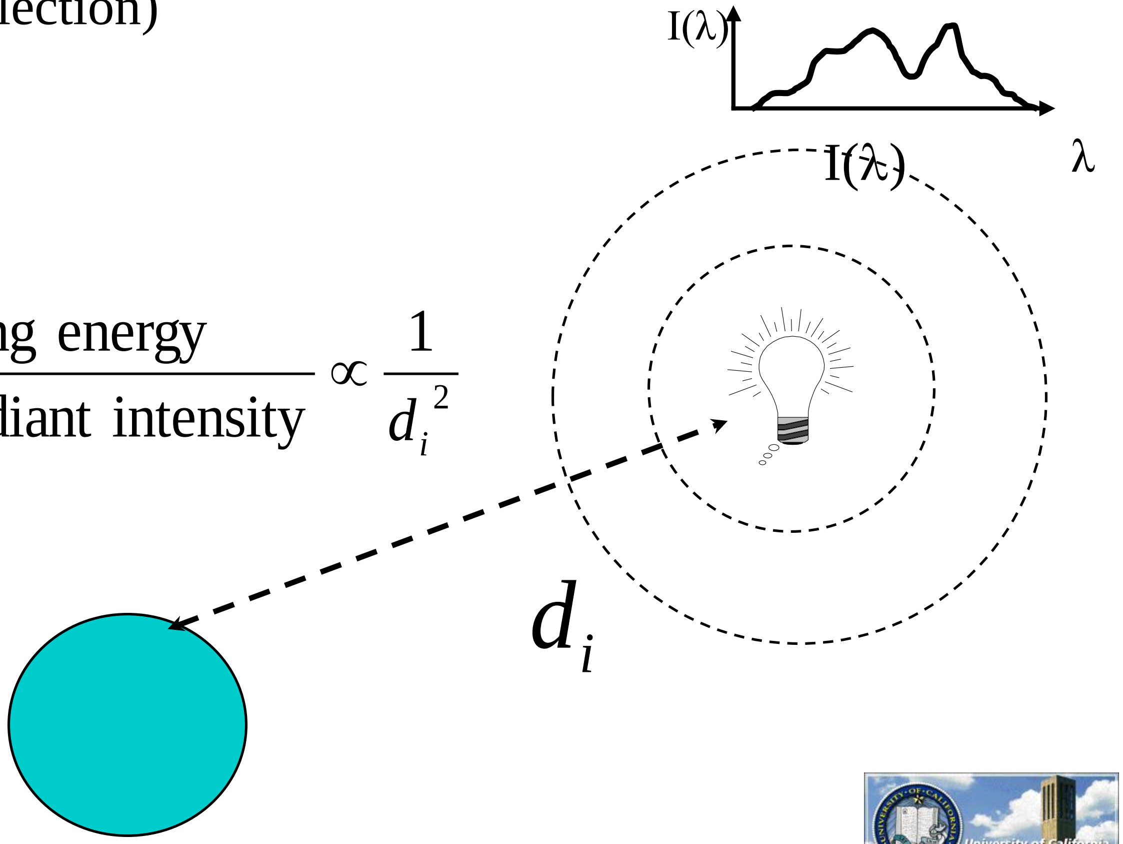


(a) (b) (c) (d) (e)

Arriving Light

- Light *arriving* at a surface
  - Strength: characterized by its irradiance (joules/sec m^2, watts/m^2, energy/time-area
  - Distance: how much emitted energy actually gets to the object (no attenuation, no reflection)

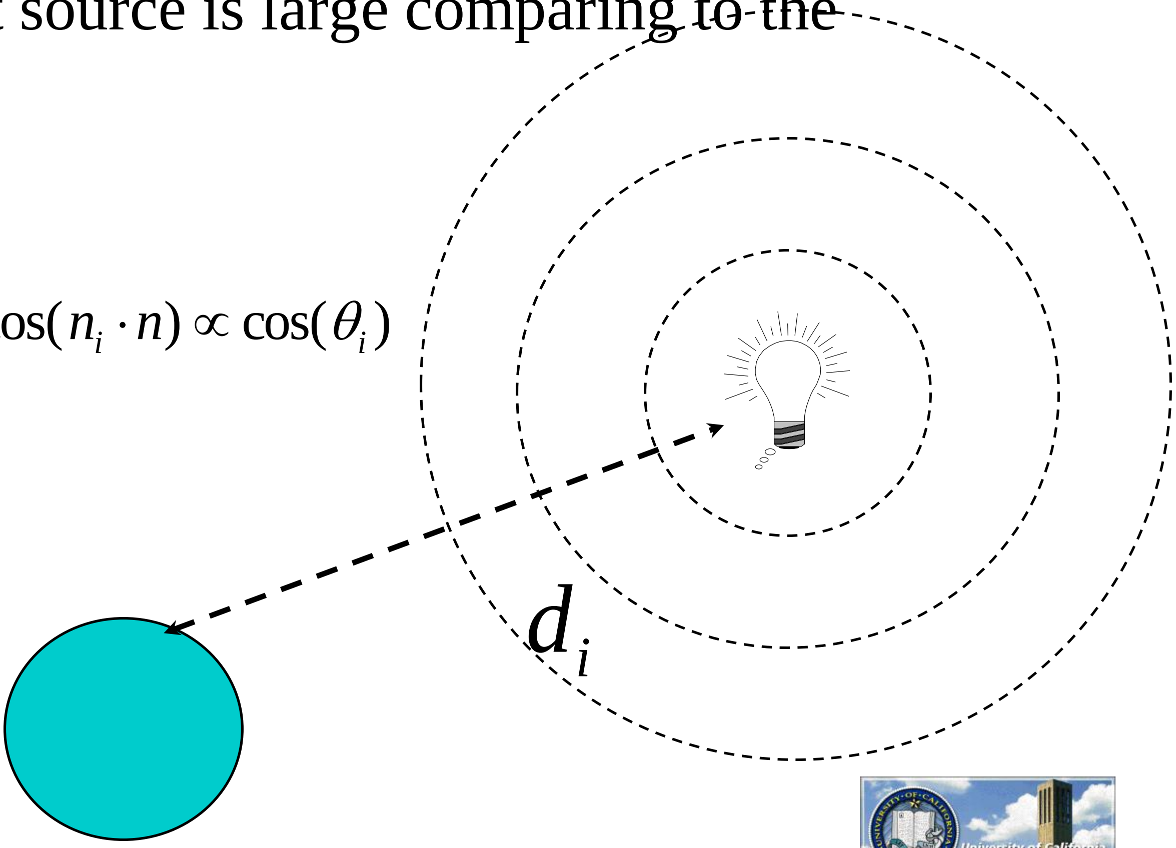
$$\frac{\text{arriving energy}}{\text{source radiant intensity}} \propto \frac{1}{d_i^{\,2}}$$



Incident Light

- Relative orientation: how much emitted energy actually incident on the object
- Follow cosine law $\cos(n_i \cdot n) = \cos(\theta_i)$
- Distance to the light source is large comparing to the object size

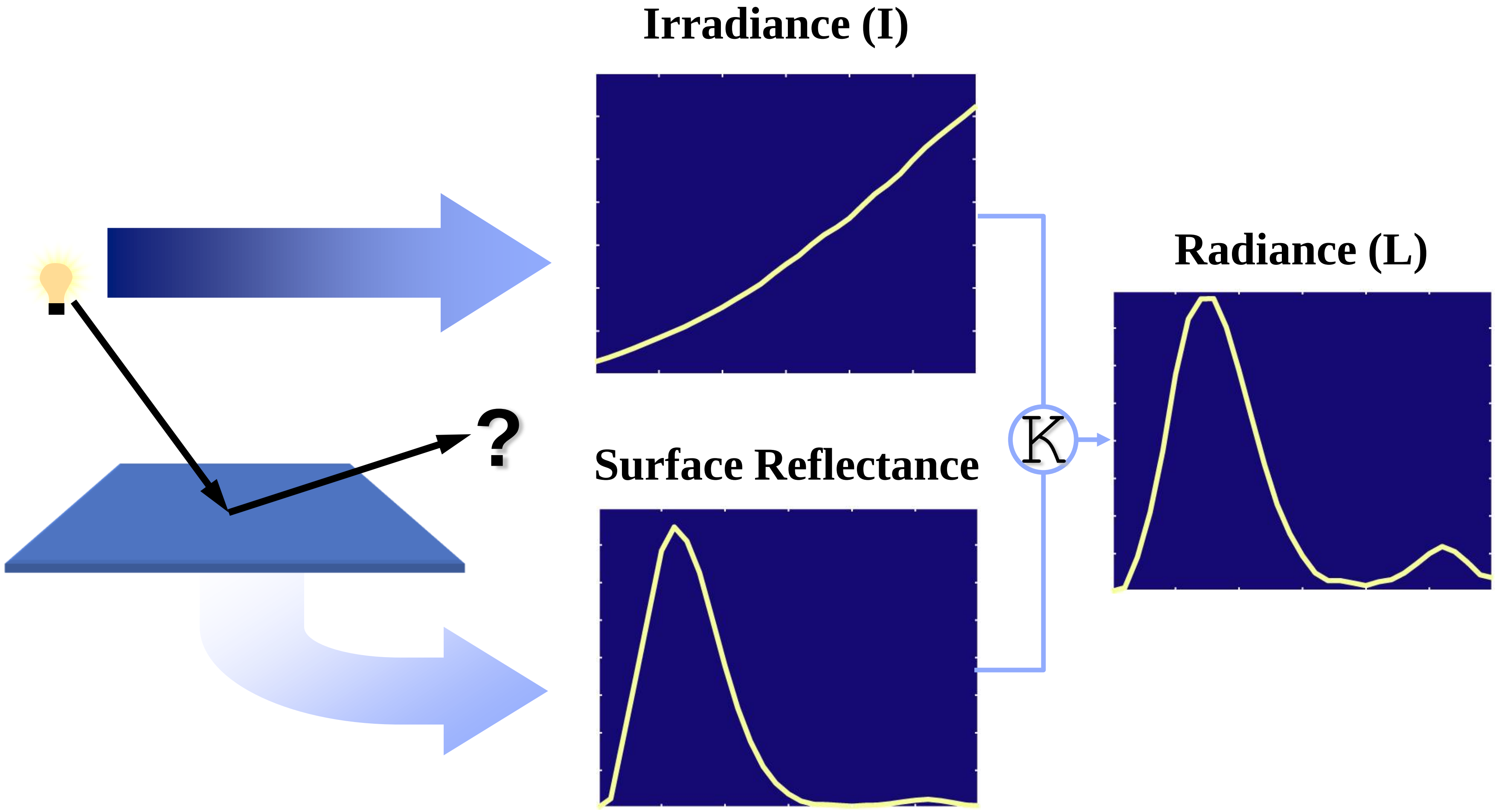
$$\frac{\text{incident energy}}{\text{arriving energy}} \propto \cos(n_i \cdot n) \propto \cos(\theta_i)$$



