Artificial Intelligence

CS 165A

Oct 29, 2020

Instructor: Prof. Yu-Xiang Wang

→ Examples of heuristics in A*-search
→ Games and Adversarial Search
Recap: Search algorithms

- State-space diagram vs Search Tree

- Uninformed Search algorithms
  - BFS / DFS
  - Depth Limited Search
  - Iterative Deepening Search.
  - Uniform cost search.

- Informed Search (with an heuristic function h):
  - Greedy Best-First-Search. (not complete / optimal)
  - A* Search (complete / optimal if h is admissible)
Recap: Summary table of uninformed search

<table>
<thead>
<tr>
<th>Criteria</th>
<th>BFS</th>
<th>Uniform-cost</th>
<th>DFS</th>
<th>Depth-limited</th>
<th>IDS</th>
<th>Bidirectional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>Yes#</td>
<td>Yes#&amp;</td>
<td>No</td>
<td>No</td>
<td>Yes#</td>
<td>Yes#+</td>
</tr>
<tr>
<td>Time</td>
<td>$O(b^d)$</td>
<td>$O(b^{1+[C/e]})$</td>
<td>$O(b^m)$</td>
<td>$O(b^l)$</td>
<td>$O(b^d)$</td>
<td>$O(b^{d/2})$</td>
</tr>
<tr>
<td>Space</td>
<td>$O(b^d)$</td>
<td>$O(b^{1+[C/e]})$</td>
<td>$O(bm)$</td>
<td>$O(bl)$</td>
<td>$O(bd)$</td>
<td>$O(b^{d/2})$</td>
</tr>
<tr>
<td>Optimal?</td>
<td>Yes$\dagger$</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes$\dagger$</td>
<td>Yes$\dagger$ +</td>
</tr>
</tbody>
</table>

- $b$: Branching factor
- $d$: Depth of the shallowest goal
- $l$: Depth limit
- $m$: Maximum depth of search tree
- $e$: The lower bound of the step cost

# Complete if $b$ is finite
&A Complete if step cost $\geq e$
$\$ Optimal if all step costs are identical
+ If both direction use BFS

(Section 3.4.7 in the AIMA book.)
Recap: A* Search (Pronounced “A-Star”)

- Uniform-cost search minimizes $g(n)$ (“past” cost)

- Greedy search minimizes $h(n)$ (“expected” or “future” cost)

- “A* Search” combines the two:
  - Minimize $f(n) = g(n) + h(n)$
  - Accounts for the “past” and the “future”
  - Estimates the cheapest solution (complete path) through node $n$

function A*-SEARCH($problem$, $h$) returns a solution or failure
return BEST-FIRST-SEARCH($problem$, $f$)
Recap: Avoiding Repeated States using A* Search

• Is GRAPH-SEARCH optimal with A*?

Try with TREE-SEARCH and GRAPH-SEARCH
Recap: Avoiding Repeated States using A* Search

• Is GRAPH-SEARCH optimal with A*?

Graph Search
Step 1: Among B, C, E, Choose C
Step 2: Among B, E, D, Choose B
Step 3: Among D, E, Choose E. (you are not going to select C again)
Recap: Consistency (Monotonicity) of heuristic $h$

- A heuristic is consistent (or monotonic) provided
  - for any node $n$, for any successor $n'$ generated by action $a$ with cost $c(n,a,n')$
    - $h(n) \leq c(n,a,n') + h(n')$
    - akin to triangle inequality.
    - guarantees admissibility (proof?).
    - values of $f(n)$ along any path are non-decreasing (proof?).
  - Contours of constant $f$ in the state space
- GRAPH-SEARCH using consistent $f(n)$ is optimal.
- Note that $h(n) = 0$ is consistent and admissible.
This lecture

• Example of heuristics / A* search
  – Effective branching factor

• Games

• Adversarial Search
Heuristics

• What’s a heuristic for
  – Driving distance (or time) from city A to city B?
  – 8-puzzle problem?
  – M&C?
  – Robot navigation?

- Straight line dist.
- # of misplaced tiles
- # of ppl on the LHS
- # of dirty tiles
Heuristics

• What’s a heuristic for
  – Driving distance (or time) from city A to city B?
  – 8-puzzle problem?
  – M&C?
  – Robot navigation?

