Active Contours
(Snakes)

Jungah Son

International Journal of Computer Vision, 321–331 (1988)

Snakes: Active Contour Models

MICHAEL KASS, ANDREW WITKIN, and DEMETRI TERZOPOULOS
Schlumberger Palo Alto Research, 3340 Hillview Ave., Palo Alto, CA 94304

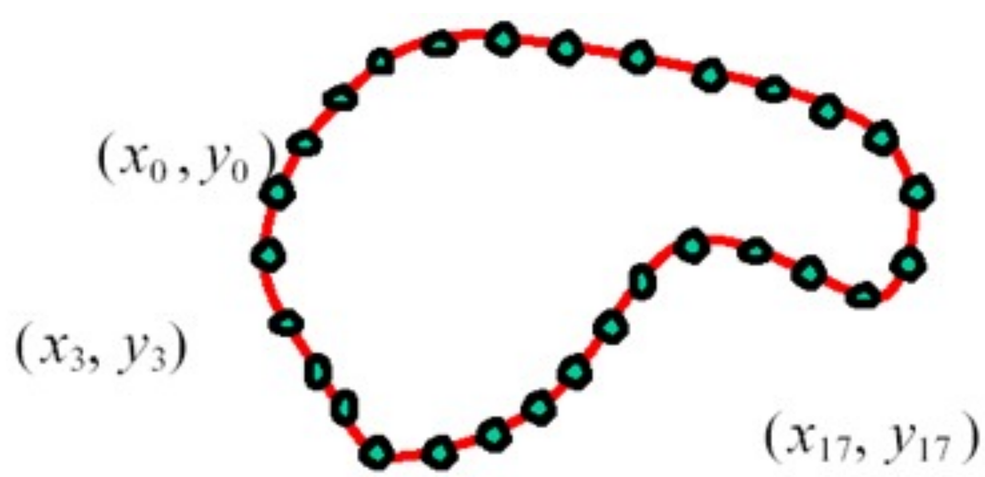
Abstract

A snake is an energy-minimizing spline guided by external constraint forces and influenced by image forces that pull it toward features such as lines and edges. Snakes are active contour models: they lock onto nearby edges, localizing them accurately. Scale-space continuation can be used to enlarge the capture region surrounding a feature. Snakes provide a unified account of a number of visual problems, including detection of edges, lines, and subjective contours; motion tracking; and stereo matching. We have used snakes successfully for interactive interpretation, in which user-imposed constraint forces guide the snake near features of interest.

Initialization

1. Use ginput function to pick initial contour points

2. Use spline function to interpolate with a spline curve



Energy formulation

1. Internal energy

$$E_{\text{int}} = (\alpha(s)|\mathbf{v}_s(s)|^2 + \beta(s)|\mathbf{v}_{ss}(s)|^2)/2$$

2. Image energy

$$E_{\text{image}} = w_{\text{line}} E_{\text{line}} + w_{\text{edge}} E_{\text{edge}}$$

3. Constraint energy

Energy formulation

1. Internal energy

$$E_{\text{int}} = (\alpha(s)|\mathbf{v}_s(s)|^2 + \beta(s)|\mathbf{v}_{ss}(s)|^2)/2$$

2. Image energy

$$E_{\text{image}} = w_{\text{line}}E_{\text{line}} + w_{\text{edge}}E_{\text{edge}}$$

3. Constraint energy

$$\left.\right\} E_{\text{ext}} = E_{\text{image}} + E_{\text{con}}$$

Problem formulation

Minimize

$$E^*_{\text{snake}} = \int_0^1 E_{\text{snake}}(\mathbf{v}(s))\,ds$$
$$= \int_0^1 E_{\text{int}}(\mathbf{v}(s)) + E_{\text{ext}}(\mathbf{v}(s))\,ds$$