Exiting Light

- How much comes out and in what direction?
- Three things can happen
    - absorption
    - reflection (the same side)
        - diffuse (no dominant direction e.g. chalk, cloth)
        - specular (w. a dominant direction e.g. waxed apple, mirror)
    - refraction (the opposite side)
        - diffuse (no dominant direction)
        - specular (w. a dominant direction)
    - absorption + reflection + refraction = total incident

Reflectance example



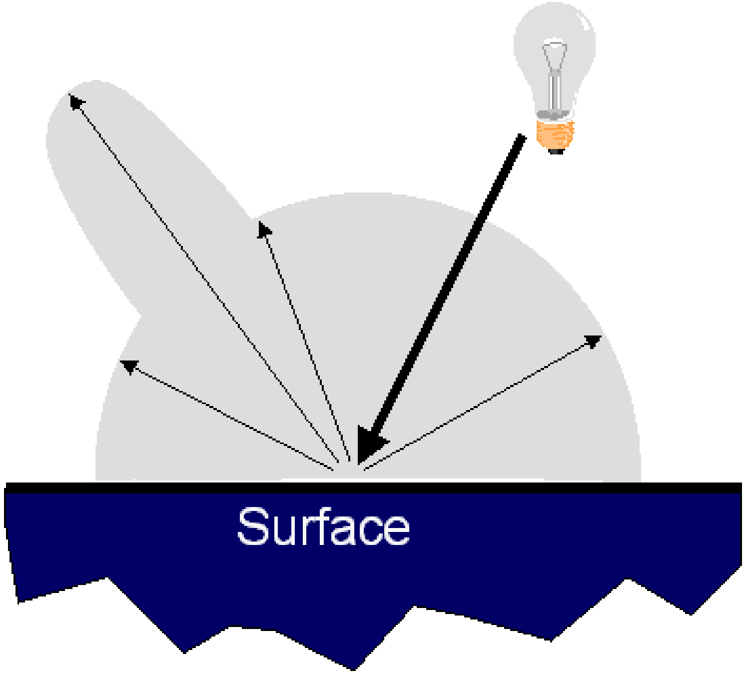
Irradiance (I)

Surface Reflectance

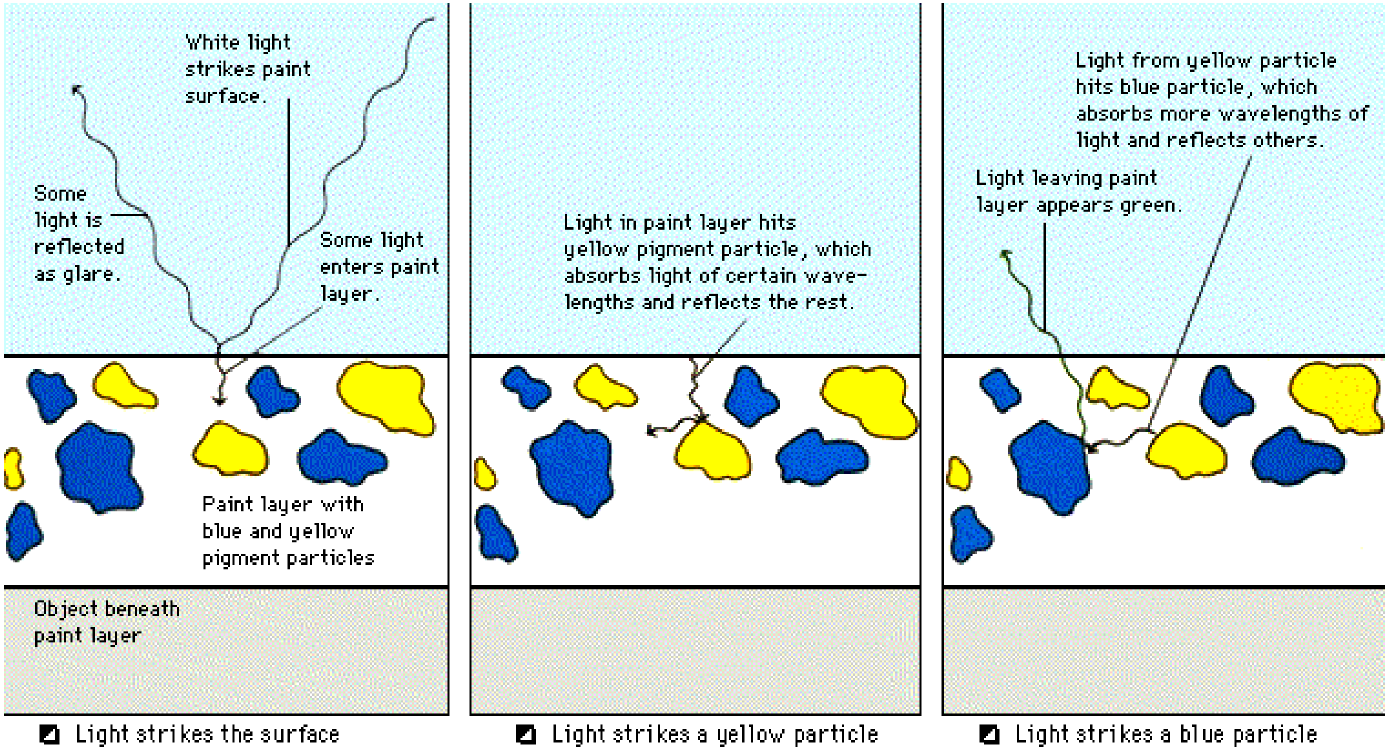
Radiance (L)

Interface Reflectance

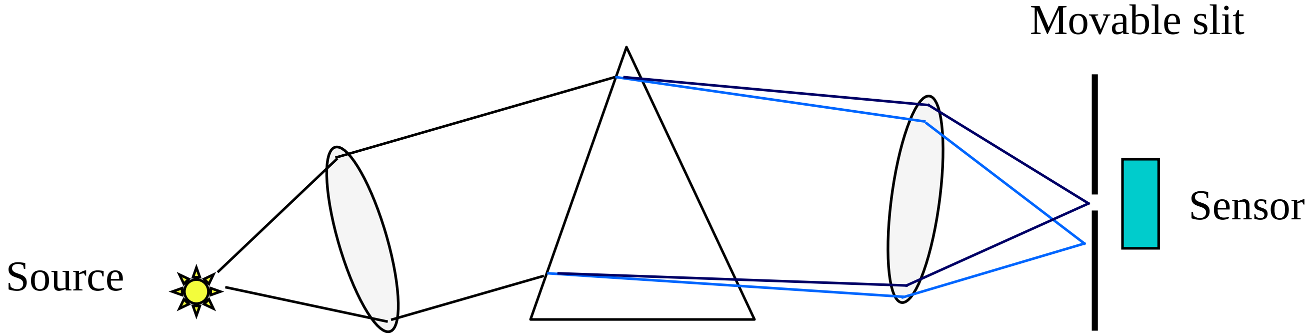
- For most non-metal objects, interface reflectance is color of the incident light
- For metal objects, different colors



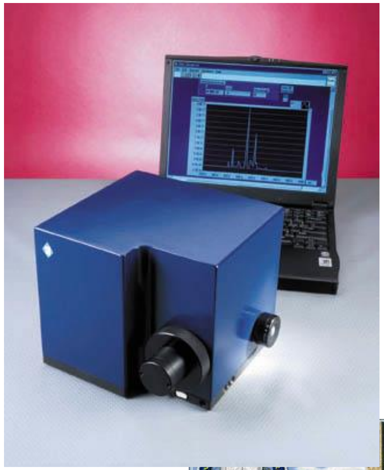
Body Reflection



Spectroradiometer



Spectral range: 250-1100 nm
Wavelength accuracy: ±0.5 nm
Repeatability: ±0.2 nm
Resolution: 0.2 nm



University of California Santa Barbara

Bidirectional reflectance distribution function (BRDF)

$f(\theta_i, \theta_e, \theta_g, \lambda)$

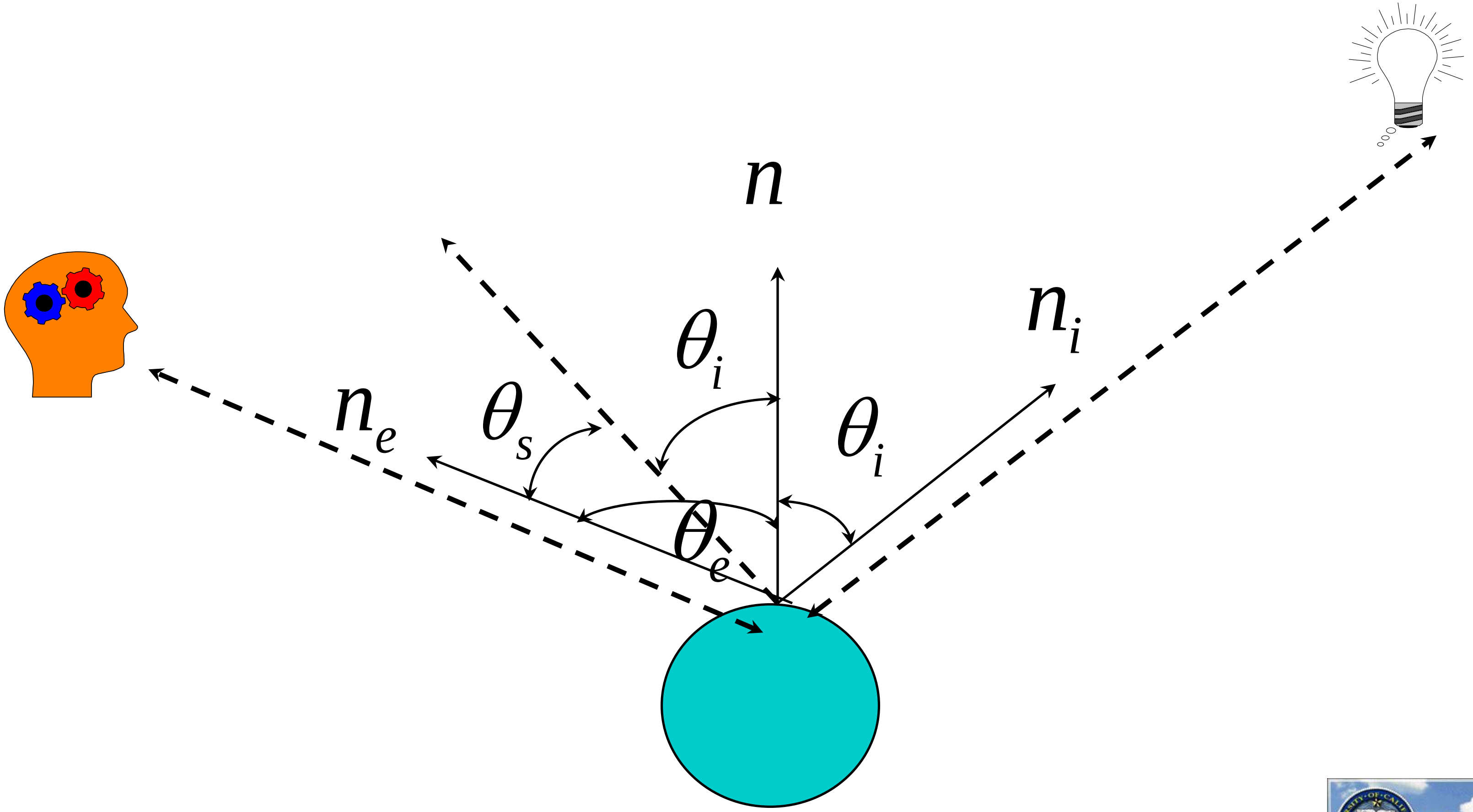
- Fraction of incident light from the incident direction to the viewing direction per unit surface area per unit viewing angle
- Diffuse (Lambertian) reflection
- Ideal specular (Mirror) reflection
- It is a function of wavelength too!