• **Admissible** heuristic
  – Does not overestimate the cost to reach the goal
  – “Optimistic”

• **Consistent** heuristic:
  – Satisfy a triangular inequality: \( h(n) \leq c(n, a, n') + h(n') \)
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• Are the above heuristics admissible? Consistent?
Example: 8-Puzzle

Start State

Goal State
Comparing and combining heuristics

- Heuristics generated by considering relaxed versions of a problem.
- Heuristic $h_1$ for 8-puzzle
  - Number of out-of-order tiles
- Heuristic $h_2$ for 8-puzzle
  - Sum of Manhattan distances of each tile
- $h_2$ dominates $h_1$ provided $h_2(n) \geq h_1(n)$.
  - $h_2$ will likely prune more than $h_1$.
- $\max(h_1, h_2, \ldots, h_n)$ is
  - admissible if each $h_i$ is
  - consistent if each $h_i$ is
- Cost of sub-problems and pattern databases
  - Cost for 4 specific tiles
  - Can these be added for disjoint sets of tiles?
Effective Branching Factor
Effective Branching Factor

• Though informed search methods may have poor worst-case performance, they often do quite well if the heuristic is good
  – Even if there is a huge branching factor
Effective Branching Factor

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  – Even if there is a huge branching factor

• One way to quantify the effectiveness of the heuristic: the effective branching factor, $b^*$
  – N: total number of nodes expanded
  – d: solution depth
  – $N = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$
Effective Branching Factor

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  - $N = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$

- For a good heuristic, $b^*$ is close to 1
## Example: 8-puzzle problem

Averaged over 100 trials each at different solution lengths

<table>
<thead>
<tr>
<th>Solution length</th>
<th>Search Cost ( A^*(h_1) )</th>
<th>Search Cost ( A^*(h_2) )</th>
<th>Effective Branching Factor ( A^*(h_1) )</th>
<th>Effective Branching Factor ( A^*(h_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>6</td>
<td>2.45</td>
<td>1.79</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
<td>13</td>
<td>2.87</td>
<td>1.48</td>
</tr>
<tr>
<td>6</td>
<td>680</td>
<td>20</td>
<td>2.73</td>
<td>1.34</td>
</tr>
<tr>
<td>8</td>
<td>6384</td>
<td>39</td>
<td>2.80</td>
<td>1.33</td>
</tr>
<tr>
<td>10</td>
<td>47127</td>
<td>93</td>
<td>2.79</td>
<td>1.38</td>
</tr>
<tr>
<td>12</td>
<td>364404</td>
<td>227</td>
<td>2.78</td>
<td>1.42</td>
</tr>
<tr>
<td>14</td>
<td>3473941</td>
<td>539</td>
<td>2.83</td>
<td>1.44</td>
</tr>
<tr>
<td>16</td>
<td>–</td>
<td>1301</td>
<td>–</td>
<td>1.45</td>
</tr>
<tr>
<td>18</td>
<td>–</td>
<td>3056</td>
<td>–</td>
<td>1.46</td>
</tr>
<tr>
<td>20</td>
<td>–</td>
<td>7276</td>
<td>–</td>
<td>1.47</td>
</tr>
<tr>
<td>22</td>
<td>–</td>
<td>18094</td>
<td>–</td>
<td>1.48</td>
</tr>
<tr>
<td>24</td>
<td>–</td>
<td>39135</td>
<td>–</td>
<td>1.48</td>
</tr>
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</table>

Ave. # of nodes expanded

Solution length
Memory Bounded Search
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- Memory, not computation, is usually the limiting factor in search problems
  - Certainly true for A* search
Memory Bounded Search

• Memory, not computation, is usually the limiting factor in search problems
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• Why? What takes up memory in A* search?
Memory Bounded Search

• Memory, not computation, is usually the limiting factor in search problems
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• Why? What takes up memory in A* search?

• Solution: Memory-bounded A* search
  – Iterative Deepening A* (IDA*)
  – Simplified Memory-bounded A* (SMA*)
  – (Read the textbook for more details.)
Summary of informed search

- How to use a heuristic function to improve search
  - Greedy Best-first search + Uniform-cost search = A* Search

- When is A* search optimal?
  - h is Admissible (optimistic) for Tree Search
  - h is Consistent for Graph Search

- Choosing heuristic functions
  - A good heuristic function can reduce time/space cost of search by orders of magnitude.
  - Good heuristic function may take longer to evaluate.
Games and Adversarial Search

- Games: problem setup
- Minimax search
- Alpha-beta pruning
Illustrative example of a simple game (1 min discussion)

Example: game 1

You choose one of the three bins. I choose a number from that bin. Your goal is to maximize the chosen number.

A
-50  50

B
1   3

C
-5  15

(Example taken from Liang and Sadigh)
Game as a search problem
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- $S_0$ The initial state
Game as a search problem

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- PLAYER(s): Returns which player has the move
Game as a search problem

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- TERMINAL-TEST(s): Returns True if the game is over.
Game as a search problem

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- ACTIONS(s): Returns the legal moves.

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- TERMINAL-TEST(s): Returns True if the game is over.

