Note for homework 1 of CS 165A

Problem 2.
(c) The answer is Yes

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\[ \begin{align*}
X_1 &< 3 \\
& \quad \text{Yes} \\
& \quad \text{No} \\
& \quad X_2 < 4 \\
& \quad \text{Yes} \\
& \quad \text{No} \\
& \quad X_2 < 3 \\
& \quad \text{Yes} \\
& \quad \text{No} \\
\end{align*} \]
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* Comparison between decision tree and linear classifier: None of them dominate each other.

Case 1. \[ \quad 0 \] Can be perfectly classified by linear classifier, while decision tree with arbitrary depth can not give perfect classification.

Case 2. \[ \quad x \quad 0 \] can be perfectly classified by decision tree with depth 2, while linear classifier can not give perfect classification.

(c) This is a non-parametric classifier.

\[ \# \text{ data} \uparrow \quad \text{complexity of the classifier} \uparrow \]

(d) \[ |X| = 2^d \]

So the answer is \[ 2^{|X|} = 2^{2^d} \]

(We say \( h \models h' \) if and only if there exists \( x \in X \) such that \( h(x) \neq h'(x) \))
Problem 3.

(a) \( \text{Err}(\hat{h}) \leq \text{Err}(\hat{h}^*) + 2 \text{GenErr} \)

Proof: \( \text{Err}(\hat{h}) \leq \text{Err}(\hat{h}^*) + \text{GenErr} \)

\[ \leq \text{Err}(\hat{h}^*) + \text{GenErr} \]

\[ \leq \text{Err}(\hat{h}^*) + 2 \text{GenErr} \]

\( \text{Q.E.D.} \)

\( \text{Q.E.D.} \):

\[ \text{GenErr} = \max_{h \in H} | \text{Err}(h) - \hat{\text{Err}}(h) | \]

\[ \geq \max_{h \in H} \{ | \text{Err}(h) - \hat{\text{Err}}(h^*) |, | \text{Err}(h^*) - \hat{\text{Err}}(h^*) | \} \]

\( \text{Q.E.D.} \):

\[ \text{Err}(\hat{h}) \leq \text{Err}(\hat{h}^*) \text{ as } \hat{h} = \arg \min_{h \in H} \hat{\text{Err}}(h) \]

(b) It suffices to prove that if we randomly pick \( n \) numbers from \( \{ x_1, \ldots, x_m \} \), and get \( \{ x_{o1}, x_{o2}, \ldots, x_{on} \} \), then \( E_n \left[ x_{o1} + \ldots + x_{on} \right] = \frac{1}{m} \sum_{i=1}^{n} x_i \).

Pf. \( E_n \left[ x_{o1} + \ldots + x_{on} \right] = \frac{1}{n} E \left( \sum_{i=1}^{n} x_i : 1 \{ x_i \text{ is picked} \} \right) \)

\[ = \frac{1}{n} \sum_{i=1}^{n} x_i \cdot P \{ x_i \text{ is picked} \} \]

\[ = \frac{1}{n} \sum_{i=1}^{n} x_i \cdot \frac{n}{m} \]

\[ = \frac{1}{m} \sum_{i=1}^{n} x_i \]