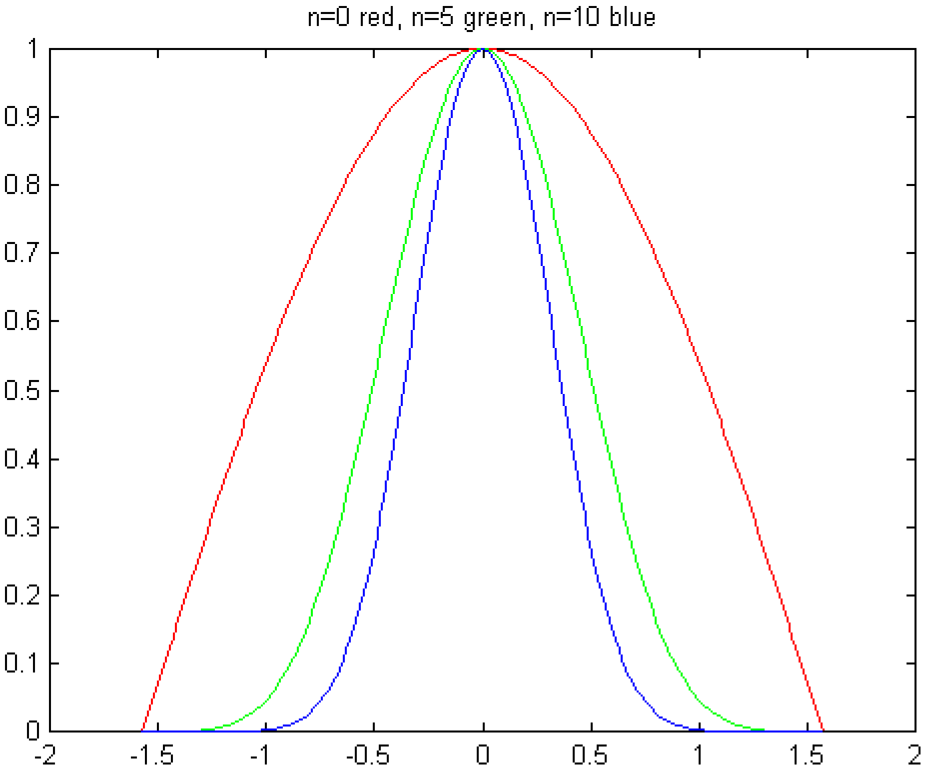
$$f(\theta_i, \theta_e, \theta_g, \lambda) = k_d$$

$$f(\theta_i, \theta_e, \theta_g, \lambda) = \begin{cases} k_s & \theta_i = \theta_e, \theta_g = 2\theta_i = 2\theta_e = \theta_i + \theta_e \\ 0 & otherwise \end{cases}$$

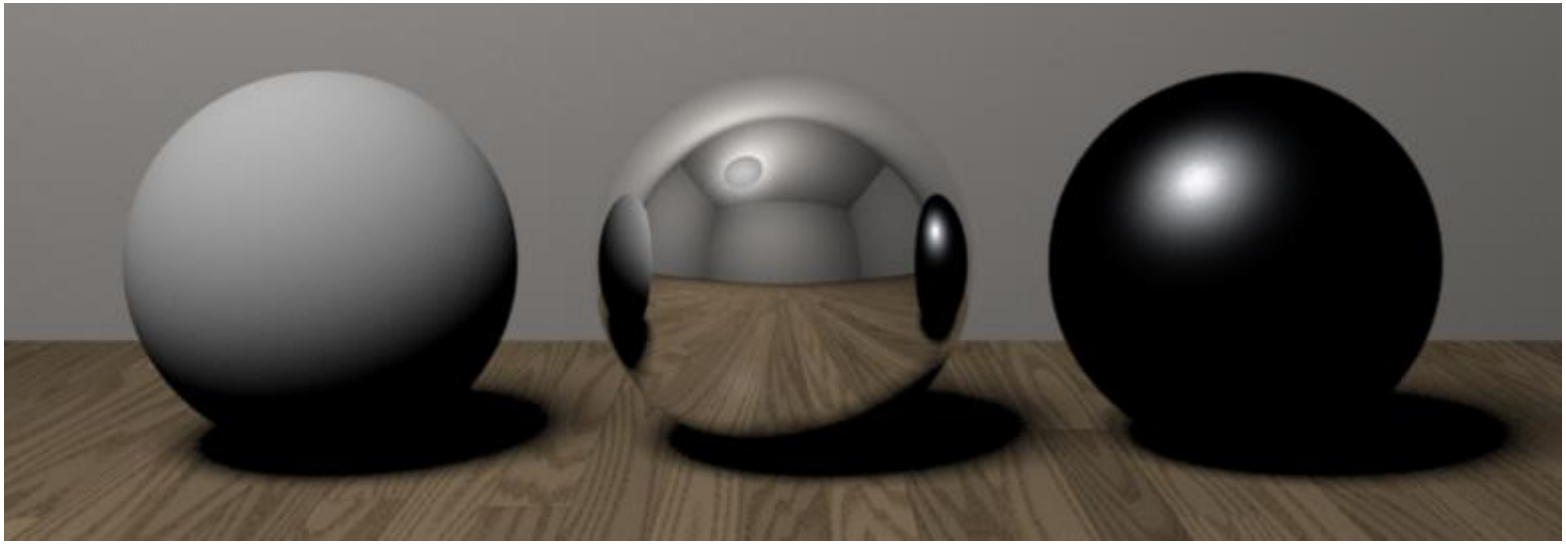
Specular (Mirror) reflection

$$f(\theta_i, \theta_e, \theta_g) = k_s \cos^n(\theta_s) \propto k_s(2\cos(\theta_i)\cos(\theta_e) - \cos(\theta_g))^n$$




n=0 red, n=5 green, n=10 blue
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

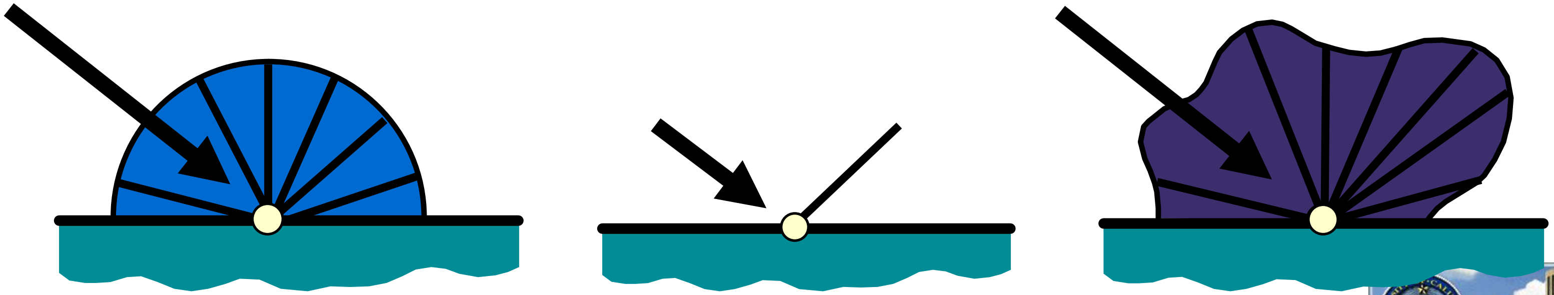
Three surface reflectance functions/models



Ideal diffuse (Lambertian)

Ideal specular

Directional diffuse



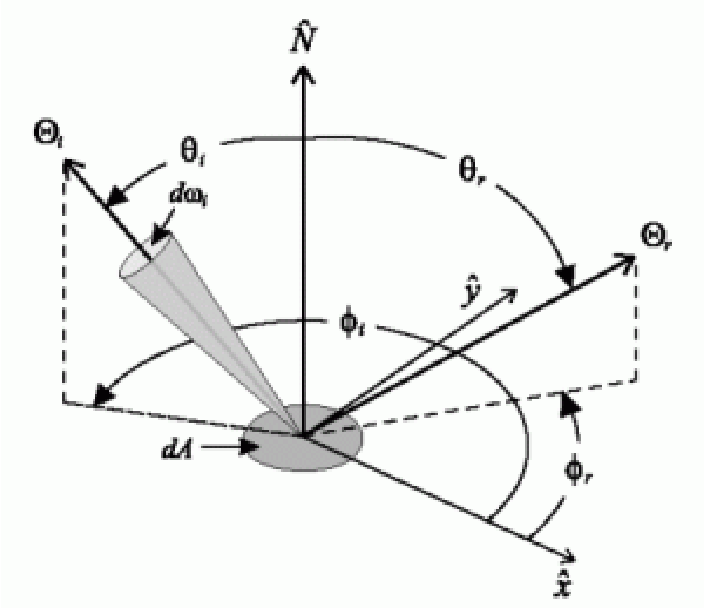
Cook-Torrance Model

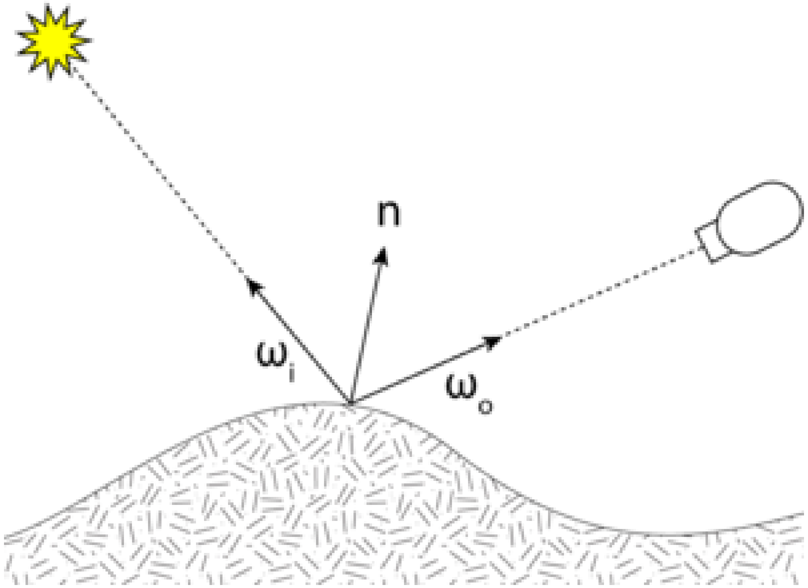
Different parameters give different surface appearances