- UTILITY(s,p): The payoff of player p at terminal state s.
Two-player, Turn-based, Perfect information, Deterministic, Zero-Sum Game

• Two-player: Tic-Tac-Toe, Chess, Go!

• Turn-based: The players take turns in round-robin fashion.

• Perfect information: The State is known to everyone

• Deterministic: Nothing is random

• Zero-sum: The total payoff for all players is a constant.

• The 8-puzzle is a one-player, perfect info, deterministic, zero-sum game.
• How about Rock-Paper-Scissors?
• How about Monopoly?
• How about Starcraft?
Tic-Tac-Toe

• The first player is X and the second is O
• Object of game: get three of your symbol in a horizontal, vertical or diagonal row on a 3x3 game board
• X always goes first
• Players alternate placing Xs and Os on the game board
• Game ends when a player has three in a row (a wins) or all nine squares are filled (a draw)
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What’s the state, action, transition, payoff for Tic-Tac-Toe?
Partial game tree for Tic-Tac-Toe

Start

X’s turn

O’s turn

X’s turn

O’s turn

X’s turn

O’s turn

X’s turn

X’s wins

X’s turns
Game trees

• A game tree is like a search tree in many ways …
  – nodes are search states, with full details about a position
    • characterize the arrangement of game pieces on the game board
  – edges between nodes correspond to moves
  – leaf nodes correspond to a set of goals
    • \{ win, lose, draw \}
    • usually determined by a score for or against player
    – at each node it is one or other player’s turn to move

• A game tree is not like a search tree because you have an opponent!
Two players: MIN and MAX

• In a zero-sum game:
  – payoff to Player 1 = - payoff to Player 2

• The goal of Player 1 is to maximizing her payoff.

• The goal of Player 2 is to maximizing her payoff as well
  – Equivalent to minimizing Player 1’s payoff.
Minimax search

• Assume that both players play perfectly
  – do not assume player will miss good moves or make mistakes

• Score(s): The score that MAX will get towards the end if both player play perfectly from s onwards.

• Consider MIN’s strategy
  – MIN’s best strategy:
    • choose the move that minimizes the score that will result when MAX chooses the maximizing move
  – MAX does the opposite
Minimaxing

- Your opponent will choose smaller numbers
- If you move left, your opponent will choose 3
- If you move right, your opponent will choose -8
- Thus your choices are only 3 or -8
- You should move left
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Each move is called a “ply”. One round is K-plies for a K-player game.
Minimax example

Which move to choose?
Minimax example

Which move to choose?

MAX

MIN
Minimax example

Which move to choose?
Minimax example

Which move to choose?

MAX

MIN

A_{11}  A_{12}  A_{13}

A_{21}  A_{22}  A_{23}

A_{31}  A_{32}  A_{33}

3  12  8

2  4  6

14  5  2
Minimax example

Which move to choose?
Minimax example

Which move to choose?

The *minimax decision* is move $A_1$
Another example

• In the game, it’s your move. Which move will the minimax algorithm choose – A, B, C, or D? What is the minimax value of the root node and nodes A, B, C, and D?
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```
MAX