Discrete Representation

Minimize

$$E^*_{\text{snake}} = \int_0^1 E_{\text{snake}}(\mathbf{v}(s))\, ds$$

$$= \int_0^1 E_{\text{int}}(\mathbf{v}(s)) + E_{\text{ext}}(\mathbf{v}(s))\, ds$$

$$E^*_{\text{snake}} = \sum_{i=1}^{n} E_{\text{int}}(i) + E_{\text{ext}}(i)$$

Discrete Representation

Minimize

$$E^*_{\text{snake}} = \sum_{i=1}^{n} \underline{E_{\text{int}}(i)} + E_{\text{ext}}(i)$$

$$E_{\text{int}} = (\alpha(s)|\mathbf{v}_s(s)|^2 + \beta(s)|\mathbf{v}_{ss}(s)|^2)/2$$

finite differences

$$E_{\text{int}}(i) = \alpha\,|\mathbf{v}_i - \mathbf{v}_{i-1}|^2/2 + \beta\,|\mathbf{v}_{i-1} - 2\mathbf{v}_i + \mathbf{v}_{i+1}|^2/2$$

Discrete Representation

Minimize

$$E^*_{\text{snake}} = \sum_{i=1}^{n} E_{\text{int}}(i) + E_{\text{ext}}(i)$$

$$= \sum_{i=1}^{n} \alpha\,|\mathbf{v}_i - \mathbf{v}_{i-1}|^2/2 + \beta\,|\mathbf{v}_{i-1} - 2\mathbf{v}_i + \mathbf{v}_{i+1}|^2/2 + E_{\text{ext}}(i)$$

$$\frac{\partial E^*_{\text{snake}}}{\partial \mathbf{v}_i} \Rightarrow$$

$$\alpha\,(\mathbf{v}_i - \mathbf{v}_{i-1}) - \alpha\,(\mathbf{v}_{i+1} - \mathbf{v}_i)$$
$$+ \beta\,[\mathbf{v}_{i-2} - 2\mathbf{v}_{i-1} + \mathbf{v}_i] - 2\beta\,[\mathbf{v}_{i-1} - 2\mathbf{v}_i + \mathbf{v}_{i+1}]$$
$$+ \beta\,[\mathbf{v}_i - 2\mathbf{v}_{i+1} + \mathbf{v}_{i+2}] + \underline{(f_x(i), f_y(i))} = 0$$

$$(f_x(i), f_y(i)) \leftarrow \left(\frac{\partial E_{\text{ext}}}{\partial x_i}, \frac{\partial E_{\text{ext}}}{\partial y_i}\right)$$

Rearrange terms

$$\alpha(\mathbf{v}_i - \mathbf{v}_{i-1}) - \alpha(\mathbf{v}_{i+1} - \mathbf{v}_i)$$
$$+ \beta[\mathbf{v}_{i-2} - 2\mathbf{v}_{i-1} + \mathbf{v}_i] - 2\beta[\mathbf{v}_{i-1} - 2\mathbf{v}_i + \mathbf{v}_{i+1}]$$
$$+ \beta[\mathbf{v}_i - 2\mathbf{v}_{i+1} + \mathbf{v}_{i+2}] + (f_x(i), f_y(i)) = 0$$

$$\Downarrow$$

$$\beta\mathbf{v}_{i-2} + (-\alpha - 4\beta)\mathbf{v}_{i-1}$$
$$+ (2\alpha + 6\beta)\mathbf{v}_i$$
$$+ (-\alpha - 4\beta)\mathbf{v}_{i+1} + \beta\mathbf{v}_{i+2} + \left(\boldsymbol{f_x}(\boldsymbol{i}), \boldsymbol{f_y}(\boldsymbol{i})\right) = 0$$

Write in matrix form

$$\beta \mathbf{v}_{i-2} + (-\alpha - 4\beta)\mathbf{v}_{i-1} + (2\alpha + 6\beta)\mathbf{v}_i + (-\alpha - 4\beta)\mathbf{v}_{i+1} + \beta \mathbf{v}_{i+2} + \left(f_x(i), f_y(i) \right) = 0$$

$$\mathbf{A}\mathbf{x} + \mathbf{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{A}\mathbf{y} + \mathbf{f}_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) = 0$$

