Name:

SAMPLE Midterm Exam

CS/ECE 181B – Intro to Computer Vision

February 9, 2009

2:00-3:15pm

Please space yourselves so that students are evenly distributed throughout the room. If possible, there should be no one directly next to you.

This is a **closed-book** test. You may use one page (8.5”x11”, front and back) of notes you have brought with you. There are also a few pages of equations, etc. included here for your reference.

Be sure to read each question carefully and provide all the information requested. If the question asks you to explain, do so!

Show your work. Write your answers in the spaces provided and, if necessary, on the back of the page. If you use the back, draw an arrow or write “SEE BACK” to make sure the graders don’t miss it. If you need more space, attach extra sheets of paper (available at the front).

Exams must be turned in by 3:15pm sharp.

Good luck!

Perspective projection

$$x' = f\frac{x}{z}$$
$$y' = f\frac{y}{z}$$
$$z' = f$$

Orthographic projection

$$x' = x$$
$$y' = y$$
$$z' = f$$

Weak perspective projection

$$x' = mx$$
$$y' = my$$
$$z' = f$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Parallel projection

$$x' = x - (z + f)\tan\theta_x$$
$$y' = y - (z + f)\tan\theta_y$$
$$z' = f$$

Paraperspective projection

$$x' = m(x - (z - z_{proj})\tan\theta_x)$$
$$y' = m(y - (z - z_{proj})\tan\theta_y)$$
$$z' = f$$

Snell's Law for refraction:

$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

Rigid coordinate transformation M from world coord. frame to camera coord. frame in homogeneous coordinates ($^{c}O_w$ is the translation vector):

$${}^{C}P = \begin{bmatrix} {}^{C}_{W}R & {}^{C}O_{W} \\ 0^T & 1 \end{bmatrix} {}^{W}P = M\ {}^{W}P$$

Another way to write this, where *T* is a translation vector:

$$P' = \begin{bmatrix} R & T \\ 0^T & 1 \end{bmatrix} P = M\ P$$

The solid angle subtended by a patch of area *dA*:

$$d\omega = \frac{dA\cos\theta}{r^2}$$

The solid angle subtended by a patch of angular size ($d\theta$, $d\phi$):

$$d\omega = \sin\theta \, d\theta \, d\phi$$

A surface experiencing radiance *L(q,f)* coming in from solid angle *dw* experiences irradiance:

$$\delta E = L(\theta,\phi)\cos\theta \, d\omega$$

Radiant energy	Q_e		Energy	J
Radiant flux	Φ_e	$\Phi_e = \frac{\Delta Q_e}{\Delta t}$	Energy per unit time (power)	J/s or W
Irradiance	E_e	$E_e = \frac{\Delta \Phi_e}{\Delta A}$	Power falling on unit area of target	W/m^2
Radiant intensity	I_e	$I_e = \frac{\Delta \Phi_e}{\Delta \omega}$	Source power radiated per unit solid angle	W/sr
Radiance	L_e	$L_e = \frac{\Delta I_e}{\Delta A}$	Source power radiated per unit area per unit solid angle	W/m^2-sr

Convolution:

$$R_{ij} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} H_{mn} F_{i-m,j-n}$$

Quaternion from axis/angle:

$$q = (\hat{n}\sin\frac{\theta}{2}, \cos\frac{\theta}{2})$$

Rotation matrix from quaternion:

$$R(q) = \begin{pmatrix} x^2 - y^2 - z^2 + w^2 & 2(xy - zw) & 2(xz + yw) \\ 2(xy + zw) & -x^2 + y^2 - z^2 + w^2 & 2(yz - xw) \\ 2(xz - yw) & 2(yz + xw) & -x^2 - y^2 + z^2 + w^2 \end{pmatrix}$$

Thin lens equation:

$$\frac{1}{z'} + \frac{1}{z} = \frac{1}{f}$$

$$\begin{aligned} x'_n &= \hat{x}/\hat{z} \\ y'_n &= \hat{y}/\hat{z} \\ r^2 &= {x'_n}^2 + {y'_n}^2 \end{aligned}$$

Apply radial distortion

$$\begin{aligned} x'_d &= x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4) \\ y'_d &= y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4) \end{aligned}$$

Image irradiance on the image plane as a function of the object radiance (L):

$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

Rotation about the *X* axis:

$$X_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

Rotation about the *Y* axis:

$$Y_\phi = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix}$$

Rotation about the *Z* axis:

$$Z_\psi = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Sensor responses:

$$c_1 = \int E(\lambda) R_1(\lambda) d\lambda$$
$$c_2 = \int E(\lambda) R_2(\lambda) d\lambda$$
$$c_3 = \int E(\lambda) R_3(\lambda) d\lambda$$

$$c_1 = \sum e(k) R_1(k) = e \cdot r_1 = e^T r_1$$
$$c_2 = \sum e(k) R_2(k) = e \cdot r_2 = e^T r_2$$
$$c_3 = \sum e(k) R_3(k) = e \cdot r_3 = e^T r_3$$

CIE chromaticity coordinates (x, y) from tristimulus values (X, Y, Z):

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

$$x + y + z = 1$$

SAMPLE Midterm Questions

1. [5 points] Briefly describe the relationship between computer vision and computer graphics.

2. [3 points] List three applications of computer vision.

3. [5 points] In a perspective projection with a focal distance of f=5, the scene point (-15, 10, 40) projects to what (x, y) point on the **virtual** image plane? To what (x, y) point on the **real** image plane?

4. [5 points] Of the following projection models: **perspective**, **orthographic**, **parallel**, **weak-perspective**, and **paraperspective**, for which ones do all projected points pass through the origin of the camera coordinate system?

5. [5 points] Give the 4x4 homogeneous transformation matrix that first translates a point by $[1\ 2\ 3]^T$ and then rotates it about the z axis by 45 degrees.

6. [3 points] Under perspective projection, what does a circle in the scene project to on the image plane?

7. [5 points] A video camera sends 30 images (frames) per second of 640*480 “true color” pixel values. How many bytes per second are sent?

8. [3 points] For a large depth of field, should the camera aperture be small or large?

9. [3 points] The pixel value recorded from a particular point on a camera's sensor array depends on several factors, such as the position and orientation of the object in the world that gets imaged at that point. Name at least three other factors that determine the pixel value of the point.

10. [3 points] A "directional diffuse" surface reflectance model is a combination of what two ideal surface models?

11. [5 points] The solid angle subtended by a small surface patch facing a camera, 5m from the camera, is 2 steradians. What is the solid angle subtended by the patch if it is rotated away from the camera 60 degrees?

12. [5 points] Two light sources, E_1 and E_2, are metamers – that is, they produce the exact same RGB pixel values. Now we mix them together to create two new light sources, X and Y, as such:

    X = 1/3 A + 2/3 B
    Y = 2/3 A + 1/3 B

    Are X and Y metamers? Explain why or why not.

13. [5 points] What 3D point $p = (x, y, z)$ is produced by adding the following 3D points, represented in homogeneous coordinates:

$$p_1 = \begin{bmatrix} 2 \\ 3 \\ 6 \\ 2 \end{bmatrix} \qquad p_2 = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 4 \end{bmatrix}$$

14. [N points] Two light sources, A and B, produce tristimulus values of (25, 50, 100) and (40, 80, 160), respectively. Are A and B metamers? Explain why or why not.