A

B

C

D

MIN
```

```
1  7  2

2  5  2  8

9  4  6

3  3  5
```
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Minimax search

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• Generate the tree of minimax values
  - Then choose best (maximum) move
  - Don’t need to keep all values around
    • Good memory property
Minimax search

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- Depth-first search is used to implement minimax
  - Expand all the way down to leaf nodes
  - Recursive implementation
Minimax properties

• Optimal?
  Yes, against an optimal opponent, if the tree is finite

• Complete?
  Yes, if the tree is finite

• Time complexity?
  Exponential: $O(b^m)$

• Space complexity?
  Polynomial: $O(bm)$
But this could take forever…

- Exact search is intractable
  - Tic-Tac-Toe is $9! = 362,880$
  - For chess, $b \approx 35$ and $m \approx 100$ for “reasonable” games
  - Go is $361! \approx 10^{750}$
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- Idea 1: Pruning
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- **Idea 1: Pruning**

- **Idea 2: Cut off early and use a heuristic function**
Pruning

• What’s really needed is “smarter,” more efficient search
  – Don’t expand “dead-end” nodes!
• **Pruning** – eliminating a branch of the search tree from consideration
Pruning

• What’s really needed is “smarter,” more efficient search
  – Don’t expand “dead-end” nodes!
• Pruning – eliminating a branch of the search tree from consideration
• Alpha-beta pruning, applied to a minimax tree, returns the same “best” move, while pruning away unnecessary branches
  – Many fewer nodes might be expanded
  – Hence, smaller effective branching factor
  – …and deeper search
  – …and better performance
    • Remember, minimax is *depth-first* search
Alpha pruning
Alpha pruning

Notes:
1. You still have to expand on all B, C, D
2. You may be lucky and expand on only one of C and D's children.
Beta pruning
Beta pruning

The diagram illustrates a tree structure with nodes labeled as MIN and MAX. The tree is pruned using the beta pruning method, where the nodes with values less than or equal to 25 are pruned because they cannot improve the overall value of the node A, which is greater than 25. The pruning process is shown by the dashed lines.
Improvements via alpha/beta pruning
Improvements via alpha/beta pruning

- Depends on the ordering of expansion
Improvements via alpha/beta pruning

• Depends on the ordering of expansion

• Perfect ordering \( O\left(\frac{b^m}{2}\right) \)
Improvements via alpha/beta pruning

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• Perfect ordering $O\left(b^{m/2}\right)$

• Random ordering $O\left(b^{3m/4}\right)$
Improvements via alpha/beta pruning

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• For specific games like Chess, you can get to almost perfect ordering.
Heuristic (Evaluation function)

- It is usually impossible to solve games completely

- Rather, cut the search off early and apply a heuristic evaluation function to the leaves
  - $h(s)$ estimates the expected utility of the game from a given position (node/state) $s$
  - like depth bounded depth first, lose completeness
  - Explore game tree using combination of evaluation function and search
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• The performance of a game-playing program depends on the quality (and speed!) of its evaluation function
Heuristics (Evaluation function)

- Typical evaluation function for game: weighted linear function
  - $h(s) = w_1 f_1(s) + w_2 f_2(s) + ... + w_d f_d(s)$
  - weights $\cdot$ features [dot product]
Heuristics (Evaluation function)

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  \[ h(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_d f_d(s) \]
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- For example, in chess
  - \( W = \{ 1, 3, 3, 5, 8 \} \)
  - \( F = \{ \# \text{ pawns advantage}, \# \text{ bishops advantage}, \# \text{ knights advantage}, \# \text{ rooks advantage}, \# \text{ queens advantage} \} \)
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  – Is this what Deep Blue used?
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  - What are some problems with this?
Heuristics (Evaluation function)

• Typical evaluation function for game: weighted linear function
  – \( h(s) = w_1f_1(s) + w_2f_2(s) + \ldots + w_df_d(s) \)
  – \( \text{weights} \cdot \text{features} \) [dot product]

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  – Is this what Deep Blue used?
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• More complex evaluation functions may involve learning
  – Adjusting weights based on outcomes
  – Perhaps non-linear functions
  – How to choose the features?
Tic-Tac-Toe revisited

A partial game tree for Tic-Tac-Toe
Evaluation function for Tic-Tac-Toe

• A simple evaluation function for Tic-Tac-Toe
  – count the number of rows where \textbf{X} can win
  – subtract the number of rows where \textbf{O} can win

• Value of evaluation function at start of game is zero
  – on an empty game board there are 8 possible winning rows for both \textbf{X} and \textbf{O}

\[
8 - 8 = 0
\]
Evaluating Tic-Tac-Toe

evalX = (number of rows where X can win) – (number of rows where O can win)

• After X moves in center, score for X is +4
• After O moves, score for X is +2
• After X’s next move, score for X is +4

8-8 = 0  8-4 = 4  6-4 = 2  6-2 = 4
Evaluating Tic-Tac-Toe

evalO = (number of rows where O can win) - (number of rows where X can win)

• After X moves in center, score for O is -4
• After O moves, score for O is +2
• After X’s next move, score for O is -4

8-8 = 0  4-8 = -4  4-6 = -2  2-6 = -4
Search depth cutoff

Tic-Tac-Toe with search depth 2

Evaluations shown for X
Expectimax: Playing against a benign opponent

• Sometimes your opponents are not clever.
  – They behave randomly.
  – You can take advantage of that by modeling your opponent.

• Example of game of chance:
  – Slot machines
  – Tetris
Expectimax example

- Your opponent behave randomly with a given probability distribution,
- If you move left, your opponent will select actions with probability $[0.5,0.5]$
- If you move right, your opponent will select actions with $[0.6,0.4]$
Expectimax example

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Note: pruning becomes tricky in expectimax… think about why.
Summary of game playing

- Minimax search
- Game tree
- Alpha-beta pruning
- Early stop with an evaluation function
- Expectimax
More reading / resources about game playing

• Required reading: AIMA 5.1-5.3

• Stochastic game / Expectiminimax: AIMA 5.5
  – Backgammon. TD-Gammon
  – Blackjack, Poker

• Famous game AI: Read AIMA Ch. 5.7 (or in the “Historical notes” of the AIMA 4th Edition)
  – Deep blue
  – TD Gammon

• AlphaGo: https://www.nature.com/articles/nature16961