Matrix A

$$\mathbf{A}\mathbf{x} + \mathbf{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{A}\mathbf{y} + \mathbf{f}_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\begin{aligned}\beta\mathbf{v}_{i-2} &+ (-\alpha - 4\beta)\mathbf{v}_{i-1} \\ &+ (2\alpha + 6\beta)\mathbf{v}_i \\ &+ (-\alpha - 4\beta)\mathbf{v}_{i+1} + \beta\mathbf{v}_{i+2} + \left(\boldsymbol{f}_x(\boldsymbol{i}), \boldsymbol{f}_y(\boldsymbol{i})\right) = 0\end{aligned}$$

$$\begin{bmatrix} 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & 0 & \cdots & \beta & -\alpha-4\beta \\ -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & \cdots & 0 & \beta \\ \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & \cdots & 0 & 0 \\ 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & \cdots & 0 & 0 \\ 0 & 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta & 0 & 0 & 0 & 0 & 0 & \cdots & 2\alpha+6\beta & -\alpha-4\beta \\ -\alpha-4\beta & \beta & 0 & 0 & 0 & 0 & \cdots & -\alpha-4\beta & 2\alpha+6\beta \end{bmatrix}$$

Matrix A

$$\mathbf{Ax} + \mathbf{f_x}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{Ay} + \mathbf{f_y}(\mathbf{x}, \mathbf{y}) = 0$$

$$\begin{aligned}\beta \mathbf{v}_{-1} &+ (-\alpha - 4\beta)\mathbf{v}_0 \\ &+ (2\alpha + 6\beta)\mathbf{v}_1 \\ &+ (-\alpha - 4\beta)\mathbf{v}_2 + \beta \mathbf{v}_3 + \left(f_x(\boldsymbol{i}), f_y(\boldsymbol{i})\right) = 0\end{aligned}$$

$$\begin{matrix} i = 1: \\ \\ \\ \\ \\ \\ \\ \\ \end{matrix}\begin{bmatrix} 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & 0 & \cdots & \beta & -\alpha-4\beta \\ -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & \cdots & 0 & \beta \\ \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & \cdots & 0 & 0 \\ 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & \cdots & 0 & 0 \\ 0 & 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta & 0 & 0 & 0 & 0 & 0 & \cdots & 2\alpha+6\beta & -\alpha-4\beta \\ -\alpha-4\beta & \beta & 0 & 0 & 0 & 0 & \cdots & -\alpha-4\beta & 2\alpha+6\beta \end{bmatrix}$$

Matrix A

$$\mathbf{Ax} + \mathbf{f_x}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{Ay} + \mathbf{f_y}(\mathbf{x}, \mathbf{y}) = 0$$

$$\beta \mathbf{v}_{n-1} + (-\alpha - 4\beta)\mathbf{v}_n$$
$$+ (2\alpha + 6\beta)\mathbf{v}_1$$
$$+ (-\alpha - 4\beta)\mathbf{v}_2 + \beta \mathbf{v}_3 + \left(f_x(i), f_y(i)\right) = 0$$

1. Snake is circular!

$$\begin{bmatrix} 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & 0 & \cdots & \beta & -\alpha-4\beta \\ -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & \cdots & 0 & \beta \\ \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & \cdots & 0 & 0 \\ 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & \cdots & 0 & 0 \\ 0 & 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta & 0 & 0 & 0 & 0 & 0 & \cdots & 2\alpha+6\beta & -\alpha-4\beta \\ -\alpha-4\beta & \beta & 0 & 0 & 0 & 0 & \cdots & -\alpha-4\beta & 2\alpha+6\beta \end{bmatrix}$$

Matrix A

$$\mathbf{A}\mathbf{x} + \mathbf{f}_\mathbf{x}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{A}\mathbf{y} + \mathbf{f}_\mathbf{y}(\mathbf{x}, \mathbf{y}) = 0$$

$$\begin{aligned}\beta \mathbf{v}_{i-2} &+ (-\alpha - 4\beta)\mathbf{v}_{i-1} \\ &+ (2\alpha + 6\beta)\mathbf{v}_i \\ &+ (-\alpha - 4\beta)\mathbf{v}_{i+1} + \beta \mathbf{v}_{i+2} + \left(f_x(i), f_y(i)\right) = 0\end{aligned}$$

2. Use circshift function!

$$\begin{bmatrix} 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & 0 & \cdots & \beta & -\alpha-4\beta \\ -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & \cdots & 0 & \beta \\ \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & \cdots & 0 & 0 \\ 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & \cdots & 0 & 0 \\ 0 & 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta & 0 & 0 & 0 & 0 & 0 & \cdots & 2\alpha+6\beta & -\alpha-4\beta \\ -\alpha-4\beta & \beta & 0 & 0 & 0 & 0 & \cdots & -\alpha-4\beta & 2\alpha+6\beta \end{bmatrix}$$