BRDF

- Bi-directional reflectance distribution function
- 4-dimensional function (angles are parameterized by azimuth and zenith angles)

$$f_r(\omega_i, \omega_o) = \frac{dL_r(\omega_o)}{dE_i(\omega_i)} = \frac{dL_r(\omega_o)}{L_i(\omega_i)\cos\theta_i \, d\omega_i}$$

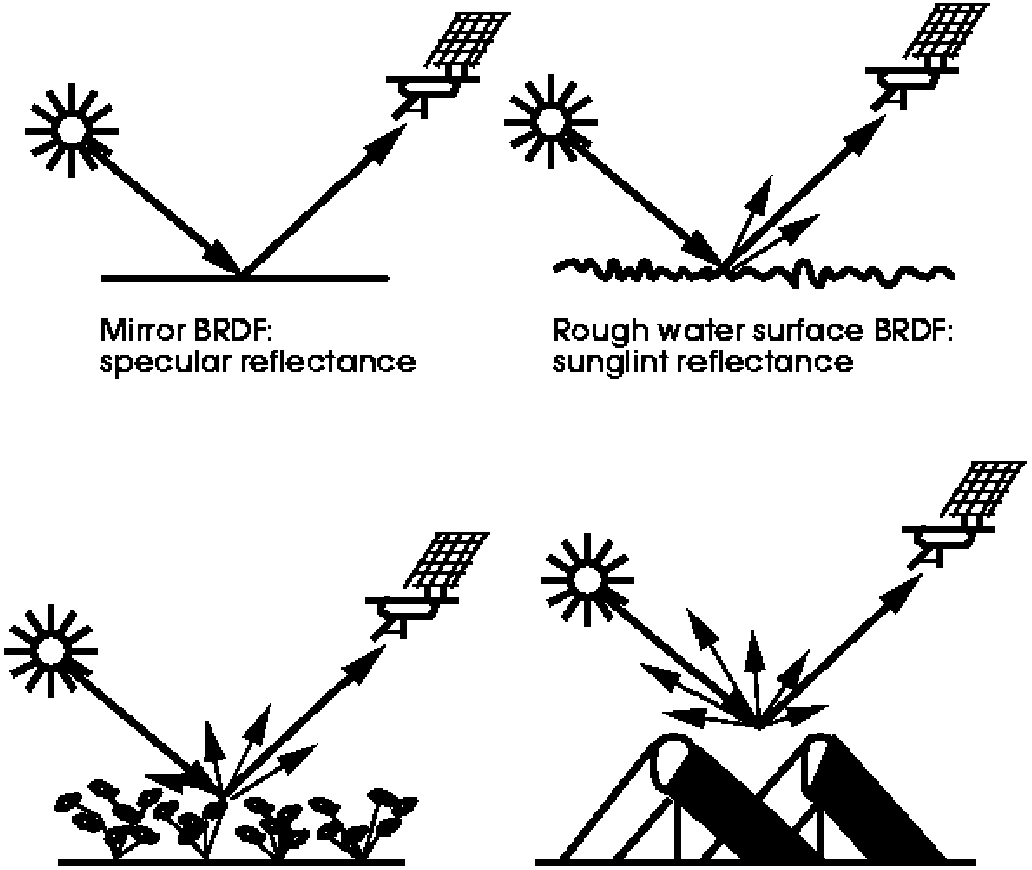




Example

Bidirectional Reflectance Distribution Functions: Causes

Wolfgang Lucht, 1997

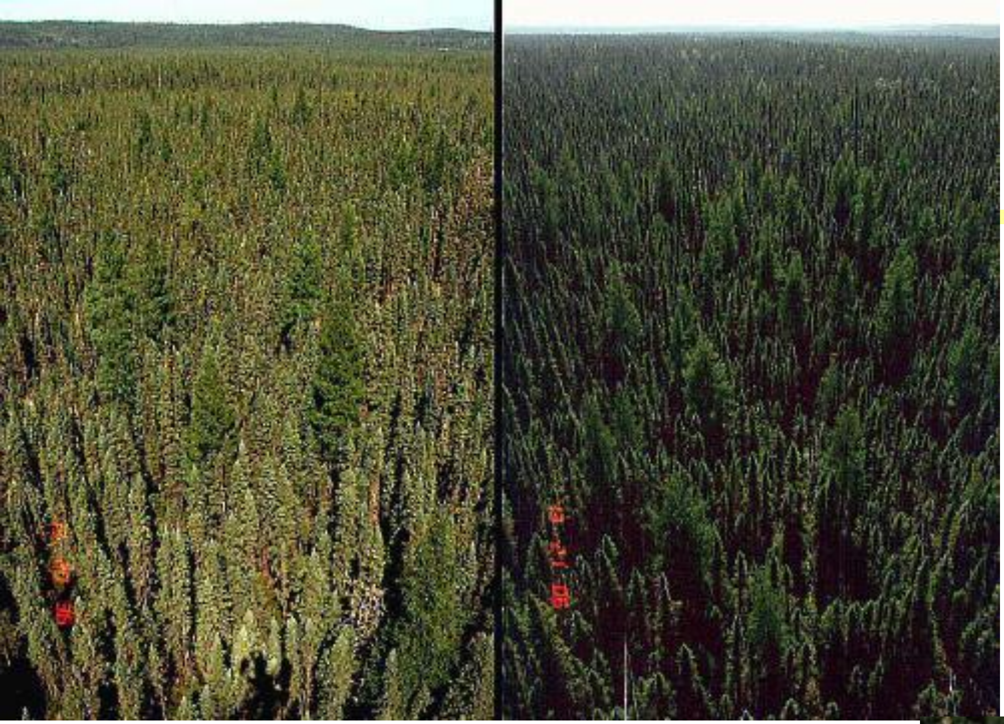


Mirror BRDF:
specular reflectance

Rough water surface BRDF:
sunglint reflectance

Volume scattering BRDF:
leaf/vegetation reflectance

Gap-driven BRDF (Forest):
shadow-driven reflectance



Left: Forward: sun behind observer

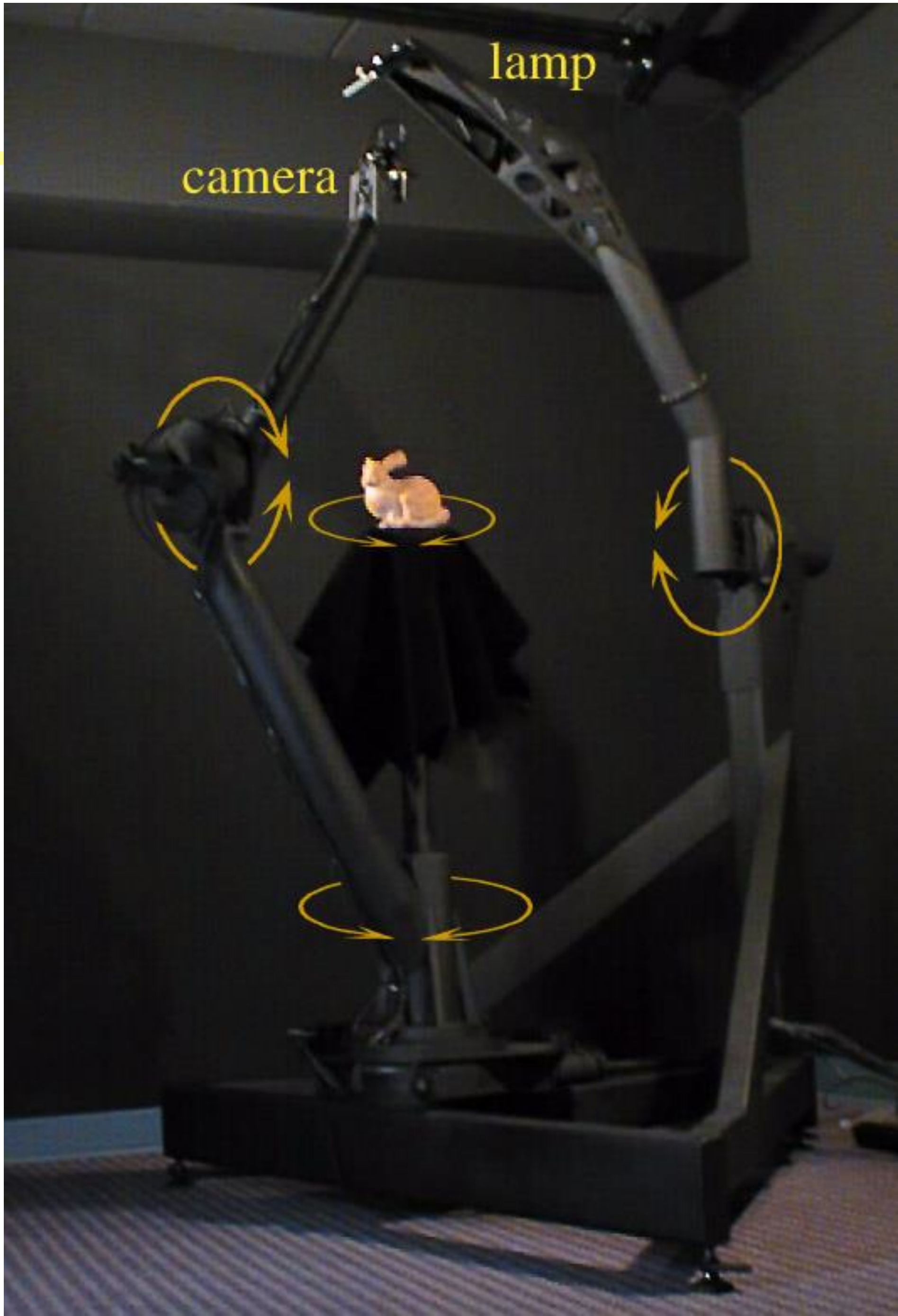
Right Backward: sun opposite observer

Left: Forward: sun behind observer

Right Backward: sun opposite observer

Caveat

- Real-world BRDF is very complicated
- Objects are mostly somewhere in between specular and diffuse
- Theoretically, it is 4-dimensional functions (certainly, simplification is possible)

