CS292F Lecture 5 Subgradient

Example (L1-Norm) $p=1$

$$f(x) = \max_{s\in\{-1,1\}^n} s^T x = \max_{s\in\{u \mid \|u\|_\infty \le 1\}} s^T x$$

$$= \text{sign}(x)\cdot x = \|x\|_1$$

$$\partial f(x) = \partial|x_1| + \partial|x_2| + \cdots + \partial|x_n|$$

$$= J_1 \times J_2 \times \cdots \times J_n$$

foreach $k \in \{1,2,\dots n\}$

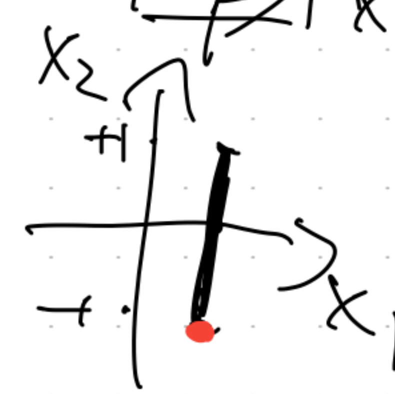
$$J_k = \begin{cases} [-1,1] & \text{if } x_k = 0 \\ 1 & \text{if } x_k > 0 \\ -1 & \text{if } x_k < 0 \end{cases}$$

Take $n=2$

$$\partial f(0,0) = [-1,1]\times[-1,1]$$

$$\partial f(0.5,0) = \{1\}\times[-1,1]$$

$$\partial f(0.1,-0.1) = \{1\}\times\{-1\}$$



Example: $f(x) = \lambda_{max}(A(x))$

$A(x) = A_0 + x_1 \cdot A_1 + \cdots + x_n \cdot A_n$ where

$\in R^{n \times n}$

$A_j \in R^{n \times n}$

$A_j = A_j^T$

$$f(x) = \sup_{\|y\|_2 \leq 1} y^T A(x) y$$

$$g_y(x) = y^T A(x) y = y^T A_0 y + x_1 \cdot y^T A_1 y + \cdots + x_n y^T A_n y$$

$$= \langle c, x \rangle$$

$$\partial f(x) = cl\left(conv\left(\bigcup_{g_y(x) = f(x)} \partial g_y(x)\right)\right)$$

$$f(x) = \max_{y : \|y\|_2 \leq 1} y^T A(x) y$$

y that are eigenvectors corresponding to λ_{max}

① find eigenvectors of $A(x)$, $y \in$ eigenvectors with eigenvalue $\lambda_{max}(A(x))$

② $(y^T A_1 y, y^T A_2 y, \cdots, y^T A_n y)^T = \nabla_x g_y(x) \in \partial f(x)$

③ $\partial f(x) = \text{Convex}\left\{ \bigcup_{y \in \circ} (y^T A_1 y, \cdots y^T A_n y) \right\}$