Solve equations

$$\mathbf{A}\mathbf{x} + \mathbf{f_x}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{A}\mathbf{y} + \mathbf{f_y}(\mathbf{x}, \mathbf{y}) = 0$$

Solve equations

$$\mathbf{A}\mathbf{x} + \mathbf{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{A}\mathbf{y} + \mathbf{f}_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\Downarrow$$

$$\mathbf{A}\mathbf{x}_t + \mathbf{f}_{\mathbf{x}}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{x}_t - \mathbf{x}_{t-1})$$

$$\mathbf{A}\mathbf{y}_t + \mathbf{f}_{\mathbf{y}}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{y}_t - \mathbf{y}_{t-1})$$

Solve equations

$$\mathbf{A}\mathbf{x} + \mathbf{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{A}\mathbf{y} + \mathbf{f}_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\Downarrow$$

$$\mathbf{A}\mathbf{x}_t + \mathbf{f}_{\mathbf{x}}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{x}_t - \mathbf{x}_{t-1})$$

$$\mathbf{A}\mathbf{y}_t + \mathbf{f}_{\mathbf{y}}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{y}_t - \mathbf{y}_{t-1})$$

$$\Downarrow$$

Use LU decomposition if needed!

$$\mathbf{x}_t = \underline{(\mathbf{A} + \gamma\mathbf{I})^{-1}}(\gamma\mathbf{x}_{t-1} - \mathbf{f}_{\mathbf{x}}(x_{t-1}, y_{t-1}))$$

$$\mathbf{y}_t = (\mathbf{A} + \gamma\mathbf{I})^{-1}(\gamma\mathbf{y}_{t-1} - \mathbf{f}_{\mathbf{y}}(x_{t-1}, y_{t-1}))$$

Solve equations

$$\mathbf{A}\mathbf{x} + \mathbf{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\mathbf{A}\mathbf{y} + \mathbf{f}_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) = 0$$

$$\Downarrow$$

$$\mathbf{A}\mathbf{x}_t + \mathbf{f}_{\mathbf{x}}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{x}_t - \mathbf{x}_{t-1})$$

$$\mathbf{A}\mathbf{y}_t + \mathbf{f}_{\mathbf{y}}(\mathbf{x}_{t-1}, \mathbf{y}_{t-1}) = -\gamma(\mathbf{y}_t - \mathbf{y}_{t-1})$$

$$\Downarrow$$

$$\mathbf{x}_t = (\mathbf{A} + \gamma\mathbf{I})^{-1}(\gamma\mathbf{x}_{t-1} - \underline{\mathbf{f}_{\mathbf{x}}(x_{t-1}, y_{t-1})})$$

$$\mathbf{y}_t = (\mathbf{A} + \gamma\mathbf{I})^{-1}(\gamma\mathbf{y}_{t-1} - \mathbf{f}_{\mathbf{y}}(x_{t-1}, y_{t-1}))$$

Use interp2 function

Last

1. Don’t forget to smooth images to enlarge “potential field”

2. Start with a simple case first

3. Snake needs parameter adjustment

Codes that we used in the discussion session

```
clc;
clear all;
close all;

img = imread('lena.jpg');
figure;
imshow(img);
hold on;

button = 1;
n = 0;
while button == 1
    [xi,yi,button] = ginput(1);
    plot(xi,yi,'go')
    n = n+1;
    xy(:,n) = [xi;yi];
end

n = n+1;
xy(:,n) = [xy(1,1);xy(2,1)];
```

```
% Original curve
xys = xy;

xs = xys(1,:);
ys = xys(2,:);

hold on;
plot(xs, ys, 'r-');

% Interpolate with a spline curve
t = 1:n;
ts = 1: 0.1: n;
xys = spline(t,xy,ts);

xs = xys(1,:);
ys = xys(2,:);

hold on;
plot(xs, ys, 'b-');
```

Codes that we used in the discussion session

```
% Read multiple images from a folder

srcFiles = dir('/Winter 2015/liptracking2/*.jpg');
for i = 1 : length(srcFiles)
    filename = strcat('/Winter 2015/liptracking2/',srcFiles(i).name);
    I = imread(filename);
end
```