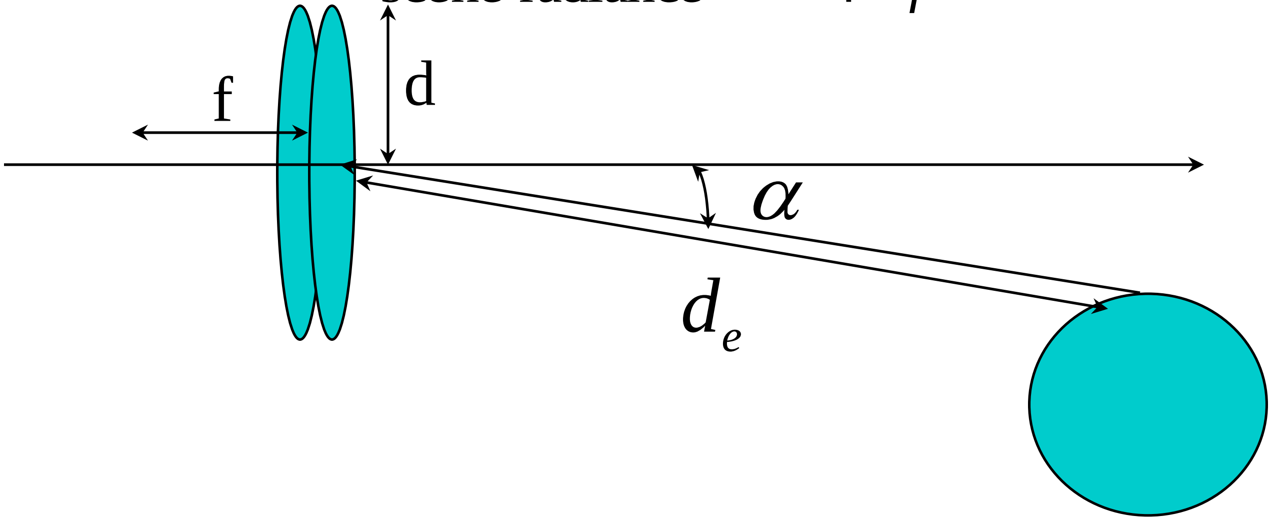


Light reaching the viewer

- How much actually being detected?
- Attenuated by distance
- Attenuated by the lens mechanism

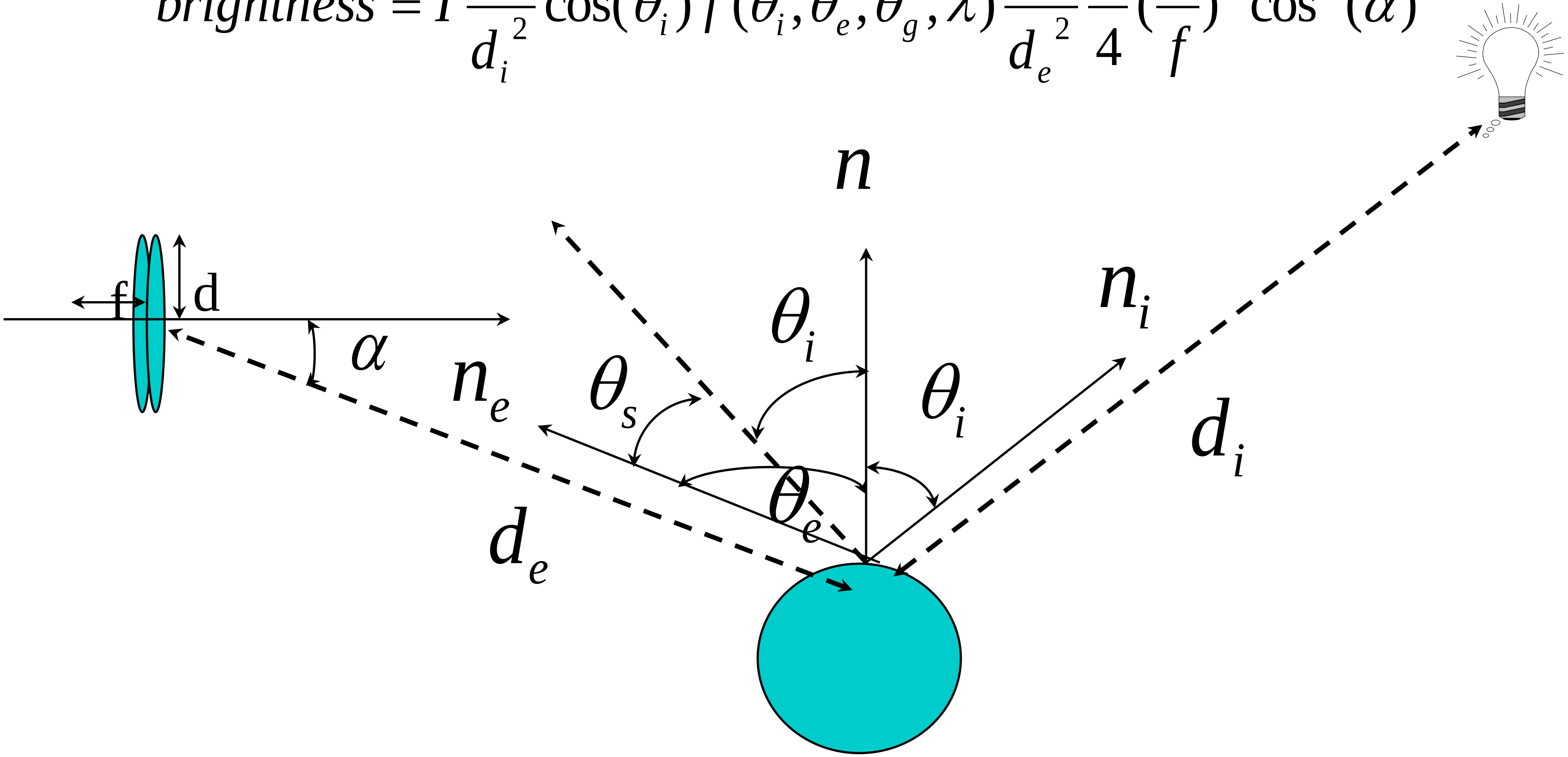
$$\frac{\text{scene radiance}}{\text{object radiance}} = \frac{1}{d_e^{\,2}}$$

$$\frac{\text{image irradiance}}{\text{scene radiance}} = \left(\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4 \alpha\right)$$



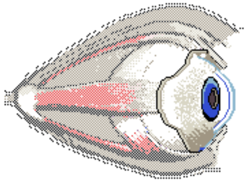
Putting It All Together

$$brightness = I\,\frac{1}{d_i^{\,2}}\cos(\theta_i)\,f(\theta_i,\theta_e,\theta_g,\lambda)\,\frac{1}{d_e^{\,2}}\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2\cos^4(\alpha)$$



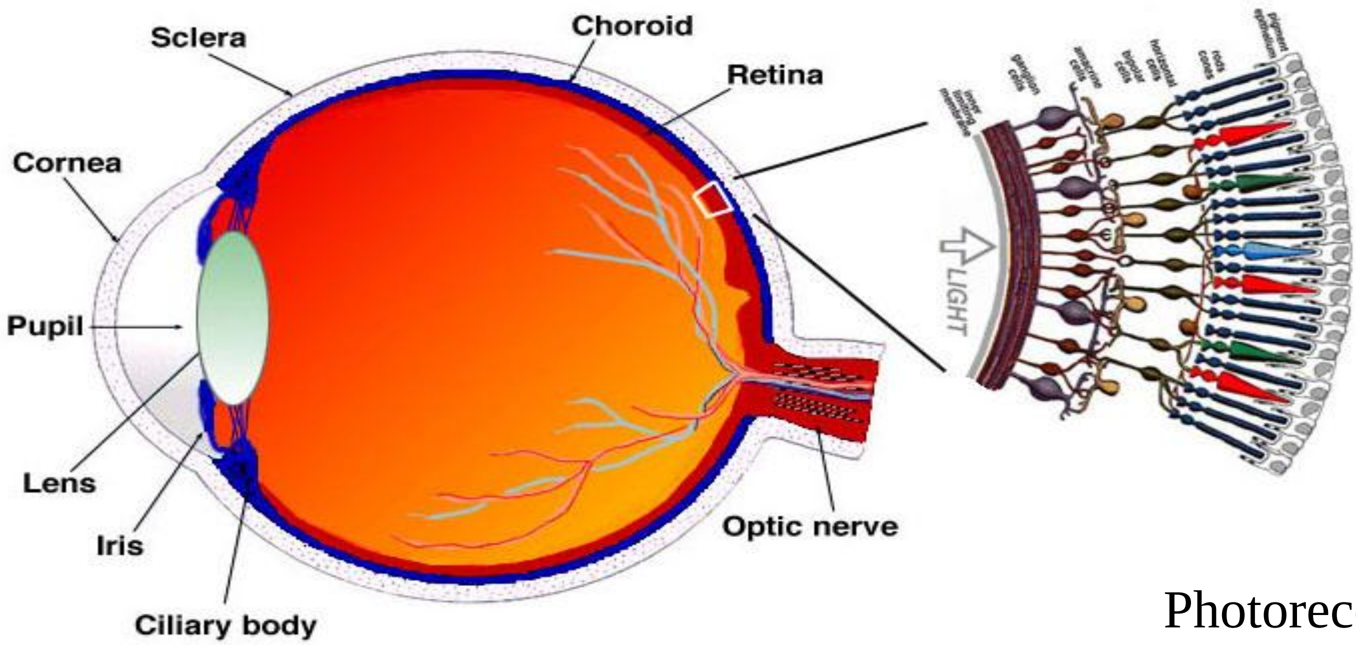
Caveat: only the *primary* ray is considered here!

The Eye



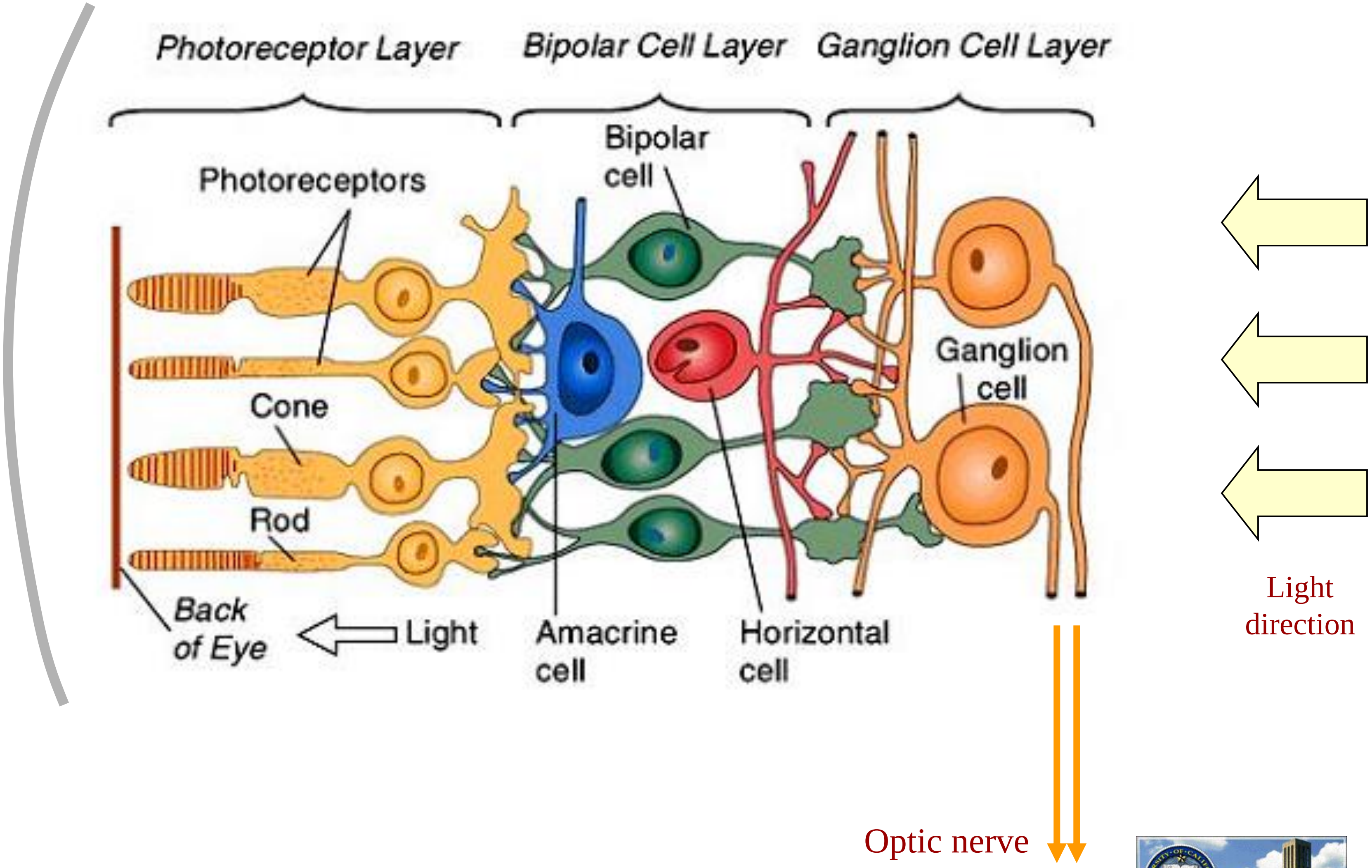
Refractive System
- Cornea (70%)
- Lens (30%)

The lens is also responsible for focusing (adjusting the amount of refraction)



Photoreceptors – sensitive to light ~400-700 nm wavelength

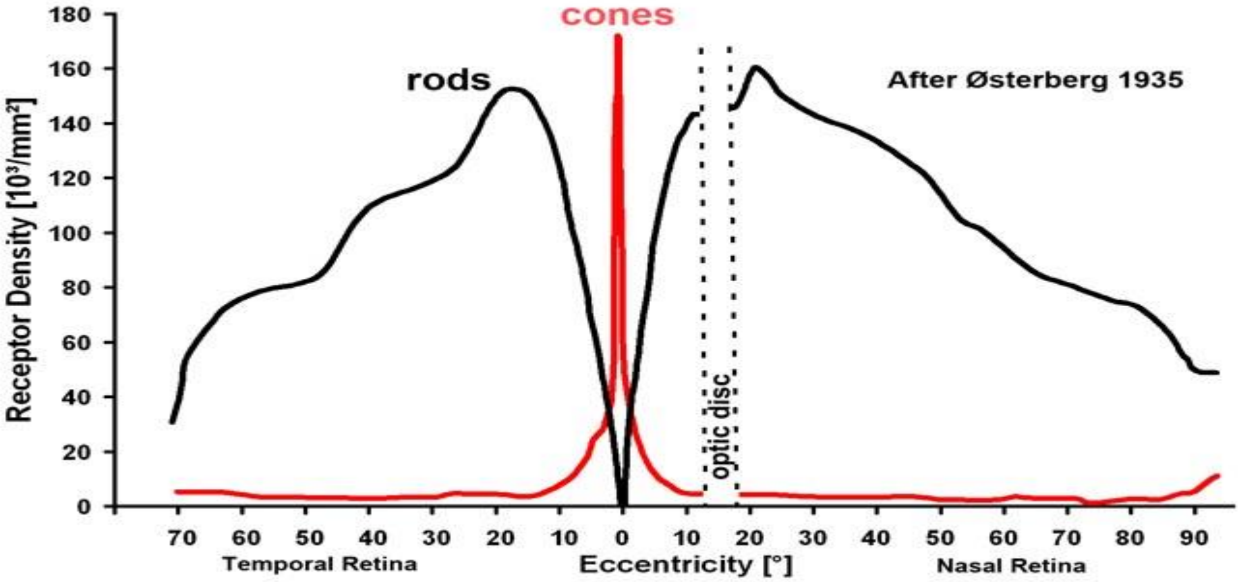
The Retina



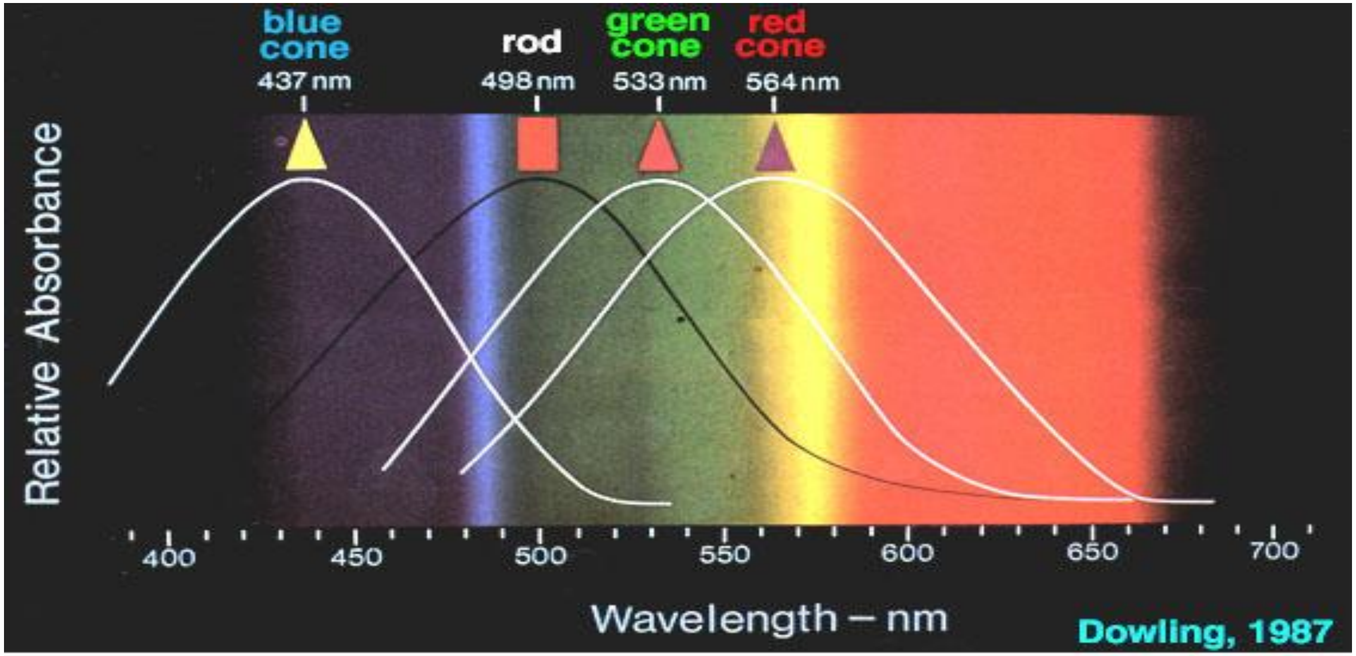
Light direction

Optic nerve (to brain)

Distribution of Rods and Cones



Rods and Cones



Frequency Dependency

- Human vision to given irradiance:

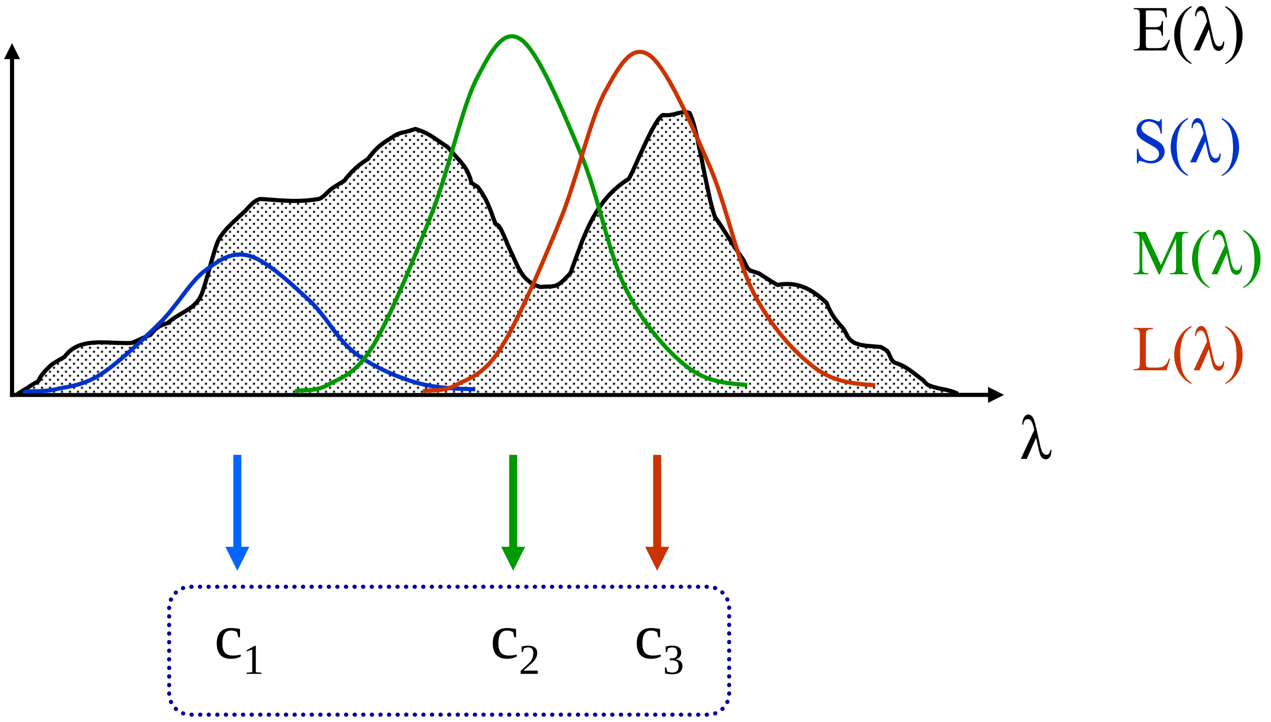
Receptor responses

$$\left\{\begin{aligned} c_1 &= \int E(\lambda)R_1(\lambda)d\lambda \\ c_2 &= \int E(\lambda)R_2(\lambda)d\lambda \\ c_3 &= \int \underbrace{E(\lambda)}\underbrace{R_3(\lambda)}d\lambda \end{aligned}\right.$$

Receptor sensitivities

Spectral power distribution
of light source
(irradiance * area)

Responses to a source



Receptor responses

Variations

- Distance attenuation
  - square drop-off too drastic
  - adding the ambient light term
  - changing the square drop-off term
  - directional source – no attenuation

$$\max(\frac{1}{c_1 + c_2 d + c_3 d^2},1)$$

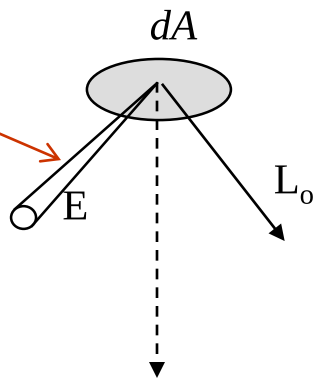
- Lens model difficult to ascertain
  - use a constant term to absorb it
- Color instead of gray scale
  - three equations instead of one

Total Incident Light

- So far, we have considered a *single* light source
- In reality, light comes from every possible direction in the hemisphere

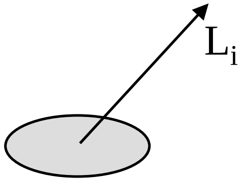
Incoming direction (θ_i,ϕ_i)

Solid angle ω



Surface patch *dA* in a solid angle $d\omega$ receives irradiance

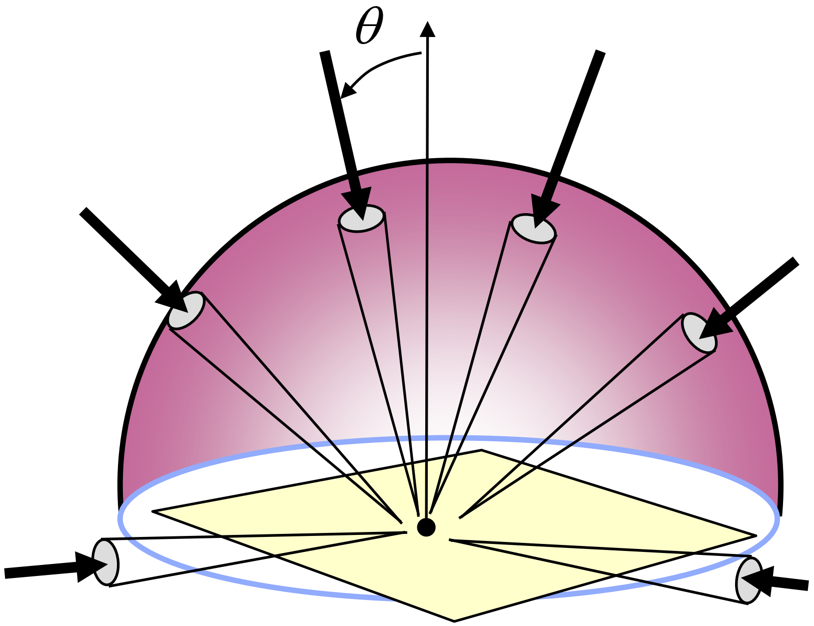
$$L_i \cos\theta_i \, d\omega$$



Irradiance

- A surface experiencing radiance *L(θ,ϕ)* coming in from solid angle *dω* experiences irradiance

$$\delta E = \underbrace{L(\theta,\phi)}_{\text{radiance}}\underbrace{\cos\theta}_{\uparrow\ \text{foreshortening factor}}\underbrace{d\omega}_{\text{solid angle}}$$



Total irradiance

$$E = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} L(\theta,\phi)\cos\theta\,\underbrace{\sin\theta\,d\theta\,d\phi}_{\text{solid angle}}$$

Caveats

- Unlike geometry, radiometry is much harder
- Modeling the true paths of all light sources (including surface interreflections, atmospheric effects, etc.) is impossible – our models are approximations
- Can we determine the shape of an object by its image? By detecting shadows?
- What about color?

Popular Models

- Diffuse models

$$Directional:$$

$$I = I_d k_d \cos(\theta_i) = I_d k_d \cos(n_i \cdot n)$$

$$I = I_d k_d \cos(\theta_i) + I_a k_a = I_d k_d \cos(n_i \cdot n) + I_a k_a$$

$$Positional:$$

$$I = \frac{I_d k_d \cos(\theta_i)}{\max(c_1 + c_2 d + c_3 d^2, 1)} + I_a k_a = \frac{I_d k_d \cos(n_i \cdot n)}{\max(c_1 + c_2 d + c_3 d^2, 1)} + I_a k_a$$

- Specular models

$$Directional:$$

$$I = I_s k_s \cos^n(\theta_s) \cos(\theta_i)$$

$$I = I_s k_s \cos^n(\theta_s) \cos(\theta_i) + I_a k_a$$

$$Positional:$$

$$I = \frac{I_s k_s \cos^n(\theta_s) \cos(\theta_i)}{\max(c_1 + c_2 d + c_3 d^2, 1)} + I_a k_a$$

Popular Models (cont.)

- Combined models

$$I = I_d \cos(\theta_i)(\alpha k_d + \beta k_s \cos^n(\theta_s))$$

$$I = I_d \cos(\theta_i)(\alpha k_d + \beta k_s \cos^n(\theta_s)) + I_a k_a$$

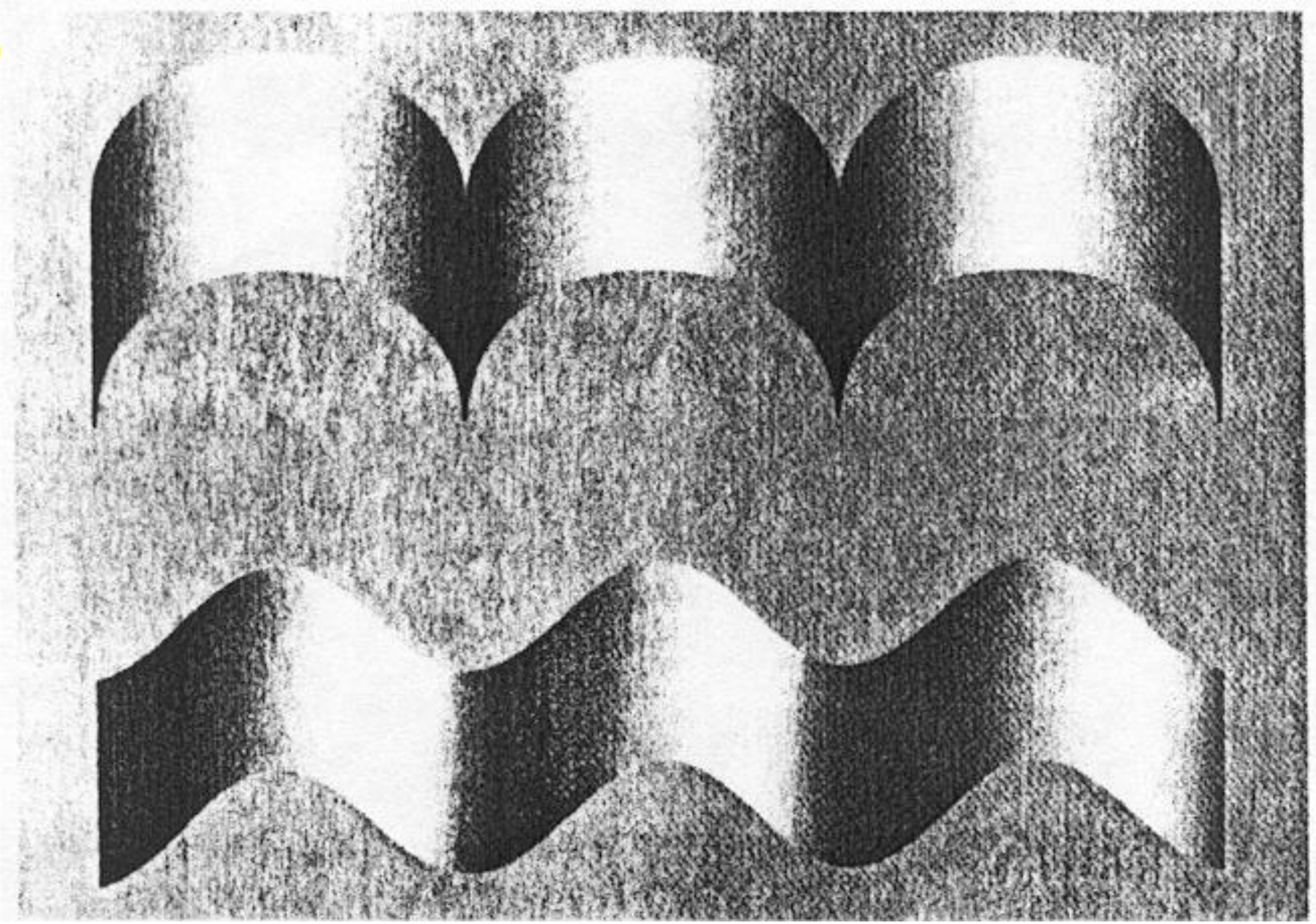
$$I = \frac{I_d \cos(\theta_i)(\alpha k_d + \beta k_s \cos^n(\theta_s))}{\max(c_1 + c_2 d + c_3 d^2, 1)} + I_a k_a$$

- Color models

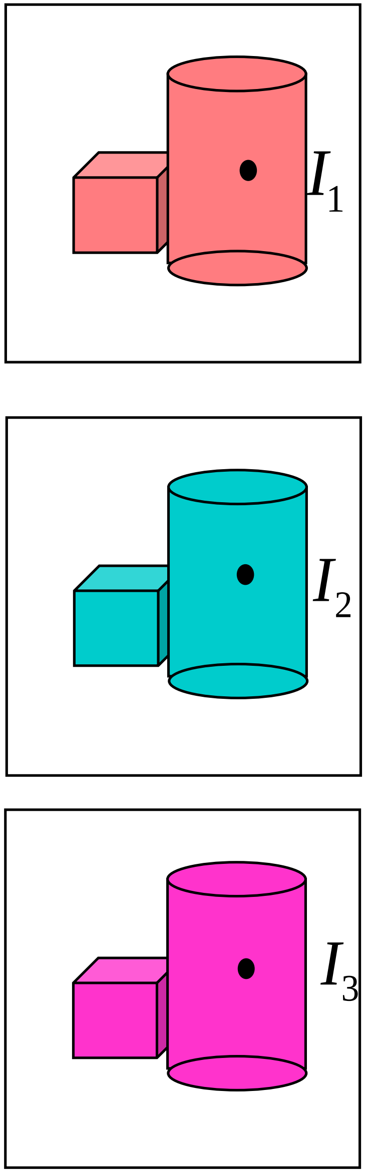
$$I_{\{r,g,b\}} = I_{d\{r,g,b\}} \cos(\theta_i)(k_{d\{r,g,b\}} + k_{s\{r,g,b\}} \cos^n(\theta_s))$$

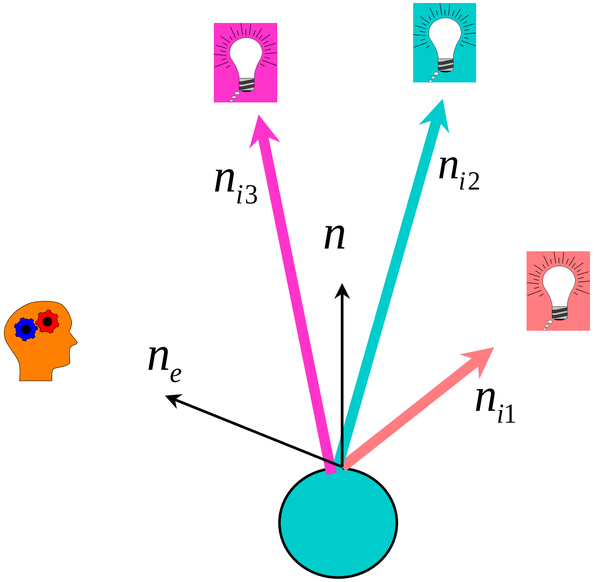
$$I_{\{r,g,b\}} = I_{d\{r,g,b\}} \cos(\theta_i)(k_{d\{r,g,b\}} + k_{s\{r,g,b\}} \cos^n(\theta_s)) + I_{a\{r,g,b\}} k_{a\{r,g,b\}}$$

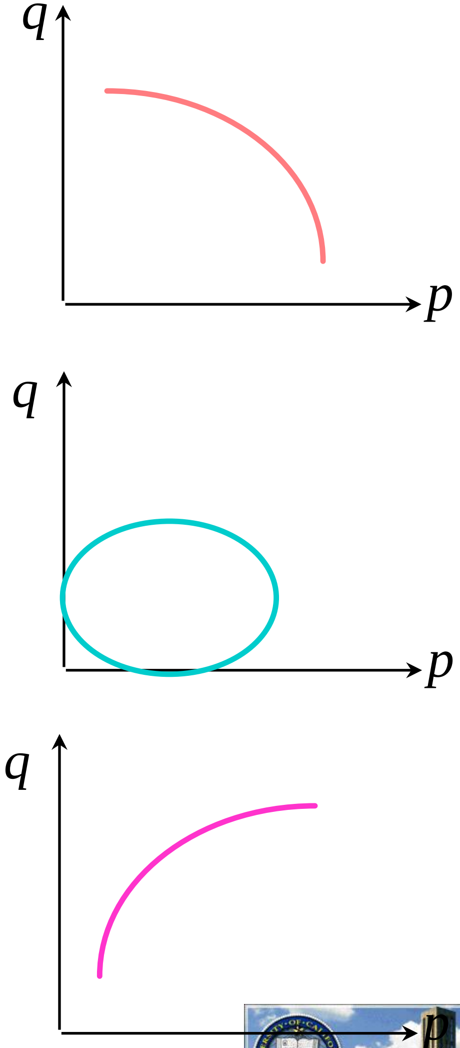
$$I_{\{r,g,b\}} = \frac{I_{d\{r,g,b\}} \cos(\theta_i)(k_{d\{r,g,b\}} + k_{s\{r,g,b\}} \cos^n(\theta_s))}{\max(c_1 + c_2 d + c_3 d^2, 1)} + I_{a\{r,g,b\}} k_{a\{r,g,b\}}$$



Photometric Stereo







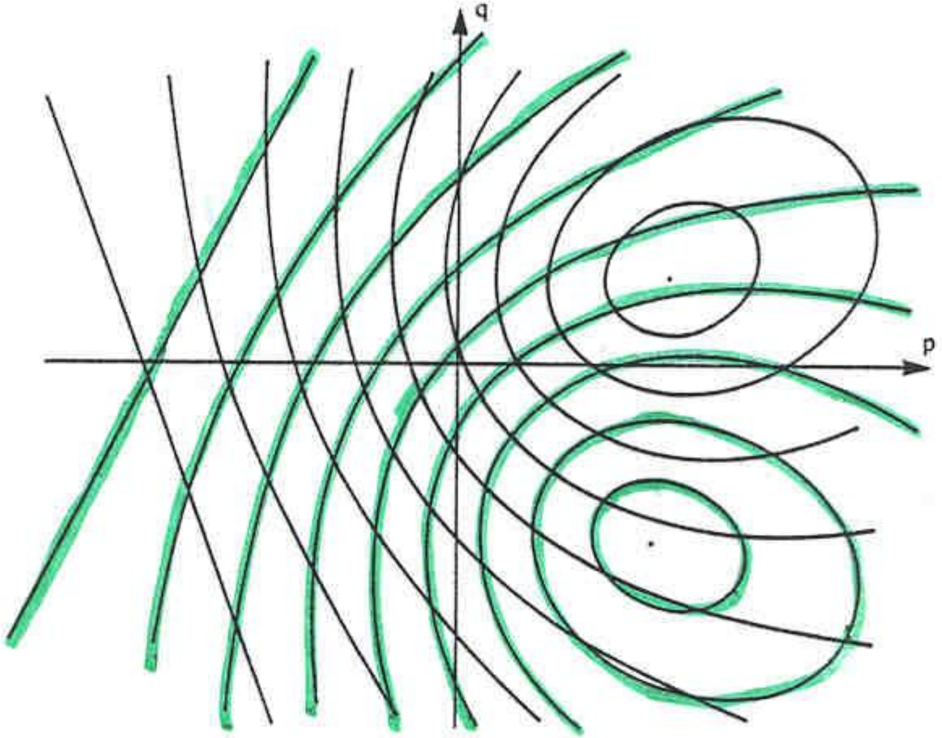


Figure 10-21. In the case of a Lambertian surface illuminated successively by two different point sources, there are at most two surface orientations that produce a particular pair of brightness values. These are found at the intersection of the corresponding contours in two superimposed reflectance maps.

Photometric Stereo

- In general, three images with three different lighting give linear equations for photometric stereo

$$I_1 = \rho_d k_1 (n_{i1} \cdot n) + k_0$$

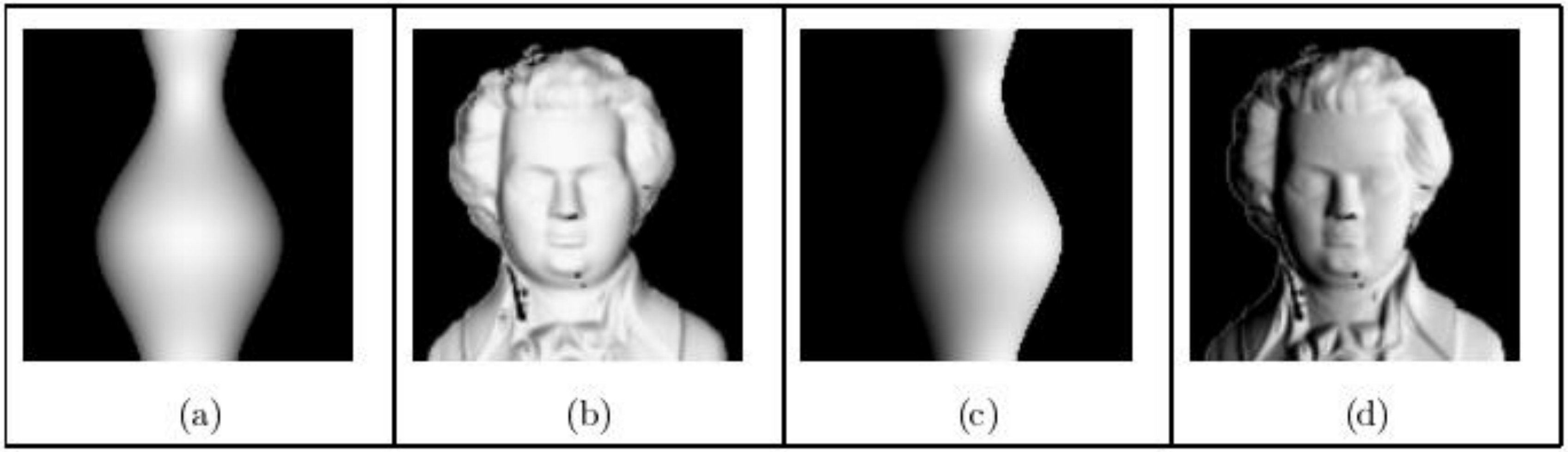
$$I_2 = \rho_d k_1 (n_{i2} \cdot n) + k_0$$

$$I_3 = \rho_d k_1 (n_{i3} \cdot n) + k_0$$

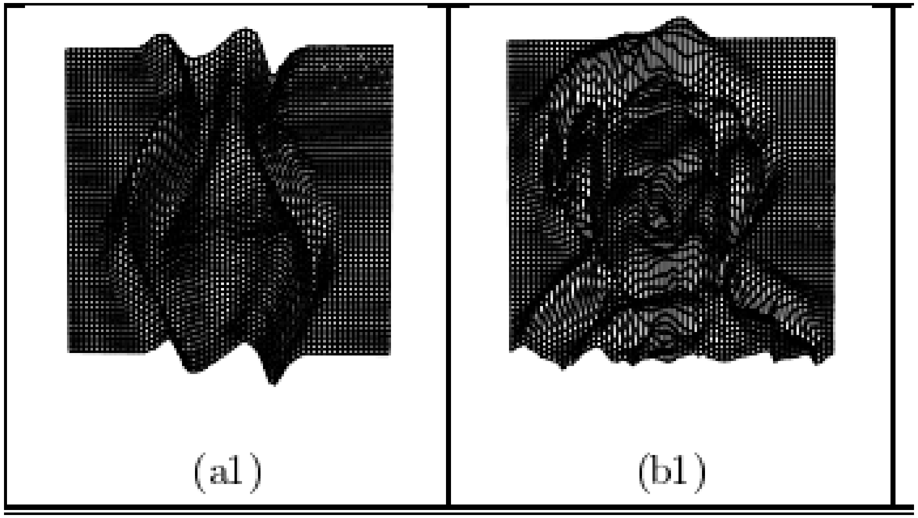
$$\begin{bmatrix} I_1 - k_0 \\ I_2 - k_0 \\ I_3 - k_0 \end{bmatrix} = \rho_d k_1 \begin{bmatrix} n_{i1}{}^x & n_{i1}{}^y & n_{i1}{}^z \\ n_{i2}{}^x & n_{i2}{}^y & n_{i2}{}^z \\ n_{i3}{}^x & n_{i3}{}^y & n_{i3}{}^z \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

$$I = \rho_d k_1 N_i n$$

$$\rho_d n = \frac{1}{k_1} N_i{}^{-1} I \begin{cases} \rho_d = \dfrac{1}{k_1} | N_i{}^{-1} I | \\ n = \dfrac{N_i{}^{-1} I}{| N_i{}^{-1} I |} \end{cases}$$



(a) (b) (c) (d)



(a1) (b1)